Online Appendix

Flexibility and Frictions in Multisector Models

By Jorge Miranda-Pinto and Eric R. Young

I. Mathematical Appendix

A. Model's implied regression to estimate elasticities

Let's start by defining $\rho_{Q_j} = \frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}$. To derive the Equation (**??**) we solve the cost minimization problem for firms in sector j, subject to the working capital constraint in the use of value-added and intermediates $\theta_j^v P_j^v V_j + \theta_j^m P_j^M M_j \leq \eta_j P_j Q_j$. The Lagrangian of this problem is (max - (cost))

$$\begin{aligned}\mathcal{L} &= -P_j^v V_j - P_j^M M_j - \lambda_j^1 \left(Q_j - Z_j \left[a_j^{\frac{1}{\epsilon_{Q_j}}} V_j^{\rho_{Q_j}} + (1-a_j)^{\frac{1}{\epsilon_{Q_j}}} M_j^{\rho_{Q_j}} \right]^{\frac{1}{\rho_{Q_j}}} \right) \\ & \quad -\mu_j^C \left(\theta_j^v P_j^v V_j + \theta_j^m P_j^M M_j - \eta_j P_j Q_j \right).\end{aligned}$$

The first-order necessary and sufficient conditions for M_j is

$$-P_j^M + \lambda_j^1 \frac{\partial Q_j}{\partial M_j} + \mu_j^C \eta_j P_j \frac{\partial Q_j}{\partial M_j} - \mu_j^C \theta_j^m P_j^M = 0.$$

Rearranging, using the fact that $\frac{\partial Q_j}{\partial M_j} = Z_j^{\rho_{Q_j}} \left(\frac{a_j Q_j}{M_j} \right)^{\frac{1}{\epsilon_{Q_j}}}$ and that in competitive markets the marginal cost of production in sector j (λ_j^1) is the price of good P_j, we have

$$P_j^M = Z_j^{\rho_{Q_j}} \left(\frac{a_j Q_j}{M_j} \right)^{\frac{1}{\epsilon_{Q_j}}} P_j \overline{\vartheta}_j, \tag{1}$$

where $0 \leq \overline{\vartheta}_j = \frac{1+\mu_j^C \eta_j}{1+\mu_j^C \theta_j^m} \leq 1$ is the wedge that reduces the value of the marginal product of intermediates. Raising the previous equation to the power of ϵ_{Q_j}, taking logs, and rearranging we obtain

$$\log\left(\frac{P_{jt}^M M_{jt}}{P_{jt} Q_{jt}}\right) = \log(a_j) + (1-\epsilon_{Q_j}) \log\left(\frac{P_{jt}^M}{P_{jt}}\right) + (\epsilon_{Q_j} - 1)\log Z_{jt} + \epsilon_{Q_j} \log \overline{\vartheta}_{jt}. \tag{2}$$

Now, we minimize the cost of the intermediate input bundle $\sum_{i=1}^N P_i M_{ij}$ subject to $M_j = \left(\sum_{i=1}^N \omega_{ij}^{\frac{1}{\epsilon_{M_j}}} M_{ij}^{\rho_{M_j}} \right)^{\frac{1}{\rho_{M_j}}}$. The Lagrangian for this problem is

$$\mathcal{L} = -\sum_{i=1}^N P_i M_{ij} - \lambda_j^2 \left(M_j - \left(\sum_{i=1}^N \omega_{ij}^{\frac{1}{\epsilon_{M_j}}} M_{ij}^{\rho_{M_j}} \right)^{\frac{1}{\rho_{M_j}}} \right).$$

Taking first order conditions with respect to M_{ij}, using the fact that in competitive markets $\lambda_j^2 = P_j^M$, and rearranging yields

$$\Delta \log\left(\frac{P_{it} M_{ijt}}{P_{jt}^M M_{jt}}\right) = (1-\epsilon_{M_j}) \Delta \log\left(\frac{P_{it}}{P_{jt}^M}\right). \tag{3}$$

Combining Equations (2) and (3) yields Equation (**??**).

B. Two-sector model solutions

We proceed to find an analytical expression for sector's 2 Lagrange multiplier μ_2. To this end, we need to solve for sectoral prices and input demand, using input optimality conditions, binding working capital constraints, and market clearing conditions.

Assume the wage rate is the numeraire ($w = 1$). From the production function of sector 1 ($Q_1 = Z_1 L_1$) and from the binding constraint in sector 1 ($L_1 = \eta_1 P_1 Q_1$), we obtain

$$P_1 = \frac{1}{\eta_1 Z_1}.$$

Using the market clearing condition for the consumption good ($Q_2 = C$), the market clearing condition for (inelastic) labor ($\bar{L} = L_1 + L_2 = 1$), and the household budget constraint $\bar{L} + \Pi = P_2 C$, we obtain

$$P_2 = \frac{1 + \Pi}{Q_2}.$$

The binding constraint of sector 2 and the market clearing condition for sector 1's goods ($Q_1 = M_{12}$) imply

$$\theta_2^w L_2 + \theta_{12}^m P_1 Q_1 = \eta_2 P_2 Q_2,$$

$$\theta_2^w L_2 + \theta_{12}^m \frac{1 - L_2}{\eta_1} = \eta_2 (1 + \Pi),$$

and that

$$L_2 = \frac{\eta_1 \eta_2 (1 + \Pi) - \theta_{12}^m}{\eta_1 \theta_2^w - \theta_{12}^m} = \frac{\eta_1 \eta_2 (1 + \Pi) - \theta_{12}^m}{\phi_1},$$

implying

$$L_1 = 1 - \left(\frac{\eta_1 \eta_2 (1 + \Pi) - \theta_{12}^m}{\eta_1 \theta_2^w - \theta_{12}^m}\right) = \frac{\eta_1 \big(\theta_2^w - \eta_2 (1 + \Pi)\big)}{\eta_1 \theta_2^w - \theta_{12}^m} = \frac{\eta_1 \big(\theta_2^w - \eta_2 (1 + \Pi)\big)}{\phi_1},$$

in which $\phi_1 = \eta_1 \theta_2^w - \theta_{12}^m$. We solve for profit and the Lagrange multiplier below.

Having solved for L_1, L_2 we obtain

$$Q_1 = M_{12} = Z_1 L_1$$

and

$$Q_2 = Z_2 \big(a^{1-\rho_Q} L_2^{\rho_Q} + (1-a)^{1-\rho_Q} M_{12}^{\rho_Q} \big)^{\frac{1}{\rho_Q}},$$

where $\rho_Q = (\epsilon_Q - 1)/\epsilon_Q$. Finally, using first order and necessary condition (FONC) in the use of labor or intermediates for firms in sector 2:

$$P_2 Z_2^{\rho_Q} \big(\frac{a Q_2}{L_2} \big)^{1-\rho_Q} - \frac{(1 + \mu_2 \theta_2^w)}{(1 + \mu_2 \eta_2)} = 0,$$

$$P_2 Z_2^{\rho_Q} \big(\frac{(1-a) Q_2}{M_{12}} \big)^{1-\rho_Q} - P_1 \frac{(1 + \mu_2 \theta_{12}^m)}{(1 + \mu_2 \eta_2)} = 0,$$

we can solve for μ_2.

PROOF OF PROPOSITION 1:

Constraint on intermediates: set $\theta_2^w = 0$ and $\theta_{12}^m = 1$, which implies $L_2 = 1 - \eta_1 \eta_2 (1 + \Pi)$ and $Q_1 = Z_1 \eta_1 \eta_2 (1 + \Pi)$. From the FONC for L_2, and from the fact that $P_2 = \frac{1+\Pi}{Q_2}$, we obtain

$$\big(\frac{Q_2}{Z_2} \big)^{\rho_Q} = (1 + \mu_2 \eta_2) \big(\frac{a_2}{L_2} \big)^{1-\rho_Q} (1 + \Pi).$$

Similarly, using the production function for sector 2 we obtain

$$\big(\frac{Q_2}{Z_2} \big)^{\rho_Q} = a_2^{1-\rho_Q} L_2^{\rho_Q} + (1 - a_2)^{1-\rho_Q} Q_1^{\rho_Q},$$

implying

$$(1+\mu_2\eta_2)\big(\frac{a_2}{L_2}\big)^{1-\rho_Q}(1+\Pi) = a_2^{1-\rho_Q}L_2^{\rho_Q} + (1-a_2)^{1-\rho_Q}Q_1^{\rho_Q},$$

$$(1+\mu_2\eta_2)\big(\frac{a_2}{(1-\eta_1\eta_2(1+\Pi))}\big)^{1-\rho_Q}(1+\Pi) = a_2^{1-\rho_Q}(1-\eta_1\eta_2(1+\Pi))^{\rho_Q}+(1-a_2)^{1-\rho_Q}(Z_1\eta_1\eta_2(1+\Pi))^{\rho_Q},$$

and

$$\mu_2 = \left(\frac{(1-\eta_1\eta_2(1+\Pi))\,(1-a_2)}{a_2\eta_2(1+\Pi)}\right)^{1-\rho_Q}(\eta_1 Z_1)^{\rho_Q} + \frac{1}{\eta_2(1+\Pi)} - \eta_1 - \frac{1}{\eta_2}.$$

To solve for profits Π we divide the FONCs for L_2 with the FONCs for M_{12}

$$\mu_2 = \left(\frac{(1-\eta_1\eta_2(1+\Pi))\,(1-a_2)}{a_2\eta_2(1+\Pi)}\right)^{1-\rho_Q}(\eta_1 Z_1)^{\rho_Q} - 1,$$

$$\Pi = \frac{(1-\eta_1)\eta_2}{1-(1-\eta_1)\eta_2} = \bar{\eta},$$

implying

$$\mu_2 = \left(\frac{(1-\eta_2)\,(1-a_2)}{Z_1\eta_1\eta_2 a_2}\right)^{1-\rho_Q}\eta_1 Z_1 - 1,$$

Therefore,

$$\frac{\partial\mu_2}{\partial\epsilon_Q} = -\frac{1}{\epsilon_Q^2}(\eta_1 Z_1)^{\rho_Q}\phi_m^{1-\rho_Q}\ln\phi_m$$

where $\phi_m = \frac{(1-\eta_2)(1-a_2)}{Z_1\eta_1\eta_2 a_2}$. If $\phi_m > 1$ the derivative is negative, otherwise it is positive. From the binding constraint we have that

$$\mu_2 = (\phi_m)^{1-\rho_Q}\,\eta_1 Z_1 - 1 > 0,$$

implying that $\phi_m > \frac{1}{(\eta_1 Z_1)^{\epsilon_Q}}$. Hence, evaluated at $Z_1 = 1$ (steady state productivity value), it is always the case that, as long as firms in sector 1 and sector

2 are constrained ($\eta_1 < 1$ and $\mu_2 > 0$), $\phi_m > 1$. Therefore, more flexible firms are less constrained $\frac{\partial \mu_2}{\partial \epsilon_Q} < 0$. The premium for production flexibility is larger when ϕ_m is larger (due to lower collateral constraint parameters η_1, η_2 , or lower productivity Z_1, or larger intermediate input share $(1 - a_2)$)

$$\frac{\partial \mu_2}{\partial \phi_m} = \frac{1}{\epsilon_Q} \phi_m^{-\rho_Q} \eta_1 Z_1 > 0,$$

PROOF OF PROPOSITION 2:

Her we study how the Lagrange multiplier μ_2 changes with financial shocks to sector 1 and 2, and then how the elasticity affects the change in the Lagrange multiplier. Following from Proposition 1, we have

$$\frac{\partial \mu_2}{\partial \eta_2} = (1 - \rho_Q) \phi_m^{-\rho_Q} \eta_1 Z_1 \frac{\partial \phi_m}{\partial \eta_2}$$

$$\frac{\partial \mu_2}{\partial \eta_2} = (1 - \rho_Q) \phi_m^{-\rho_Q} \eta_1 Z_1 \frac{(a_2 - 1)}{Z_1 \eta_1 \eta_2^2 a_2} = \frac{1}{\epsilon_Q} \phi_m^{-\rho_Q} \frac{(a_2 - 1)}{\eta_2^2 a_2} < 0.$$

We then have that

$$\frac{\partial (\partial \mu_2 / \partial \eta_2)}{\partial \epsilon_Q} = \frac{\phi_m^{-\rho_Q} (1 - a_2)}{\epsilon_Q^2 a_2 \eta_2^2} \big(1 + \frac{1}{\epsilon_Q} \ln \phi_m\big),$$

which is positive as long as $1 + \frac{1}{\epsilon_Q} \ln \phi_m > 0$. As long as $\phi_m > 1$, the condition for $\partial \mu_2 / \partial \epsilon_Q < 0$, it then holds that $\frac{\partial (\partial \mu_2 / \partial \eta_2)}{\partial \epsilon_Q} > 0$, which implies that a more flexible sector displays smaller increases in μ_2 due to tightening credit constraints.

