

Online Appendix

Interactions and Coordination between Monetary and Macro-Prudential Policies

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The online Appendix has two parts. The first part derives the system of equations that analytically characterize the solution to the model. As an aside, it also derives the socially optimal inflation rate in the economy without funding frictions or macro-prudential policy intervention and the equilibrium quantities of inflation and price dispersion in the economy with a constant employment gap. The second part develops a simple model based on differences in monitoring to rationalize the productivity edge of financial intermediaries.

1 Analytical Characterization

This part proceeds by steps. First, I solve the portfolio problems of financial intermediaries and households in that order. Second, I derive the aggregate production function in equilibrium and the labor wedge. Third, as an aside, I derive the socially optimal inflation rate in the economy without funding frictions or macro-prudential policy intervention and the equilibrium quantities of inflation and price dispersion in the economy with a constant employment gap. Fourth, I derive the ODEs for mappings $\{\frac{\omega}{A}q, v\}$ and the Kolmogorov-Chapman forward equation for invariant distribution g . Lastly, I derive the forward-looking equation for inflation π and a PDEs that analytically characterize π .

1.1 Portfolio Problems

Financial Intermediaries By definition value function V_t satisfies

$$\int_0^t \gamma e^{-\gamma s} \Lambda_s n_{f,s} ds + e^{-\gamma t} \Lambda_t V_t = E_t \int_0^\infty \gamma e^{-\gamma s} \Lambda_s n_{f,s} ds . \quad (1)$$

The conjectures made in Subsection II.B of the paper ensure that the LHS evolves over time stochastically according to a diffusion process with the same Brownian shock dZ_t as that in the law of motion of A_t . (Recall $V_t = v_t n_{f,t}$.) Because the RHS is a conditional expectation of a random variable, the drift process of the LHS equals zero. I apply Ito's

Lemma to the LHS and then equalize the resulting drift process to zero. I obtain as an outcome the following Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \max_{\phi_t \geq 0} \left\{ \frac{\gamma}{v_t} + \mu_{\Lambda,t} + \mu_{v,t} + \mu_{n_f,t} + \sigma_{\Lambda,t} \sigma_{v,t} + \sigma_{\Lambda,t} \sigma_{n_f,t} + \sigma_{v,t} \sigma_{n_f,t} - \gamma \right\}, \quad (2)$$

subject to : $\phi_t \leq \min \{ \lambda v_t, \Phi_t \}$,

with the optimization problem on the RHS being the portfolio problem of financial intermediaries. Note that this problem is the same as that laid out in the paper, but expressed in a different manner.

The law of motion of net worth $n_{f,t}$ laid out in the paper implies

$$\mu_{n_f,t} = \left[\frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t) \right] \phi_t + (i_t - \pi_t), \quad (3)$$

$$\sigma_{n_f,t} = \phi_t \sigma_{q,t}. \quad (4)$$

None of the processes $r_{k,t}$, $i_t - \pi_t$, q_t , $\mu_{q,t}$, or $\sigma_{q,t}$ depends on leverage multiple ϕ_t because individual financial intermediaries take rates of returns, asset prices, and their dynamics as given. Neither do processes v_t , $\mu_{v,t}$, or $\sigma_{v,t}$ depend on ϕ_t because in Subsection II.B of the paper I conjecture that