We now study how the Lagrange multiplier changes with a financial shock to sector 1

$$\frac{\partial \mu_2}{\partial \eta_1} = \phi_m^{1-\rho_Q} Z_1 \frac{\epsilon_Q - 1}{\epsilon_Q},$$

which implies that declines in η_1 increase (decrease) the shadow cost of working capital when $\epsilon_Q < 1$ ($\epsilon_Q > 1$). Note that for Cobb-Douglas technologies, tightening credit conditions for sector 1 have no effect on sector 2's shadow cost of debt. If $\frac{\partial(\partial\mu_2/\partial\eta_1)}{\partial\epsilon_Q} > 0$, more flexible firms would experience a larger decline or a smaller increase in the Lagrange multiplier followed by a credit tightening in sector 1. We have that

$$\frac{\partial(\partial\mu_2/\partial\eta_1)}{\partial\epsilon_Q} = \frac{\phi_m^{1-\rho_Q} Z_1}{\epsilon_Q^2}\big(1 - \frac{(\epsilon_Q-1)}{\epsilon_Q}\ln\phi_m\big),$$

which is positive as long as $\big(1 - \frac{(\epsilon_Q-1)}{\epsilon_Q}\ln\phi_m > 0\big)$. When labor and intermediates are substitutes, $\frac{\partial(\partial\mu_2/\partial\eta_1)}{\partial\epsilon_Q} > 0$ is positive.

C. Constraint on labor

PROOF:

Set $\theta_2^w = 1$ and $\theta_{12}^m = 0$, which implies $L_2 = \eta_2(1{+}\Pi)$ and $Q_1 = Z_1\left(1 - \eta_2(1+\Pi)\right)$. From the FONC for M_{12}, and from the fact that $P_2 = \frac{1+\Pi}{Q_2}$ and $P_1 = \frac{1}{Z_1\eta_1}$, we obtain

$$(\frac{Q_2}{Z_2})^{\rho_Q} = Z_1\eta_1(1+\mu_2\eta_2)(\frac{(1-a_2)}{M_{12}})^{1-\rho_Q}(1+\Pi).$$

Again using the production function we obtain

$$(\frac{Q_2}{Z_2})^{\rho_Q} = a_2^{1-\rho_Q}L_2^{\rho_Q} + (1-a_2)^{1-\rho_Q}M_{12}^{\rho_Q},$$

which implies

$$(1+\mu_2\eta_2)(\frac{(1-a_2)}{M_{12}})^{1-\rho_Q}Z_1\eta_1(1+\Pi) = a_2^{1-\rho_Q}L_2^{\rho_Q} + (1-a_2)^{1-\rho_Q}M_{12}^{\rho_Q},$$

$$(1{+}\mu_2\eta_2)(\frac{(1-a_2)}{Z_1\big(1-\eta_2(1+\Pi)\big)})^{1-\rho_Q}Z_1\eta_1(1{+}\Pi) = a_2^{1-\rho_Q}\big(\eta_2(1{+}\Pi)\big)^{\rho_Q}{+}(1{-}a_2)^{1-\rho_Q}Z_1^{\rho_Q}\big(1{-}\eta_2(1{+}\Pi)\big)^{\rho_Q},$$

and

$$\mu_2 = \frac{1}{Z_1\eta_1}\left(\frac{\big(1-\eta_2(1+\Pi)\big)a_2Z_1}{(1-a_2)\,\eta_2(1+\Pi)}\right)^{1-\rho_Q} + \frac{\big(1-(1+\Pi)(\eta_1+\eta_2)\big)}{\eta_1\eta_2(1+\Pi)}.$$

To solve for profits Π we divide the FONCs for L_2 with the FONCs for M_{12}

$$\mu_2 = \frac{1}{Z_1\eta_1}\left(\frac{\big(1-\eta_2(1+\Pi)\big)a_2Z_1}{(1-a_2)\,\eta_2(1+\Pi)}\right)^{1-\rho_Q} - 1,$$

implying

$$\Pi = \frac{1}{\eta_1+\eta_2-\eta_1\eta_2}$$

and

$$\mu_2 = \frac{1}{Z_1\eta_1}\left(\frac{\eta_1(1-\eta_2)a_2Z_1}{(1-a_2)\eta_2}\right)^{1-\rho_Q} - 1.$$

Therefore,

$$\frac{\partial\mu_2}{\partial\epsilon_Q} = -\frac{1}{\epsilon_Q^2}\frac{1}{Z_1\eta_1}\phi_w^{1-\rho_Q}\ln\left(\phi_w\right),$$

where $\phi_w = \frac{\eta_1(1-\eta_2)a_2Z_1}{(1-a_2)\eta_2}$. If $\phi_w > 1$ the derivative is negative, otherwise it is positive. For the constraint to be binding, we require $\mu_2 > 0$, implying

$$\phi_w > (Z_1\eta_1)^{\epsilon_Q}.$$

Therefore, only for high values of Z_1 and η_1, the model can replicate the negative relationship between elasticities and the shadow cost of debt.

Let us see how sector 1's constraint affects sector 2's wedge, when sector 2's constraint tightens. We have that

$$\frac{\partial\phi_w}{\partial\eta_2} = -\frac{\eta_1Z_1a_2(1-a_2)}{\big((1-a_2)\eta_2\big)^2}$$

a tightening of sector 2's constraint raises the cost of labor (constrained input). On the other hand, we have that

$$\frac{\partial(\partial\phi_w/\partial\eta_2)}{\partial\eta_1} < 0$$

implying that a tighter constraint in sector 1 mitigates the increase in ϕ_w due to a tightening in η_2 (it makes $\frac{\partial \phi_w}{\partial \eta_2}$ less negative).

PROOF OF PROPOSITION 3:

Let us define $\rho_{Q_j} = \frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}$ and assume $\epsilon_{M_j} = \epsilon_{Q_j}$ for all j. To obtain real GDP in this economy, use the cost minimizing problem

$$Min \quad \sum_{j=1}^{N} P_j C_j,$$

subject to

$$C = \prod_{j=1}^{N} C_j^{\beta_j},$$

which yields

$$P_j C_j = \beta_j \sum_{j=1}^{N} P_j C_j.$$

Combining the previous condition with the household budget constraint

$$\sum_{j=1}^{N} P_j C_j = WL + \Pi,$$

gives

$$P_j C_j = \beta_j (WL + \Pi),$$

and the fact that labor is inelastically supplied $L = 1$ and the wage rate is the numeraire

$$C_j = \frac{\beta_j (1 + \Pi)}{P_j}$$

$$C = \prod_{j=1}^{N} C_j^{\beta_j} = \prod_{j=1}^{N} \big(\frac{\beta_j(1+\Pi)}{P_j}\big)^{\beta_j}$$

$$\log C = \sum_{j=1}^{N} \beta_j \log\big(\frac{\beta_j(1+\Pi)}{P_j}\big)$$

$$\log C = \sum_{j=1}^{N} \beta_j \log\big(\frac{\beta_j}{P_j}\big) + \sum_{j=1}^{N} \beta_j \log(1+\Pi),$$

using the fact that $\sum_{j=1}^{N} \beta_j = 1$ we have that real GDP in this economy is

$$\log C = \sum_{j=1}^{N} \beta_j \log\big(\frac{\beta_j}{P_j}\big) + \log(1+\Pi).$$

We need to solve for sectoral prices. We first modify the production function

$$Z_j^{-\rho_{Q_j}} = a_j^{1-\rho_{Q_j}} \big(\frac{L_j}{Q_j}\big)^{\rho_{Q_j}} + (1-a_j)^{1-\rho_{Q_j}} \big(\frac{M_j}{Q_j}\big)^{\rho_{Q_j}},$$

define wedges as follows

$$\vartheta_j^m = \frac{(1+\mu_j\eta_j)}{(1+\mu_j\theta_j^m)},$$
$$\vartheta_j^w = \frac{(1+\mu_j\eta_j)}{(1+\mu_j\theta_j^w)},$$

and use the first order conditions for labor and intermediates

$$P_j Z_j^{\rho_{Q_j}} \big(\frac{a_j Q_j}{L_j}\big)^{1-\rho_{Q_j}} = \frac{(1+\mu_j\theta_j^w)}{(1+\mu_j\eta_j)} = (\vartheta_j^w)^{-1},$$

$$P_j Z_j^{\rho_{Q_j}} \big(\frac{(1-a_j) Q_j}{M_j}\big)^{1-\rho_{Q_j}} = P_j^M \frac{(1+\mu_j\theta_j^m)}{(1+\mu_j\eta_j)} = P_j^M (\vartheta_j^m)^{-1}.$$

This definition of wedge implies that a decline in η_j decreases the wedge ϑ_j. A decline in η_j increases μ_j. Therefore, the denominator increases more than the numerator. A decline in η_j corresponds to tighter credit, which is isomorphic to an increase in sectoral spreads (or EBP to be more precise). Thus, increases in sectoral spread decrease ϑ_j.

To solve for real GDP we first need to solve for sectoral prices. We use sectoral first order conditions

$$(\frac{L_j}{Q_j})^{\rho_{Q_j}} = P_j^{\epsilon_{Q_j}-1} Z_j^{\frac{(\epsilon_{Q_j}-1)^2}{\epsilon_{Q_j}}} a_j^{\frac{(\epsilon_{Q_j}-1)}{\epsilon_{Q_j}}} (\vartheta_j^w)^{\epsilon_{Q_j}-1},$$

$$(\frac{M_j}{Q_j})^{\rho_{Q_j}} = (\frac{P_j}{P_j^M})^{\epsilon_{Q_j}-1} Z_j^{\frac{(\epsilon_{Q_j}-1)^2}{\epsilon_{Q_j}}} (1-a_j)^{\frac{(\epsilon_{Q_j}-1)}{\epsilon_{Q_j}}} (\vartheta_j^m)^{\epsilon_{Q_j}-1},$$

implying (now allowing for heterogeneous elasticities)

$$P_j^{1-\epsilon_{Q_j}} = a_j Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^w)^{\epsilon_{Q_j}-1} + (1-a_j) Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^m)^{\epsilon_{Q_j}-1} (P_j^M)^{1-\epsilon_{Q_j}},$$

$$P_j^{1-\epsilon_{Q_j}} = a_j Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^w)^{\epsilon_{Q_j}-1} + (1-a_j) Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^m)^{\epsilon_{Q_j}-1} \Big(\sum_{i=1}^N \omega_{ij} P_i^{1-\epsilon_{M_j}}\Big)^{\frac{1-\epsilon_{Q_j}}{1-\epsilon_{M_j}}}.$$

Now assume that $\epsilon_{Q_j} = \epsilon_{M_j}$ for all j implies

$$P_j^{1-\epsilon_{Q_j}} = a_j Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^w)^{\epsilon_{Q_j}-1} + (1-a_j) Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^m)^{\epsilon_{Q_j}-1} \sum_{i=1}^N \omega_{ij} P_i^{1-\epsilon_{Q_j}},$$

and in matrix form

$$P^{1-\epsilon_Q} = a \circ (Z \circ \vartheta^w)^{\circ \epsilon_Q - 1} + \big((1-a) \circ (Z \circ \vartheta^m)^{\circ \epsilon_Q - 1} 1'\big) \circ (\Omega \circ (P1')^{\circ((1-\epsilon_{Q_j})1')'})'1.$$

Note here that the term $\sum_{i=1}^N \omega_{ij} P_i^{1-\epsilon_{Q_j}}$ has all sectoral prices and intermediates shares, from i to N, raised to the power of sector's j elasticity. With common elasticity expressing these terms in matrix form is trivial: $\Omega' P^{1-\epsilon_Q}$. Nevertheless, the matrix form with heterogeneous elasticities is $\Omega \circ (P1')^{\circ((1-\epsilon_{Q_j})1')'})'1$.

We now solve for sectoral sale shares. We multiply sectoral market clearing condition for sector j by sectoral price P_j we obtain

$$S_j = P_j C_j + \sum_{i=1}^{N} P_j M_{ji},$$

where S_j is sectoral sales. Let's use the household optimal consumption share for each good (with $\epsilon_D = 1$ we have $P_j C_j = \beta_j P_c C$) and rearrange the firm optimality condition for M_{ji}

$$P_j M_{ji}^{1-\rho_{Q_i}} = \vartheta_i^m Z_i^{\rho_{Q_i}} \big((1-a_i)\omega_{ji}\big)^{1-\rho_{Q_i}} M_i^{\rho_{Q_i}-\rho_{M_i}} P_i Q_i^{1-\rho_{Q_i}},$$

which combined with the FONC for M_i

$$M_i = (\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1} \frac{P_i^{\epsilon_{Q_i}}}{(P_i^M)^{\epsilon_{Q_i}}} (1-a_i) Q_i,$$

yields

$$P_j M_{ji}^{1-\rho_{Q_i}} = \vartheta_i^m Z_i^{\rho_{Q_i}} \big((1-a_i)\omega_{ji}\big)^{1-\rho_{Q_i}} \Big((\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1} \frac{P_i^{\epsilon_{Q_i}}}{(P_i^M)^{\epsilon_{Q_i}}} (1-a_i) Q_i\Big)^{\rho_{Q_i}-\rho_{M_i}} P_i Q_i^{1-\rho_{Q_i}},$$

Note that unlike the case $\epsilon_{Q_j} = \epsilon_{M_j}$, when $\epsilon_{Q_j} \neq \epsilon_{M_j}$ there is no linear closed-form solution for sales shares (given prices).

Assuming that $\epsilon_{Q_j} = \epsilon_{M_j}$

$$P_j M_{ji}^{1-\rho_{Q_i}} = \vartheta_i^m Z_i^{\rho_{Q_i}} \big((1-a_i)\omega_{ji}\big)^{1-\rho_{Q_i}} P_i Q_i^{1-\rho_{Q_i}},$$

$$P_j M_{ji} = \big(\frac{P_i}{P_j}\big)^{\epsilon_{Q_i}-1} (\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1} (1-a_i)\omega_{ji} P_i Q_i,$$

to get

$$S_j = \beta_j P_c C + \sum_{i=1}^{N} P_j^{1-\epsilon_{Q_i}} P_i^{\epsilon_{Q_i}-1} (\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1} (1-a_i)\omega_{ji} S_i.$$

$$\frac{S_j}{P_c C} = \beta_j + \sum_{i=1}^{N} P_j^{1-\epsilon_{Q_i}} P_i^{\epsilon_{Q_i}-1} (\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1} (1-a_i)\omega_{ji} \frac{S_i}{P_c C},$$

$$s = [I - \left((P1')^{\circ((1-\epsilon_Q)1')'}\right) \circ \left((\vartheta^m)^{\circ\epsilon_Q} \circ (Z \circ P)^{\circ(\epsilon_Q - 1)} 1'\right)' \circ ((1-a)1')' \circ \Omega]^{-1}\beta,$$

in which $s = \frac{S_j}{P_c C} = \frac{S_j}{1+\Pi}$. Note that with common elasticity the matrix form solution simplifies to

$$s = [I - (P^{\circ(1-\epsilon_Q)}1') \circ \left((\vartheta^m)^{\circ\epsilon_Q} \circ (Z \circ P)^{\circ(\epsilon_Q-1)} 1'\right)' \circ ((1-a)1')' \circ \Omega]^{-1}\beta,$$

Having solved for prices and sales shares we can solve for profits. Combining the firms FONCs for input we have

$$P_j Z_j^{\rho_{Q_j}} \left(\frac{(1-a_j)Q_j}{M_j}\right)^{1-\rho_{Q_j}} = \frac{P_j^M}{\vartheta_j^m},$$

$$M_j^{1-\rho_{Q_j}} = \vartheta_j^m \frac{P_j}{P_j^M} Z_j^{\rho_{Q_j}} \left((1-a_j)Q_j\right)^{1-\rho_{Q_j}},$$

$$M_j^{1-\rho_{Q_j}} = \vartheta_j^m \frac{P_j^{\rho_{Q_j}}}{P_j^M} Z_j^{\rho_{Q_j}} \left((1-a_j)\right)^{1-\rho_{Q_j}} (P_j Q_j)^{1-\rho_{Q_j}},$$

$$M_j = (\vartheta_j^m)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} \frac{P_j^{\epsilon_{Q_j}-1}}{(P_j^M)^{\epsilon_{Q_j}}} (1-a_j) \frac{P_j Q_j}{P_c C} P_c C,$$

where $P_c C = 1 + \Pi$ and $s_j = \frac{P_j Q_j}{P_c C}$, implying

$$M_j = (\vartheta_j^m)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} \frac{P_j^{\epsilon_{Q_j}-1}}{(P_j^M)^{\epsilon_{Q_j}}} (1-a_j) s_j (1+\Pi),$$

which combined with the ratio between the labor and intermediates first order condition

$$L_j = \left(\frac{P_j^M \vartheta_j^w}{\vartheta_j^m}\right)^{\epsilon_{Q_j}} \frac{a_j M_j}{(1-a_j)},$$

yields

$$L_j = \left(\frac{P_j^M \vartheta_j^w}{\vartheta_j^m}\right)^{\epsilon_{Q_j}} \frac{a_j}{(1-a_j)} (\vartheta_j^m)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} \frac{P_j^{\epsilon_{Q_j}-1}}{(P_j^M)^{\epsilon_{Q_j}}} (1-a_j) s_j (1+\Pi).$$

$$L_j = (\vartheta_j^w)^{\epsilon_{Q_j}} a_j Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j (1+\Pi).$$

We then use the labor market clearing condition, the solution for prices, and the solution for sale shares, to solve for profits

$$(1+\Pi) \sum_{j=1}^{N} a_j (\vartheta_j^w)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j = 1.$$

$$(1+\Pi) = \frac{1}{\sum_{j=1}^{N} a_j (\vartheta_j^w)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j}.$$

Solution two-sector model with heterogeneous CES

In the Island economy (suppose $Z_j = 1$ for all j and $a_j = a$ for all j), the solution for prices, sales shares, and profits is

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2-1} \end{bmatrix} \circ \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \circ \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \right)' \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2-1} \end{bmatrix} \circ \begin{bmatrix} P_1^{1-\epsilon_1} & 0 \\ 0 & P_2^{1-\epsilon_2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} P_1^{1-\epsilon_1} & 0 \\ 0 & \vartheta_2^{\epsilon_2-1} P_2^{1-\epsilon_2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} P_1^{1-\epsilon_1} \\ \vartheta_2^{\epsilon_2-1} P_2^{1-\epsilon_2} \end{bmatrix},$$

implying

$$\begin{aligned} P_1^{1-\epsilon_{Q_1}} &= \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}, \\ P_2^{1-\epsilon_{Q_2}} &= \frac{a}{\vartheta_2^{1-\epsilon_{Q_2}} - (1-a)}. \end{aligned}$$

To obtain sales, we have

$$= [I - \left(s(P1')^{\circ((1-\epsilon_Q)1')'}\right) \circ \left((\vartheta^m)^{\epsilon_Q} \circ (Z \circ P)^{\epsilon_Q - 1} 1'\right)' \circ ((1-a)1')' \circ \Omega]^{-1} \beta,$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \circ \begin{bmatrix} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \end{bmatrix}' \begin{bmatrix} 1-a & 0 \\ 0 & 1-a \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_1} \end{bmatrix} \circ \begin{bmatrix} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \\ \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \end{bmatrix} \begin{bmatrix} 1-a & 0 \\ 0 & 1-a \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} (1-a)P_1^{1-\epsilon_1}\vartheta_1^{\epsilon_1}P_1^{\epsilon_1-1} & 0 \\ 0 & (1-a)P_2^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1-(1-a)\vartheta_1^{\epsilon_1} & 0 \\ 0 & 1-(1-a)\vartheta_2^{\epsilon_2} \end{bmatrix}^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{(1-(1-a)\vartheta_1^{\epsilon_1})(1-(1-a)\vartheta_2^{\epsilon_2})} \begin{bmatrix} 1-(1-a)\vartheta_2^{\epsilon_2} & 0 \\ 0 & 1-(1-a)\vartheta_1^{\epsilon_1} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{(1-(1-a)\vartheta_1^{\epsilon_1})(1-(1-a)\vartheta_2^{\epsilon_2})} \begin{bmatrix} \beta_1(1-(1-a)\vartheta_2^{\epsilon_2}) \\ \beta_2(1-(1-a)\vartheta_1^{\epsilon_1}) \end{bmatrix},$$

which yields

$$\begin{aligned} s_1 =& \frac{\beta_1}{1-(1-a)\vartheta_1^{\epsilon_1}}, \\ s_2 =& \frac{\beta_2}{1-(1-a)\vartheta_2^{\epsilon_2}}. \end{aligned}$$

In the Star Supplier Economy (suppose $Z_j = 1$ for all j and $a_j = a$ for all j), the solution for prices, sales shares, and profits is

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2-1} \end{bmatrix} \circ \Big(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \circ \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \Big)' \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2-1} \end{bmatrix} \circ \begin{bmatrix} P_1^{1-\epsilon_1} & 0 \\ P_1^{1-\epsilon_2} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} P_1^{1-\epsilon_1} & 0 \\ \vartheta_2^{\epsilon_2-1} P_1^{1-\epsilon_2} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} P_1^{1-\epsilon_1} \\ \vartheta_2^{\epsilon_2-1} P_1^{1-\epsilon_2} \end{bmatrix},$$

implying

$$\begin{aligned} P_1^{1-\epsilon_{Q_1}} =& \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}, \\ P_2^{1-\epsilon_{Q_2}} =& a\vartheta_2^{\epsilon_{Q_2}-1} + (1-a)\vartheta_2^{\epsilon_{Q_2}-1}\Big(\frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}\Big)^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}} \end{aligned}$$

To obtain sales, we have

$$s = [I - \big((P1')^{\circ((1-\epsilon_Q)1')'}\big)\big) \circ \big((\vartheta^m)^{\epsilon_Q} \circ (Z \circ P)^{\epsilon_Q - 1} 1'\big)' \circ ((1-a)1')' \circ \Omega]^{-1}\beta,$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \circ \begin{bmatrix} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \end{bmatrix}' \begin{bmatrix} 1-a & 1-a \\ 0 & 0 \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \circ \begin{bmatrix} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \\ \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \end{bmatrix} \begin{bmatrix} 1-a & 1-a \\ 0 & 0 \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} (1-a)P_1^{1-\epsilon_1}\vartheta_1^{\epsilon_1}P_1^{\epsilon_1-1} & (1-a)P_1^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \\ 0 & 0 \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1-(1-a)\vartheta_1^{\epsilon_1} & -(1-a)P_1^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{1-(1-a)\vartheta_1^{\epsilon_1}} \begin{bmatrix} 1 & (1-a)P_1^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \\ 0 & 1-(1-a)\vartheta_1^{\epsilon_1} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{1-(1-a)\vartheta_1^{\epsilon_1}} \begin{bmatrix} \beta_1 + \beta_2(1-a)P_1^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \\ \beta_2(1-(1-a)\vartheta_1^{\epsilon_1}) \end{bmatrix},$$

which yields the following solutions for the Star supplier economy

$$\begin{aligned}
(P_1^S)^{1-\epsilon_{Q_1}} &= \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}, \\
(P_2^S)^{1-\epsilon_{Q_2}} &= a\vartheta_2^{\epsilon_{Q_2}-1} + (1-a)\vartheta_2^{\epsilon_{Q_2}-1}\Big(\frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}\Big)^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}, \\
s_1^S &= \frac{\beta}{1-(1-a)\vartheta_1^{\epsilon_1}} + \frac{\beta_2(P_1^S)^{1-\epsilon_2}(P_2^S)^{\epsilon_2-1}\vartheta_2^{\epsilon_2}(1-a)}{1-(1-a)\vartheta_1^{\epsilon_1}} \\
s_2^S &= 1-\beta,
\end{aligned}$$

and the following solutions for the Island economy

$$
\begin{aligned}
P_1^{1-\epsilon_{Q_1}} =& \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}, \\
(P_2^I)^{1-\epsilon_{Q_2}} =& \frac{a}{\vartheta_2^{1-\epsilon_{Q_2}} - (1-a)}, \\
s_1^I =& \frac{\beta}{1-(1-a)\vartheta_1^{\epsilon_1}} \\
s_2^I =& \frac{1-\beta}{1-(1-a)\vartheta_2^{\epsilon_2}}
\end{aligned}
$$

D. *Sectoral shock: heterogeneous elasticities*

PROOF PROPOSITION 4:

We start by defining the input-output multiplier (IOM) as

$$
IOM = \frac{\partial \log C^S}{\partial \vartheta_1} - \frac{\partial \log C^I}{\partial \vartheta_1},
$$

in which C^S and C^I stand for real GDP in the Star supplier and Island economies, respectively.

From the definition of real GDP, we have

$$
\frac{\partial \log C^I}{\partial \vartheta_1} = \underbrace{-\beta \frac{\partial \log P_1}{\partial \vartheta_1}}_{\text{Real wage channel}} \underbrace{-(1+\Pi^I)a\Big[\epsilon_1 s_1^I \vartheta_1^{\epsilon_1 - 1} P_1^{\epsilon_1 - 1} + s_1^I \vartheta_1^{\epsilon_1} \frac{\partial P_1^{\epsilon_1-1}}{\partial \vartheta_1} + \vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1} \frac{\partial s_1^I}{\partial \vartheta_1}\Big]}_{\text{Rents channel}}
$$

$$
\frac{\partial \log C^S}{\partial \vartheta_1} = \underbrace{-\beta \frac{\partial \log P_1}{\partial \vartheta_1} - (1-\beta)\frac{\partial \log P_2^S}{\partial \vartheta_1}}_{\text{Real wage channel}} \underbrace{-(1+\Pi^S)a\Big[\epsilon_1 s_1^S \vartheta_1^{\epsilon_1 - 1} P_1^{\epsilon_1 - 1} + s_1^S \vartheta_1^{\epsilon_1} \frac{\partial P_1^{\epsilon_1-1}}{\partial \vartheta_1} + \vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1} \frac{\partial s_1^S}{\partial \vartheta_1} + s_2^S \vartheta_2^{\epsilon_2} \frac{\partial (P_2^S)^{\epsilon_2 - 1}}{\partial \vartheta_1}\Big]}_{\text{Rents channel}},
$$

where we differentiate the effects of distortions on the real wage and on the rents rebated to the household. Using the fact that $\frac{\partial \log P_2}{\vartheta_1} = \frac{1}{P_2}\frac{\partial P_2}{\vartheta_1}$, that $\frac{\partial P_2^{\epsilon_2 - 1}}{\vartheta_1} = (\epsilon_2 - 1)P_2^{\epsilon_2 - 2}\frac{\partial P_2}{\vartheta_1}$, and that $s_2^S = 1-\beta$, we reorganize the IOM as follows

$$IOM = \underbrace{-\frac{\partial \log P_2^S}{\partial \vartheta_1}(1-\beta)\Big(1 + a\vartheta_2^{\epsilon_2}(P_2^S)^{\epsilon_2-1}(1+\Pi^S)(\epsilon_2-1)\Big)}_{\text{Term 1}}$$
$$\underbrace{-a\epsilon_1\vartheta_1^{\epsilon_1-1}P_1^{\epsilon_1-1}\big[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\big]}_{\text{Term 2}}$$
$$\underbrace{-a\vartheta_1^{\epsilon_1}(\epsilon_1-1)P_1^{\epsilon_1-2}\frac{\partial P_1}{\partial \vartheta_1}\big[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\big]}_{\text{Term 3}}$$
$$\underbrace{-a\vartheta_1^{\epsilon_1}P_1^{\epsilon_1-1}\big[(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta_1} - (1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big]}_{\text{Term 4}}.$$

We first analyze Term 1

$$\text{Term 1} = -\frac{\partial \log P_2^S}{\partial \vartheta_1}(1-\beta)\Big(1 + a\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1}(1+\Pi^S)(\epsilon_2-1)\Big),$$

$$\text{Term 1} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}(1-\beta)\Big(1+a\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1}(1+\Pi^S)(\epsilon_2-1)\Big),$$

where $\phi = \frac{a}{\vartheta_1^{1-\epsilon_1}-(1-a)}$ (> 0 so prices are positive) and

$$(P_2^S)^{1-\epsilon_2} = \vartheta_2^{\epsilon_2-1}\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)$$

$$(P_2^S)^{\epsilon_2-1} = \vartheta_2^{1-\epsilon_2}\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)^{-1},$$

implying

$$\text{Term 1} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}(1-\beta)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}\Bigg[1+\frac{a\vartheta_2(1+\Pi^S)(\epsilon_2-1)}{\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)}\Bigg],$$

$$\text{Term 1} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}(1-\beta)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2} + \frac{(1-a)a\vartheta_1^{-\epsilon_1}(1-\beta)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}a\vartheta_2(1+\Pi^S)(\epsilon_2-1)}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)^2\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}$$

$$\text{Term 1} = \frac{\psi_1^{t1}(\epsilon_1)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}} + \frac{\psi_2^{t1}(\epsilon_1)(\epsilon_2-1)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)^2},$$

$$\text{Term 1} = \psi_1^{t1}(\epsilon_1,\epsilon_2) + (\epsilon_2-1)\psi_2^{t1}(\epsilon_1,\epsilon_2),$$

where $\psi_1^{t1}(\epsilon_1,\epsilon_2)$ and $\psi_2^{t1}(\epsilon_1,\epsilon_2)$ are positive and non-linear functions of ϵ_1 and ϵ_2.

Based on the last term, Term 1 is positive and increasing in ϵ_2 (whenever $\epsilon_2 > 1$), implying that larger flexibility of the downstream sector generates a smaller increase in rents (downstream rents), from shrinking production more given the shock to the supplier, compared to the Island economy. It is also the case though that a larger downstream elasticity mitigates the price increase in sector 2, which in turn mitigates the reduction in real wage due to the shock. Nevertheless, the first effect dominates. It could be the case that the last term becomes negative for sufficiently low ϵ_2. To analyze that possibility assume $\epsilon_2 = 0$. In this case, Term 1 becomes

$$\text{Term } 1_{\epsilon_2=0} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}\phi^{\frac{\epsilon_1}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}(1-\beta)\Big(\frac{\Big(a+(1-a)\phi_1^{\frac{1}{1-\epsilon_1}}\Big)-a\vartheta_2(1+\Pi^S)}{\Big(a+(1-a)\phi_1^{\frac{1}{1-\epsilon_1}}\Big)}\Big),$$

$$\text{Term } 1_{\epsilon_2=0} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}\phi^{\frac{\epsilon_1}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}(1-\beta)\Big(\frac{a(1-\vartheta_2(1+\Pi^S))+(1-a)\phi_1^{\frac{1}{1-\epsilon_1}}\Big)}{\Big(a+(1-a)\phi_1^{\frac{1}{1-\epsilon_1}}\Big)}\Big),$$

which is still positive as long as $a(1-\vartheta_2(1+\Pi^S)) > 0$. This is the case whenever sector 2 is reasonably constrained ($\vartheta_2 << 1$).

The key difference with respect to the homogeneous elasticity case is that while the term increases monotonically with ϵ_2 or ϵ, it actually decreases with ϵ_1. Intuitively, a higher ϵ_1 reduces the price increase of sector 1, which then implies a lower increase in the marginal cost of the downstream sector, and then a smaller increase in P_2. With homogeneous elasticities, even when a higher elasticity mitigates shocks to the supplier (less price adjustment and more quantity adjustment), it also amplifies the response of the downstream sector (larger reduction in rents and, therefore, income to the household). The latter effect does not exist when we only change ϵ_1 and keep ϵ_2 fixed. In other words, it is the higher elasticity of the downstream sector, not the upstream sector, that amplifies the aggregate effects from distortions in Term 1. In any case, with a larger common elasticity, it is more likely that, through this term, the $IOM > 0$ and the Star supplier amplifies shocks to sector 1. With heterogeneous elasticities, a high ϵ_2 and a low ϵ_1 imply $IOM > 0$, all else equal.

We now analyze Term 2

$$\text{Term } 2 = -a\epsilon_1 \vartheta_1^{\epsilon_1 - 1} P_1^{\epsilon_1 - 1} \left[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\right]$$

$$\text{Term } 2 = -\epsilon_1 \psi_1^{t2}(\epsilon_1, \epsilon_2),$$

in which $\psi_1^{t2}(\epsilon_1, \epsilon_2)$ is positive and non-linear function of ϵ_1 and ϵ_2. Term 2 is negative as s_1^S is larger than s_1^I, $s_1^S = \underbrace{\frac{\beta}{1-(1-a)\vartheta_1^{\epsilon_1}}}_{s_1^I} + \underbrace{\frac{\beta_2 (P_1^S)^{1-\epsilon_2} (P_2^S)^{\epsilon_2 - 1} \vartheta_2^{\epsilon_2} (1-a)}{1-(1-a)\vartheta_1^{\epsilon_1}}}_{s_1^{S_2} > 0}$), while $\Pi^S \approx \Pi^I$. Through this, when sector 1 is slightly constrained ($\vartheta_1 \approx 1$), a shock to sector 1 is mitigated in the star supplier economy, more so the higher ϵ_1. Intuitively, if $\epsilon_1 = 0$ this term is irrelevant because the distorted sector in both networks is optimally not changing its production plan (M, L). When $\epsilon_1 > 0$ in the Star supplier economy, a larger fraction of the economy is better able to couple with the shock. However, when sector 1 is heavily distorted, this term shrinks when ϵ_1 is larger, and larger than 1. Thus, when the distortion is severe, the composition effect dominates the relocation effect, and the Star supplier economy displays a larger reduction in real GDP, all else equal.

Term 3 is

$$\text{Term } 3 = -a(\epsilon_1 - 1)\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 2} \frac{\partial P_1}{\partial \vartheta_1} \left[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\right]$$

$$\text{Term 3} = (\epsilon_1 - 1)\psi_1^{t3}(\epsilon_1, \epsilon_2).$$

where $\psi_1^{t3}(\epsilon_1, \epsilon_2)$ is positive and non-linear function of ϵ_1 and ϵ_2. Term 3 is positive when $\epsilon_1 > 1$ (as $s_1^S > s_1^I$ and $\frac{\partial P_1}{\partial \vartheta_1} < 0$), but negative when $\epsilon_1 < 1$. Here a higher elasticity amplifies further (if $1 < \epsilon_1 < \bar{\epsilon}_1$ and distortion is not too tight). This effect is not the direct effect on P_1, as that is the same for both networks, but it is the effect on sector 1's rents. When the distorted sector is very flexible, it optimally shrinks more, reducing households rents (a function of revenue). However, $\frac{\partial P_1}{\partial \vartheta_1}$ is less negative the larger ϵ_1. When the distortion is initially very tight, or the elasticity very large, a further increase in the elasticity reduces the value Term 3. A larger ϵ_1 also reduces the value of $\frac{\partial P_1}{\partial \vartheta_1}$.

Term 4 is

$$\text{Term 4} = -a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1}\big[(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta_1} - (1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big],$$

where we use the fact that $s_1^S = \underbrace{\frac{\beta}{1-(1-a)\vartheta_1^{\epsilon_1}}}_{s_1^I} + \underbrace{\frac{\beta_2 (P_1^S)^{1-\epsilon_2}(P_2^S)^{\epsilon_2 - 1}\vartheta_2^{\epsilon_2}(1-a)}{1-(1-a)\vartheta_1^{\epsilon_1}}}_{s_1^{S_2}>0}$

to obtain

$$-a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1}\big[(1+\Pi^S)(\frac{\partial s_1^I}{\partial \vartheta_1} + \frac{\partial s_1^{S_2}}{\partial \vartheta_1}) - (1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big]$$
$$-a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1}\big[(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1} + (1+\Pi^S)\frac{\partial s_1^{S_2}}{\partial \vartheta_1}\big],$$

where $(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1} = (\Pi^S - \Pi^I)\frac{\epsilon_1(1-a)\beta_1\vartheta^{\epsilon_1 - 1}}{(1-(1-a)\vartheta^{\epsilon_1})^2} \approx 0$. Regarding the term $\frac{\partial s_1^{S_2}}{\partial \vartheta_1}$, we have

$$s_1^{S_2} = \frac{\beta_2 (P_1^S)^{1-\epsilon_2}(P_2^S)^{\epsilon_2 - 1}\vartheta_2^{\epsilon_2}(1-a)}{1-(1-a)\vartheta_1^{\epsilon_1}},$$

in which

$$
\begin{aligned}
(P_1^S)^{1-\epsilon_{Q_1}} =& \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)} = \phi_1, \\
(P_2^S)^{1-\epsilon_{Q_2}} =& a\vartheta_2^{\epsilon_{Q_2}-1} + (1-a)\vartheta_2^{\epsilon_{Q_2}-1}\Big(\frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}\Big)^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}} = \vartheta_2^{\epsilon_{Q_2}-1}\Big(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big),
\end{aligned}
$$

implying

$$
s_1^{S_2} = \frac{\beta_2 \phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}} \vartheta_2(1-a)}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)},
$$

$$
\begin{aligned}
\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)} \\
&+ (\epsilon_2-1)\frac{\vartheta_2(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta_1^{-\epsilon_1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)} \\
&- (\epsilon_2-1)\frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}-1}\beta_2\vartheta_1^{-\epsilon_1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)^2},.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)} \\
&+ (\epsilon_2-1)\Big[\frac{\vartheta_2(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta_1^{-\epsilon_1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)} - \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}-1}\beta_2\vartheta_1^{-\epsilon_1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)^2}\Big],
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)} \\
&+ (\epsilon_2-1)\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1}a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}\Big]\Big[1-\frac{(1-a)a^{-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big],
\end{aligned}
$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1 - 1}\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)} \\ &+ (\epsilon_2 - 1)\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1} a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}\Big]\Big[1 - \frac{(1-a)a^{-1}(a/\phi_1)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1 - 1}\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)} \\ &+ (\epsilon_2 - 1)\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1} a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}\Big]\Big[1 - \frac{(1-a)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}-1}}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1 - 1}\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)} \\ &+ (\epsilon_2 - 1)\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1} a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}\Big]\Big[1 - \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon_1 \underbrace{\frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1 - 1}\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}}_{>0} \\ &+ (\epsilon_2 - 1)\underbrace{\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1} a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^{\frac{\epsilon_1+\epsilon_2-2}{1-\epsilon_1}}\Big]\Big[\frac{a}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big]}_{>0},\end{aligned}$$

Recall that

$$\text{Term 4} \approx -a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1}(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta_1},$$

implying

$$\text{Term 4} \approx -\epsilon_1 \psi_1^{t4}(\epsilon_1, \epsilon_2) - (\epsilon_2 - 1)\psi_2^{t4}(\epsilon_1, \epsilon_2),$$

where $\psi_1^{t4}(\epsilon_1, \epsilon_2), \psi_2^{t4}(\epsilon_1, \epsilon_2), \psi_3^{t4}(\epsilon_1, \epsilon_2)$ are positive and non-linear functions of ϵ_1 and ϵ_2

We can see that, given $\epsilon_2 > 1$, a larger ϵ_1 makes Term 4 more negative. The distorted sector shrinks more, which is bad for rents but good to relocate activity to the less distorted sector. Given ϵ_1, a larger ϵ_2 also helps mitigating the effect of the distortion as sector 1. This is because sector 2 will demand less intermediates (it shrinks more), making the distorted sector smaller.

We can then rewrite the IOM as

$$IOM \approx \psi_1^{t1}(\epsilon_1, \epsilon_2) + (\epsilon_2 - 1)\psi_2^{t1}(\epsilon_1, \epsilon_2) - \epsilon_1 \psi_1^{t2}(\epsilon_1, \epsilon_2) + (\epsilon_1 - 1)\psi_1^{t3}(\epsilon_1, \epsilon_2) - \epsilon_1 \psi_1^{t4}(\epsilon_1, \epsilon_2) - (\epsilon_2 - 1)\psi_2^{t4}(\epsilon_1, \epsilon_2),$$

$$IOM \approx \psi_1^{t1}(\epsilon_1, \epsilon_2) + (\epsilon_2 - 1)\big(\psi_2^{t1}(\epsilon_1, \epsilon_2) - \psi_2^{t4}(\epsilon_1, \epsilon_2)\big) - \epsilon_1\big(\psi_1^{t2}(\epsilon_1, \epsilon_2) + \psi_1^{t4}(\epsilon_1, \epsilon_2)\big) + (\epsilon_1 - 1)\psi_1^{t3}(\epsilon_1, \epsilon_2),$$

$$IOM \approx \psi_1^{t1}(\epsilon_1, \epsilon_2) + \psi_2^{t4}(\epsilon_1, \epsilon_2) - \psi_2^{t1}(\epsilon_1, \epsilon_2) - \psi_1^{t3}(\epsilon_1, \epsilon_2) + \epsilon_1(\psi_1^{t3}(\epsilon_1, \epsilon_2) - \psi_1^{t2}(\epsilon_1, \epsilon_2) - \psi_1^{t4}(\epsilon_1, \epsilon_2)) + \epsilon_2(\psi_2^{t1}(\epsilon_1, \epsilon_2) + \psi_1^{t6}(\epsilon_1, \epsilon_2) - \psi_2^{t4}(\epsilon_1, \epsilon_2) - \psi_1^{t5}(\epsilon_1, \epsilon_2))$$

$$IOM \approx \tilde{\psi}_1(\epsilon_1, \epsilon_2) - \tilde{\psi}_2(\epsilon_1, \epsilon_2) + \epsilon_1(\tilde{\psi}_3(\epsilon_1, \epsilon_2) - \tilde{\psi}_4(\epsilon_1, \epsilon_2)) + \epsilon_2\big(\tilde{\psi}_5(\epsilon_1, \epsilon_2) - \tilde{\psi}_6(\epsilon_1, \epsilon_2)\big)$$

where ψ_j^{ti} and $\tilde{\psi}_j$ are positive and non-linear functions of ϵ_1 and ϵ_2.

In a nutshell, depending on the exact heterogeneity in elasticities, and the severity of the distortion, the IOM can be positive (Star supplier amplifies distortions) or negative (Star supplier mitigates distortions).

E. *Sectoral shock: homogeneous elasticity*

Term 1

$$\text{Term 1} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}\phi^{\frac{\epsilon_1 - \epsilon_2}{1-\epsilon_1}}}{\Big(a + (1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^2}(1-\beta)\Bigg(\frac{\Big(a + (1-a)\phi_1^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}\Big) + a\vartheta_2(1+\Pi^S)(\epsilon_2 - 1)}{\Big(a + (1-a)\phi_1^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}\Big)}\Bigg),$$

becomes

$$\text{Term } 1_{\epsilon_1=\epsilon_2} = \frac{(1-a)a(1-\beta)\vartheta_1^{-\epsilon}}{\left(\vartheta_1^{1-\epsilon}-(1-a)\right)^2}\frac{\Big(a+(1-a)\phi_1+a\vartheta_2(1+\Pi^S)(\epsilon-1)\Big)}{\Big(a+(1-a)\phi_1\Big)^2},$$

$$\text{Term } 1_{\epsilon_1=\epsilon_2} = \frac{(1-a)a(1-\beta)\vartheta_1^{-\epsilon}\left(a+(1-a)\phi_1\right)}{\left(\vartheta_1^{1-\epsilon}-(1-a)\right)^2\Big(a+(1-a)\phi_1\Big)^2} + \frac{a\vartheta_2(1+\Pi^S)(\epsilon-1)}{\left(\vartheta_1^{1-\epsilon}-(1-a)\right)^2\Big(a+(1-a)\phi_1\Big)^2},$$

$$\text{Term } 1_{\epsilon_1=\epsilon_2} = \frac{(1-a)a(1-\beta)\vartheta_1^{-\epsilon}}{\left(\vartheta_1^{1-\epsilon}-(1-a)\right)^2\Big(a+(1-a)\phi_1\Big)} + \frac{a\vartheta_2(1+\Pi^S)(\epsilon-1)}{\left(\vartheta_1^{1-\epsilon}-(1-a)\right)^2\Big(a+(1-a)\phi_1\Big)^2},$$

$$\text{Term } 1_{\epsilon_1=\epsilon_2} = \psi_1^{t1}(\epsilon) + (\epsilon-1)\psi_2^{t2}(\epsilon),$$

in which $\psi_1^{t1} > 0, \psi_2^{t2} > 0$ and depending on ϵ in a non-linear way.

This term is positive and increasing in ϵ (whenever $\epsilon > 1$ and $\vartheta_2 << 1$), implying that larger flexibility generates a larger decline in rents (downstream rents), from shrinking production more given the shock to the supplier. From this term, a higher elasticity increases IOM and the star supplier amplifies shocks compared to the Island.

$$\text{Term } 2 = -a\epsilon\vartheta_1^{\epsilon-1}P_1^{\epsilon-1}\left[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\right]$$

$$\text{Term } 2 = -\epsilon\psi_1^{t2}(\epsilon),$$

in which $\psi_1^{t2} > 0$ and depending on ϵ in a non-linear way.

Term 2 is negative as s_1^S is larger than s_1^I, $s_1^S = \underbrace{\frac{\beta}{1-(1-a)\vartheta_1^{\epsilon}}}_{s_1^I} + \underbrace{\frac{\beta_2(P_1^S)^{1-\epsilon}(P_2^S)^{\epsilon-1}\vartheta_2^{\epsilon}(1-a)}{1-(1-a)\vartheta_1^{\epsilon}}}_{s_1^{S_2}>0}),$ while $\Pi^S \approx \Pi^I$. Through this, when sector 1 is slightly constrained ($\vartheta_1 \approx 1$), a shock to sector 1 is mitigated in the star supplier economy, more so the higher ϵ. Intuitively, if $\epsilon = 0$ this term is irrelevant because the distorted sector in both networks is optimally not changing its production plan (M, L). When $\epsilon > 0$ in the Star supplier economy, a larger fraction of the economy is better able to couple with the shock. However, when sector 1 is heavily distorted, this term shrinks when ϵ is large, and larger than 1. Thus, when the distortion is severe, the composition effect dominates the relocation effect, and the Star supplier economy displays a larger reduction in real GDP, all else equal.

Term 3 is

$$\text{Term } 3 = -a(\epsilon - 1)\vartheta_1^{\epsilon} P_1^{\epsilon-2} \frac{\partial P_1}{\partial \vartheta_1} \big[(1 + \Pi^S) s_1^S - (1 + \Pi^I) s_1^I\big]$$

is positive when $\epsilon > 1$ (as $s_1^S > s_1^I$ and $\frac{\partial P_1}{\partial \vartheta_1} < 0$), but negative when ϵ_1. Here a higher elasticity amplifies further (if $1 < \epsilon < \bar{\epsilon}$ and distortion is not too tight). This effect is not the direct effect on P_1, as that is the same for both networks, but it is the effect on sector 1's rents. When the sectors are very flexible (so the distorted sector is very flexible), it optimally shrinks more, reducing households rents (a function of revenue). However, $\frac{\partial P_1}{\partial \vartheta_1}$ is less negative the larger ϵ. When the distortion is initially very tight, or the elasticity very large, a further increase in the elasticity reduces the value Term 3. A larger ϵ also reduces the value of $\frac{\partial P_1}{\partial \vartheta_1}$.

Term 3 can be rewritten as

$$\text{Term } 3 = (\epsilon - 1)\psi_1^{t3}(\epsilon)$$

in which $\psi_1^{t3} > 0$ and depending on ϵ in a non-linear way.

Term 4 is

$$\text{Term } 4 = -a\vartheta_1^{\epsilon} P_1^{\epsilon-1} \big[(1 + \Pi^S) \frac{\partial s_1^S}{\partial \vartheta_1} - (1 + \Pi^I) \frac{\partial s_1^I}{\partial \vartheta_1}\big],$$

where we use the fact that $s_1^S = \underbrace{\frac{\beta}{1-(1-a)\vartheta_1^\epsilon}}_{s_1^I} + \underbrace{\frac{\beta_2 (P_1^S)^{1-\epsilon}(P_2^S)^{\epsilon-1}\vartheta_2^\epsilon (1-a)}{1-(1-a)\vartheta_1^\epsilon}}_{s_1^{S_2}>0}$ to obtain

$$-a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1}\big[(1+\Pi^S)\big(\frac{\partial s_1^I}{\partial \vartheta_1} + \frac{\partial s_1^{S_2}}{\partial \vartheta_1}\big) - (1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big]$$
$$-a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1}\big[(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1} + (1+\Pi^S)\frac{\partial s_1^{S_2}}{\partial \vartheta_1}\big],$$

where $(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1} = (\Pi^S - \Pi^I)\frac{\epsilon_1 (1-a)\beta_1 \vartheta^{\epsilon-1}}{(1-(1-a)\vartheta^\epsilon)^2} \approx 0$. Regarding the term $\frac{\partial s_1^{S_2}}{\partial \vartheta_1}$, we have

$$s_1^{S_2} = \frac{\beta_2 (P_1^S)^{1-\epsilon}(P_2^S)^{\epsilon-1}\vartheta_2^\epsilon (1-a)}{1-(1-a)\vartheta_1^\epsilon},$$

in which

$$\begin{aligned}
(P_1^S)^{1-\epsilon} =& \frac{a}{\vartheta_1^{1-\epsilon} - (1-a)} = \phi_1, \\
(P_2^S)^{1-\epsilon} =& a\vartheta_2^{\epsilon-1} + (1-a)\vartheta_2^{\epsilon-1}\Big(\frac{a}{\vartheta_1^{1-\epsilon} - (1-a)}\Big) = \vartheta_2^{\epsilon-1}\big(a + (1-a)\phi_1\big),
\end{aligned}$$

implying

$$s_1^{S_2} = \frac{\beta_2 \phi_1 \vartheta_2 (1-a)}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1\big)},$$

$$\begin{aligned}
\frac{\partial s_1^S}{\partial \vartheta_1} =& (1-\epsilon)\frac{\vartheta_2 (1-a)^2 a^2 \beta_2 \vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^\epsilon\big)\big(\vartheta_1^{1-\epsilon} - (1-a)\big)^3\big(a+(1-a)\phi_1\big)^2} \\
&+ \epsilon\Big[\frac{\vartheta_2 (1-a)^2 a \beta_2 \vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^\epsilon\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon} - (1-a)\big)}\Big] - (1-\epsilon)\Big[\frac{\vartheta_2 \beta_2 (1-a) a \vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^\epsilon\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon} - (1-a)\big)}\Big],
\end{aligned}$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\Big[\frac{\vartheta_2(1-a)^2a^2\beta_2\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^3\big(a+(1-a)\phi_1\big)^2} - \frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)a}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^{-1}\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2\big(a+(1-a)\phi_1\big)} - 1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)a\big(1-(1-a)\vartheta_1^{\epsilon}\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2\big(a+(1-a)\frac{a}{\vartheta_1^{1-\epsilon}-(1-a)}\big)} - 1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)a\big(1-(1-a)\vartheta_1^{\epsilon}\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2\frac{a\vartheta_1^{1-\epsilon}}{\vartheta_1^{1-\epsilon}-(1-a)}\big)} - 1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)a\big(1-(1-a)\vartheta_1^{\epsilon}\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)a\vartheta_1^{1-\epsilon}\big)} - 1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)\big(1-(1-a)\vartheta_1^{\epsilon}\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}\big)} - 1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon \underbrace{\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big]}_{>0}$$

$$+ (1-\epsilon)\underbrace{\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}}_{>0}\underbrace{\Big[\frac{(1-a)\big(1-(1-a)\vartheta_1^{\epsilon}\big)-\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}}\Big]}_{><0},$$

To figure out whether $\frac{\partial s_1^{S_2}}{\partial \vartheta_1} >< 0$ we look at the last term $\Big[\frac{(1-a)\big(1-(1-a)\vartheta_1^{\epsilon}\big)-\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}}\Big]$, which can be expressed as

$$\Big[\frac{(1-a)\big(1-(1-a)\big)-\big(1-(1-a)\big)}{\big(1-(1-a)\big)}\Big],$$

$$\Big[\frac{(1-a)a-a}{\big(1-(1-a)\big)}\Big] = \Big[\frac{a(1-a-1)}{\big(1-(1-a)\big)}\Big] = \Big[\frac{-a^2}{a}\Big] = -a < 0,$$

when ($\vartheta_1 \approx 1$). In this case, if $\epsilon > 1$, Term 4 is negative, implying a smaller, potentially negative, IOM. This effect is smaller the larger the elasticity (as ϑ^{ϵ} decreases with ϵ). Now, term ψ_3 could be positive if sector $\vartheta_1 << 1$ is very distorted. In that case, a large elasticity could imply that Term 4 is positive, in which the Star supplier network amplifies shocks. However, this effect would be mitigates by the fact that $\vartheta_1^{\epsilon} \approx 0$ in this case. In both situations, a larger elasticity would imply a smaller miitgation effect of the Star supplier, compared to the Island economy, or a mild amplification effect in the Star supplier.

Recall that

$$\text{Term 4} \approx -\underbrace{a\vartheta_1^{\epsilon}P_1^{\epsilon-1}(1+\Pi^S)}_{>0}\frac{\partial s_1^{S_2}}{\partial \vartheta_1}.$$

$$\text{Term 4} \approx -\epsilon\psi_1^{t4} - (1-\epsilon)\psi_2^{t4},$$

in which ψ_1^{t4} and a non-linear function of ϵ, while ψ_2^{t4} can be positive or negative and it is a non-linear function of ϵ.

Putting Term 1, Term 2, Term 3, and Term 4 together yields:

$$IOM \approx \psi_1^{t1}(\epsilon) + (\epsilon-1)\psi_2^{t2}(\epsilon) - \epsilon\psi_1^{t2}(\epsilon) + (\epsilon-1)\psi_1^{t3}(\epsilon) - \epsilon\psi_1^{t4}(\epsilon) - (1-\epsilon)\psi_2^{t4}(\epsilon),$$

$$IOM \approx \psi_1^{t1}(\epsilon) + (\epsilon-1)\big(\psi_2^{t2}(\epsilon) + \psi_1^{t3}(\epsilon)\big) - \epsilon\big(\psi_1^{t2}(\epsilon) + \psi_1^{t4}(\epsilon)\big) - (1-\epsilon)\psi_2^{t4}(\epsilon),$$

$$IOM \approx \psi_1^{t1}(\epsilon) - \big(\psi_2^{t3}(\epsilon) + \psi_1^{t3}(\epsilon)\big) - \psi_2^{t4}(\epsilon) + \epsilon\big(\psi_2^{t2}(\epsilon) + \psi_1^{t3}(\epsilon) + \psi_2^{t4}(\epsilon) - \psi_1^{t2}(\epsilon) - \psi_1^{t4}(\epsilon)\big)$$

$$IOM \approx \hat{\psi}_1(\epsilon) - \hat{\psi}_2(\epsilon) - \hat{\psi}_3(\epsilon) + \epsilon\big(\hat{\psi}_4(\epsilon) - \hat{\psi}_5(\epsilon) - \hat{\psi}_2(\epsilon)\big)$$

where $\psi_1^{t1}, \psi_2^{t1}, \psi_1^{t2}, \psi_1^{t3}, \psi_1^{t4}$ are positive and non-linear functions of ϵ. On the other hand, ψ_2^{t4} can be positive or negative depending on ϑ_1 and ϵ and it is a non-linear function of ϵ. Also, $\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_4, \hat{\psi}_5$ are positive and non-linear functions of ϵ. On the other hand, $\hat{\psi}_3$ is a non-linear function of ϵ and can take positive or negative values.

F. *Aggregate shock: homogeneous elasticity*

PROOF PROPOSITION 5: AGGREGATE SHOCK:

In the homogeneous elasticity case we have

$$\begin{aligned}
P_1^{1-\epsilon} =& \frac{a}{\vartheta^{1-\epsilon} - (1-a)}, \\
P_2^{1-\epsilon} =& \frac{a}{\vartheta^{1-\epsilon} - (1-a)}, \\
s_1 =& \frac{\beta_1}{1-(1-a)\vartheta^{\epsilon}} \\
s_2 =& \frac{\beta_2}{1-(1-a)\vartheta^{\epsilon}} \\
(1+\Pi) =& \frac{1}{\sum_{j=1}^{N} a_j (\vartheta_j^w)^{\epsilon Q_j} Z_j^{\epsilon Q_j - 1} P_j^{\epsilon Q_j - 1} s_j} = \frac{1-(1-a)\vartheta^{\epsilon}}{\vartheta - (1-a)\vartheta^{\epsilon}} \geq 1.
\end{aligned}$$

For the star supplier we have

$$
\begin{aligned}
P_1^{1-\epsilon} =& \frac{a}{\vartheta^{1-\epsilon} - (1-a)}, \\
P_2^{1-\epsilon} =& \frac{a}{\vartheta^{1-\epsilon} - (1-a)} \\
s_1 =& \frac{\beta_1 + \beta_2(1-a)\vartheta^{\epsilon}}{1-(1-a)\vartheta^{\epsilon}} \\
s_2 =& \beta_2, \\
(1+\Pi) =& \frac{1}{\sum_{j=1}^{N} a_j (\vartheta_j^w)^{\epsilon Q_j} Z_j^{\epsilon Q_j - 1} P_j^{\epsilon Q_j - 1} s_j} = \frac{1-(1-a)\vartheta^{\epsilon}}{\vartheta - (1-a)\vartheta^{\epsilon}} \geq 1.
\end{aligned}
$$

Note here that $\frac{\partial(1+\Pi)}{\partial\vartheta} < 0$—a tighter distortion, lower ϑ, increases rents from distortions (think of increased mark-ups or rents from financial intermediary). This effect is stronger the smaller the elasticity, as in that case firms adjust production down less but prices increase more ($\uparrow PQ$).

To obtain the IOM we compute

$$
\frac{\partial \log C^i}{\partial \vartheta} = \underbrace{-\beta 1 \frac{\partial \log P_1}{\partial \vartheta} - (1-\beta 1)\frac{\partial \log P_2}{\partial \vartheta}}_{\text{Real wage channel}} \underbrace{-(1+\Pi)a\Big[\epsilon s_1 \vartheta^{\epsilon-1} P_1^{\epsilon-1} + s_1 \vartheta^{\epsilon} \frac{\partial P_1^{\epsilon-1}}{\partial \vartheta} + \vartheta^{\epsilon} P_1^{\epsilon-1} \frac{\partial s_1}{\partial \vartheta} + s_2 \vartheta^{\epsilon} \frac{\partial P_2^{\epsilon-1}}{\partial \vartheta} + s_2 \epsilon \vartheta^{\epsilon-1} P_2^{\epsilon-1} + \vartheta^{\epsilon} P_2^{\epsilon-1} \frac{\partial s_2}{\partial \vartheta}\Big]}_{\text{Rents channel}}
$$

$$
\frac{\partial \log C^s}{\partial \vartheta} = \underbrace{-\beta 1 \frac{\partial \log P_1}{\partial \vartheta} - (1-\beta 1)\frac{\partial \log P_2}{\partial \vartheta}}_{\text{Real wage channel}} \underbrace{-(1+\Pi)a\Big[\epsilon s_1 \vartheta^{\epsilon-1} P_1^{\epsilon-1} + s_1 \vartheta^{\epsilon} \frac{\partial P_1^{\epsilon-1}}{\partial \vartheta} + \vartheta^{\epsilon} P_1^{\epsilon-1} \frac{\partial s_1}{\partial \vartheta} + s_2 \vartheta^{\epsilon} \frac{\partial P_2^{\epsilon-1}}{\partial \vartheta} + s_2 \epsilon \vartheta^{\epsilon-1} P_2^{\epsilon-1}\Big]}_{\text{Rents channel}}
$$

Implying

$$
IOM = (1+\Pi)a\Big[\epsilon\vartheta^{\epsilon-1}P_1^{\epsilon-1}\Delta s_1 + \vartheta^{\epsilon}\frac{\partial P_1^{\epsilon-1}}{\partial\vartheta}\Delta s_1 + \vartheta^{\epsilon}P_1^{\epsilon-1}\Big(\frac{\partial s_1^I}{\partial\vartheta} - \frac{\partial s_1^S}{\partial\vartheta}\Big) + \vartheta^{\epsilon}\frac{\partial P_2^{\epsilon-1}}{\partial\vartheta}\Delta s_2 + \epsilon\vartheta^{\epsilon-1}P_2^{\epsilon-1}\Delta s_2 + \vartheta^{\epsilon}P_2^{\epsilon-1}\frac{\partial s_2^I}{\partial\vartheta},\Big]
$$

where $\Delta s_j = s_j^I - s_j^S$. We now use the fact that $P_1 = P_2$ in both networks to obtain

$$IOM = (1+\Pi)a\Big[\epsilon\vartheta^{\epsilon-1}P^{\epsilon-1}(\Delta s_1+\Delta s_2)+\vartheta^{\epsilon}\frac{\partial P^{\epsilon-1}}{\partial\vartheta}(\Delta s_1+\Delta s_2)+\vartheta^{\epsilon}P_1^{\epsilon-1}(\frac{\partial s_1^I}{\partial\vartheta}-\frac{\partial s_1^S}{\partial\vartheta})+\vartheta^{\epsilon}P_2^{\epsilon-1}\frac{\partial s_2^I}{\partial\vartheta}\Big],$$

using the solution for sectoral sales, we can easily show that $\Delta s_1 = -\Delta s_2$, which implies that

$$IOM = (1+\Pi)a\Big[\vartheta^{\epsilon}P^{\epsilon-1}(\frac{\partial s_1^I}{\partial\vartheta}-\frac{\partial s_1^S}{\partial\vartheta}+\frac{\partial s_2^I}{\partial\vartheta})\Big],$$

in which $(\frac{\partial s_1^i}{\partial\vartheta}-\frac{\partial s_1^s}{\partial\vartheta}+\frac{\partial s_2^i}{\partial\vartheta})=0$, implying

$$IOM = (1+\Pi)a\Big[\vartheta^{\epsilon}P^{\epsilon-1}(\frac{\partial s_1^i}{\partial\vartheta}-\frac{\partial s_1^s}{\partial\vartheta}+\frac{\partial s_2^i}{\partial\vartheta})\Big]=0.$$

Thus, we have shown that when $\epsilon_1=\epsilon_2$ the Star supplier economy is isomorphic to the Island economy.

G. *Aggregate shock: heterogeneous elasticity*

We now study the heterogeneous elasticities case. We have in the Island economy

$$\begin{aligned}
P_1^{1-\epsilon_1} =& \frac{a}{\vartheta^{1-\epsilon_1}-(1-a)},\\
P_2^{1-\epsilon_2} =& \frac{a}{\vartheta^{1-\epsilon_2}-(1-a)},\\
s_1 =& \frac{\beta_1}{1-(1-a)\vartheta^{\epsilon_1}}\\
s_2 =& \frac{\beta_2}{1-(1-a)\vartheta^{\epsilon_2}},
\end{aligned}$$

and in the Star supplier economy

$$
\begin{aligned}
P_1^{1-\epsilon_{Q_1}} =& \frac{a}{\vartheta^{1-\epsilon_{Q_1}} - (1-a)}, \\
P_2 =& \frac{1}{\vartheta}\Big(a + (1-a)\Big(\frac{a}{\vartheta^{1-\epsilon_{Q_1}} - (1-a)}\Big)^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}\Big)^{\frac{1}{1-\epsilon_{Q_2}}} \\
s_1 =& \frac{\beta_1}{1-(1-a)\vartheta^{\epsilon_1}} + \frac{\beta_2 \phi^{\frac{1-\epsilon_2}{1-\epsilon_1}} \vartheta(1-a)}{\big(1-(1-a)\vartheta^{\epsilon_1}\big)\big(a + (1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}, \\
s_2 =& \beta_2
\end{aligned}
$$

To obtain the IOM we compute

$$
\begin{aligned}
\frac{\partial \log C^I}{\partial \vartheta} =& \underbrace{-\beta 1 \frac{\partial \log P_1}{\partial \vartheta} - \beta_2 \frac{\partial \log P_2^I}{\partial \vartheta}}_{\text{Real wage channel}} \\
& \underbrace{-(1+\Pi^I) a \Big[\epsilon_1 s_1^I \vartheta^{\epsilon_1 - 1} P_1^{\epsilon_1 - 1} + s_1^I \vartheta^{\epsilon_1} \frac{\partial P_1^{\epsilon_1 - 1}}{\partial \vartheta} + \vartheta^{\epsilon_1} P_1^{\epsilon_1 - 1} \frac{\partial s_1^I}{\partial \vartheta} + s_2^I \vartheta^{\epsilon_2} \frac{\partial (P_2^I)^{\epsilon_2 - 1}}{\partial \vartheta} + s_2^I \epsilon_2 \vartheta^{\epsilon_2 - 1} (P_2^I)^{\epsilon_2 - 1} + \vartheta^{\epsilon_2} (P_2^I)^{\epsilon_2 - 1} \frac{\partial s_2^I}{\partial \vartheta}\Big]}_{\text{Rents channel}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \log C^s}{\partial \vartheta} =& \underbrace{-\beta 1 \frac{\partial \log P_1}{\partial \vartheta} - \beta_2 \frac{\partial \log P_2^S}{\partial \vartheta}}_{\text{Real wage channel}} \\
& \underbrace{-(1+\Pi^S) a \Big[\epsilon_1 s_1^S \vartheta^{\epsilon_1 - 1} P_1^{\epsilon_1 - 1} + s_1^S \vartheta^{\epsilon_1} \frac{\partial P_1^{\epsilon_1 - 1}}{\partial \vartheta} + \vartheta^{\epsilon_1} P_1^{\epsilon_1 - 1} \frac{\partial s_1^S}{\partial \vartheta} + s_2^S \vartheta^{\epsilon_2} \frac{\partial (P_2^S)^{\epsilon_2 - 1}}{\partial \vartheta} + s_2^S \epsilon_2 \vartheta^{\epsilon_2 - 1} (P_2^S)^{\epsilon_2 - 1}\Big]}_{\text{Rents channel}}
\end{aligned}
$$

Using the fact that $\frac{\partial \log P_2}{\vartheta} = \frac{1}{P_2}\frac{\partial P_2}{\vartheta}$, that $\frac{\partial P_2^{\epsilon_2 - 1}}{\vartheta} = (\epsilon_2 - 1) P_2^{\epsilon_2 - 2}\frac{\partial P_2}{\vartheta}$, and that $s_2^S = 1 - \beta$, we reorganize the IOM as follows

$$IOM = \underbrace{-\frac{\partial \log P_2^S}{\partial \vartheta}(1-\beta)\Big(1 + a\vartheta^{\epsilon_2}(P_2^S)^{\epsilon_2-1}(1+\Pi^S)(\epsilon_2-1)\Big)}_{\text{Term 1}}$$
$$\underbrace{-a\epsilon_1\vartheta^{\epsilon_1-1}P_1^{\epsilon_1-1}\big[(1+\Pi^S)s_1^S-(1+\Pi^I)s_1^I\big]}_{\text{Term 2}}$$
$$\underbrace{-a\vartheta^{\epsilon_1}(\epsilon_1-1)P_1^{\epsilon_1-2}\frac{\partial P_1}{\partial \vartheta}\big[(1+\Pi^S)s_1^S-(1+\Pi^I)s_1^I\big]}_{\text{Term 3}}$$
$$\underbrace{-a\vartheta^{\epsilon_1}P_1^{\epsilon_1-1}\big[(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta}-(1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta}\big]}_{\text{Term 4}}$$
$$\underbrace{-\epsilon_2\vartheta^{\epsilon_2-1}\big[(1+\Pi^S)s_2^S(P_2^S)^{\epsilon_2-1}-(1+\Pi^I)s_2^I(P_2^I)^{\epsilon_2-1}\big]}_{\text{Term 5}}$$
$$\underbrace{+(1+\Pi^S)\vartheta^{\epsilon_2}(P_2^I)^{\epsilon_2-1}\frac{\partial s_2^I}{\partial \vartheta}}_{\text{Term 6}}$$

Term 1

$$-(1-\beta_1)\frac{\partial \log P_2}{\partial \vartheta} = -\frac{(1-\beta_1)}{P_2}\frac{\partial P_2}{\partial \vartheta} = -\frac{(1-\beta 1)}{P_2}\big[-\frac{P_2}{\vartheta} - P_2^{\epsilon_2}\frac{(1-a)a\vartheta^{-\epsilon_1}\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}\vartheta^{-1}}{\big(\vartheta^{1-\epsilon_1}-(1-a)\big)^2}\big],$$

$$-(1-\beta_1)\frac{\partial \log P_2}{\partial \vartheta} = \frac{(1-\beta 1)}{\vartheta} + (1-\beta_1)P_2^{\epsilon_2-1}\frac{(1-a)a\vartheta^{-\epsilon_1-1}\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\big(\vartheta^{1-\epsilon_1}-(1-a)\big)^2}.$$

$$\text{Term 1} = \psi_1^{t1}(\epsilon_1,\epsilon_2) + (\epsilon_2-1)\psi_2^{t1}(\epsilon_1,\epsilon_2),$$

in which $\psi_1^{t1}(\epsilon_1,\epsilon_2)$ and $\psi_2^{t1}(\epsilon_1,\epsilon_2)$ are positive and non-linear functions of ϵ_1 and ϵ_2.

Term 2

$$\text{Term 2} = -a\epsilon\vartheta^{\epsilon_1-1}P_1^{\epsilon_1-1}\big[(1+\Pi^S)s_1^S-(1+\Pi^I)s_1^I\big]$$

$$\text{Term } 2 = -\epsilon_1 \psi_1^{t2}(\epsilon_1, \epsilon_2),$$

in which $\psi_1^{t2} > 0$ and a non-linear function of ϵ_1 and ϵ_2.

Term 3 is

$$\text{Term } 3 = -a(\epsilon_1 - 1)\vartheta^{\epsilon_1} P_1^{\epsilon_1 - 2} \frac{\partial P_1}{\partial \vartheta} \big[(1 + \Pi^S) s_1^S - (1 + \Pi^I) s_1^I\big]$$

$$\text{Term } 3 = (\epsilon_1 - 1)\psi_1^{t3}(\epsilon_1, \epsilon_2).$$

where $\psi_1^{t3}(\epsilon_1, \epsilon_2)$ is positive and non-linear function of ϵ_1 and ϵ_2.

Let us compute Term 4

$$-a\vartheta^{\epsilon_1} P_1^{\epsilon_1 - 1} \big[(1 + \Pi^S)\big(\frac{\partial s_1^I}{\partial \vartheta} + \frac{\partial s_1^{S_2}}{\partial \vartheta}\big) - (1 + \Pi^I)\frac{\partial s_1^I}{\partial \vartheta}\big]$$

$$-a\vartheta^{\epsilon_1} P_1^{\epsilon_1 - 1} \big[(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta} + (1 + \Pi^S)\frac{\partial s_1^{S_2}}{\partial \vartheta}\big],$$

where $(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta} = (\Pi^S - \Pi^I)\frac{\epsilon_1(1-a)\beta_1\vartheta^{\epsilon_1 - 1}}{(1-(1-a)\vartheta^{\epsilon_1})^2} \approx 0$. Regarding the term $\frac{\partial s_1^{S_2}}{\partial \vartheta}$, we have

$$s_1^{S_2} = \frac{\beta_2 \phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}} \vartheta(1-a)}{\big(1 - (1-a)\vartheta^{\epsilon_1}\big)\big(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)},$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta} = & \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a + (1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} \\ & - (1-\epsilon_2)\Big[\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon} - (1-a))^{\frac{\epsilon_1+\epsilon_2-2}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}\Big] + (1-\epsilon_2)\Big[\frac{\vartheta^{1-\epsilon_1}(1-a)^2 a^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon} - (1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\big(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)^2}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta} = & \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a + (1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} \\ & + (1-\epsilon_2)\Big\{\frac{\vartheta^{1-\epsilon_1}(1-a)^2 a^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon} - (1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\big(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)^2} - \frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon} - (1-a))^{\frac{\epsilon_1+\epsilon_2-2}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}\Big\},\end{aligned}$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}$$
$$+ (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta_1^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta_1^{\epsilon})\left(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\right)}\left\{\frac{(1-a)a^{-1}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} - (\vartheta_1^{1-\epsilon}-(1-a))^{-1}\right\},$$

$$\frac{\partial s_1^S}{\partial \vartheta} = \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}$$
$$+ (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\left(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\right)}\left\{\frac{(1-a)a^{-1}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} - \frac{\phi_1}{a}\right\},$$

$$\frac{\partial s_1^S}{\partial \vartheta} = \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}$$
$$+ (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\left(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\right)}\left\{\frac{(1-a)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}} - \phi(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})a}\right\},$$

$$\frac{\partial s_1^S}{\partial \vartheta} = \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}$$
$$+ (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\left(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\right)}\left\{\frac{(1-a)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}} - \phi a - (1-a)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}})}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})a}\right\},$$

$$\frac{\partial s_1^S}{\partial \vartheta} = \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}$$
$$+ (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\left(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\right)}\left\{\frac{-\phi a}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})a}\right\},$$

$$\frac{\partial s_1^S}{\partial \vartheta} = \underbrace{\frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})}}_{>0} + \epsilon_1 \underbrace{\frac{(1-a)^2 \phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2 (a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}}_{>0}$$

$$(\epsilon_2 - 1) \underbrace{\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \left(\vartheta^{1-\epsilon} - (1-a)\right)^{\frac{\epsilon_2 - 1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\left(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\right)} \frac{\phi a}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})a}}_{>0}$$

Recall that

$$\text{Term 4} \approx -a\vartheta^{\epsilon_1} P_1^{\epsilon_1 - 1}(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta},$$

implying

$$\text{Term 4} \approx -\epsilon_1 \psi_1^{t4}(\epsilon_1, \epsilon_2) - (\epsilon_2 - 1)\psi_2^{t4}(\epsilon_1, \epsilon_2),$$

where $\psi_1^{t4}(\epsilon_1, \epsilon_2)$ and $\psi_2^{t4}(\epsilon_1, \epsilon_2)$ are positive and non-linear functions of ϵ_1 and ϵ_2

We have Term 5

$$\text{Term 5} = -\epsilon_2 \vartheta^{\epsilon_2 - 1}\left[(1+\Pi^S)s_2^S(P_2^S)^{\epsilon_2 - 1} - (1+\Pi^I)s_2^I(P_2^I)^{\epsilon_2 - 1}\right]$$

$$\text{Term 5} = -\epsilon_2 \psi_1^{t5}(\epsilon_1, \epsilon_2),$$

in which $\psi_1^{t5}(\epsilon_1, \epsilon_2)$ is a non-linear function of ϵ_1 and ϵ_2 and it could take positive or negative values.

Term 6

$$\text{Term 6} = (1+\Pi^S)\vartheta^{\epsilon_2}(P_2^I)^{\epsilon_2 - 1}\frac{\partial s_2^I}{\partial \vartheta}$$

$$\text{Term 6} = (1+\Pi^S)\vartheta^{\epsilon_2}(P_2^I)^{\epsilon_2 - 1}\frac{(1-a)\vartheta^{\epsilon_2 - 1}\beta_2 \epsilon_2}{\left(1-(1-a)\vartheta^{\epsilon_2}\right)^2}$$

$$\text{Term 6} = \epsilon_2 \psi_1^{t6}(\epsilon_2),$$

where $\psi_1^{t6}(\epsilon_2)$ is positive and a non-linear function of ϵ_2.

Putting Term 1, Term 2, Term 3, Term 4, Term 5, and Term 6 together yields:

$$IOM \approx \psi_1^{t1}(\epsilon_1,\epsilon_2)+(\epsilon_2-1)\psi_2^{t1}(\epsilon_1,\epsilon_2)-\epsilon_1\psi_1^{t2}(\epsilon_1,\epsilon_2)+(\epsilon_1-1)\psi_1^{t3}(\epsilon_1,\epsilon_2)-\epsilon_1\psi_1^{t4}(\epsilon_1,\epsilon_2)-(\epsilon_2-1)\psi_2^{t4}(\epsilon_1,\epsilon_2)-\epsilon_2\psi_1^{t5}(\epsilon_1,\epsilon_2)+\epsilon_2\psi_1^{t6}(\epsilon_2),$$

$$IOM \approx \psi_1^{t1}(\epsilon_1,\epsilon_2)+(\epsilon_2-1)\big(\psi_2^{t1}(\epsilon_1,\epsilon_2)-\psi_2^{t4}(\epsilon_1,\epsilon_2)\big)-\epsilon_1\big(\psi_1^{t2}(\epsilon_1,\epsilon_2)+\psi_1^{t4}(\epsilon_1,\epsilon_2)\big)+(\epsilon_1-1)\psi_1^{t3}(\epsilon_1,\epsilon_2)+\epsilon_2\big(\psi_1^{t6}(\epsilon_2)-\psi_1^{t5}(\epsilon_1,\epsilon_2)\big),$$

$$IOM \approx \bar{\psi}_1(\epsilon_1,\epsilon_2)-\bar{\psi}_2(\epsilon_1,\epsilon_2)+\epsilon_1\big(\bar{\psi}_3(\epsilon_1,\epsilon_2)-\bar{\psi}_4(\epsilon_1,\epsilon_2)\big)+\epsilon_2\big(\bar{\psi}_5(\epsilon_1,\epsilon_2)-\bar{\psi}_6(\epsilon_1,\epsilon_2)-\bar{\psi}_7(\epsilon_1,\epsilon_2)\big)$$

where $\psi_1^{t1},\psi_2^{t1},\psi_1^{t2},\psi_1^{t3},\psi_1^{t4},\psi_2^{t4},\psi_1^{t6}$ are positive and non-linear functions of ϵ_1 and ϵ_2. On the other hand, $\psi_1^{t5}(\epsilon_1,\epsilon_2)$ is a non-linear function of ϵ_1 and ϵ_2 and it could take positive or negative values. Also, $\bar{\psi}_1,\bar{\psi}_2,\bar{\psi}_3,\bar{\psi}_4,\bar{\psi}_5,\bar{\psi}_6$ are positive and non-linear functions of ϵ_1 and ϵ_2. On the other hand, $\bar{\psi}_7$ is a non-linear function of ϵ_1 and ϵ_2 and it could take positive or negative values.

II. Additional Empirical Results

A. *Spreads and Flexibility using statistically significant elasticity at 95% confidence*

Table 1 shows that the same negative relationship between flexibility and spreads holds when we define statistically significant point estimates based on the 95% confidence rather than 90% confidence.

B. *Spreads and Flexibility OLS Elasticities*

Table 2 shows that similar results to Table **??** hold when we use our biased OLS estimate grouping sectors by high and low flexibility. We see that high-flexibility sectors experienced an increase in spreads that was 1.09 percentage points than in low-flexibility sectors.

C. *Complementary Evidence Using Firm-Level Data on Short Term Liquidity*

In this Appendix, we use firm-level data to estimate the relationship between production flexibility and short-term liquidity. We obtain firms' working capital (current assets - current liabilities) to sales ratio. We have a balanced panel

Table 1—Spreads and Flexibility (95% confidence)

VARIABLES	(1) Δ Spread	(2) Δ Spread	(3) Δ Spread	(4) Δ Spread
$\epsilon_Q^{IV} \cdot GR$	-0.342***			
	(0.117)			
$\epsilon_Q^{IV} \cdot EBP$		-0.151**		
		(0.068)		
High $\epsilon_Q^{IV} \cdot GR$			-1.486***	
			(0.459)	
High $\epsilon_Q^{IV} \cdot EBP$				-0.744***
				(0.243)
Observations	2,917	2,917	2,917	2,917
Number of sector	53	53	53	53
Adjusted R-squared	0.434	0.435	0.436	0.440
Time FE	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes

Note: ϵ_Q^{IV} is the IV estimate of sectoral elasticity. High ϵ_Q^{IV} is a dummy that takes the value of 1 for sectors with an elasticity above median and the value of 0 otherwise. Standard errors presented in parentheses are clustered at the sector level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

2002q1-2015q4. We drop outliers (1% and 99% percentiles) in terms of sales growth, working capital to sales growth, and leverage growth during the Great Recession. The results in Table 4 show that high flexibility firms experienced growth in their working capital to sales ratio that is 59 percentage points larger than low flexibility firms. During the Great Recession, the average working capital to sales ratio growth in the sample is -3.94%, the 1st percentile is -%228, the 99th percentile is 862%, and the standard deviation is 533%.

Table 2—Spreads and Flexibility OLS

VARIABLES	(1) Δ Spread	(2) Δ Spread
High $\epsilon_Q^{FE} \cdot GR$	-1.098*	
	(0.624)	
High $\epsilon_Q^{FE} \cdot EBP$		-0.608*
		(0.316)
Observations	2,917	2,917
Adjusted R-squared	0.433	0.437
Time FE	Yes	Yes
Sector FE	Yes	Yes

Note: This table presents an OLS regression using the 4-quarters change in sectoral credit spread as the dependent variable. The independent variables are sectoral sales, the value of property and plants, inventories, leverage (total debt divided by assets), the excess bond premium (EBP), time fixed-effects, sector fixed-effect, the elasticity, the interaction between the elasticity and a Great Recession dummy, and the interaction between the elasticity and the EBP. High ϵ_Q^{FE} is a dummy that takes the value of 1 for sectors with an elasticity above median, and that takes the value of 0 otherwise. Standard errors presented in parentheses are clustered at the sector level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table 3—Average Spreads and Flexibility

VARIABLES	(1) Δ Spread	(2) Δ Spread	(3) Δ Spread	(4) Δ Spread
$\epsilon_Q^{IV} \cdot GR$	-0.343***			
	(0.115)			
$\epsilon_Q^{IV} \cdot EBP$		-0.153**		
		(0.067)		
High $\epsilon_Q^{IV} \cdot GR$			-1.147**	
			(0.486)	
High $\epsilon_Q^{IV} \cdot EBP$				-0.603**
				(0.238)
Observations	2,917	2,917	2,917	2,917
Number of sector	53	53	53	53
Adjusted R-squared	0.525	0.526	0.524	0.527
Time FE	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes

Note: This table presents an OLS regression using the four-quarter change in average sectoral credit spreads as the dependent variable. The independent variables are sectoral sales, the value of property and plants, inventories, leverage (total debt divided by assets), the excess bond premium (EBP), time fixed-effects, sector fixed-effects, the estimates sectoral elasticity of substitution, the interaction between the elasticity and a Great Recession dummy, and the interaction between the elasticity and the EBP. ϵ_Q^{IV} are the IV estimates of sectoral elasticity in Table **??**. High ϵ_Q^{IV} is a dummy that takes the value of 1 for sectors with an elasticity above median and the value of 0 otherwise. Standard errors presented in parentheses are clustered at the sector level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

VARIABLES	(1) % Δ WCS	(2) % Δ WCS	(3) % Δ WCS	(4) % Δ WCS
$\epsilon_Q^{IV} \cdot GR$	0.155**			
	(0.063)			
$\epsilon_Q^{IV} \cdot EBP$		0.052**		
		(0.026)		
High $\epsilon_Q^{IV} \cdot GR$			0.594**	
			(0.245)	
High $\epsilon_Q^{IV} \cdot EBP$				0.185*
				(0.100)
Observations	82,998	82,998	82,998	82,998
Adjusted R-squared	0.002	0.002	0.002	0.002
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes

Table 4—Working Capital to Sales (WCS) Growth and Flexibility

Note: This table presents an OLS regression using firm-level working capital to sales ratio as the dependent variable. The independent variables are sectoral sales, the value of property and plants, inventories, leverage (total debt divided by assets), the excess bond premium (EBP), time fixed-effects, firm fixed-effects, the high elasticity dummy, the interaction between the high elasticity dummy and a Great Recession dummy, and the interaction between the high elasticity dummy the EBP. Standard errors presented in parentheses are clustered at the firm level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.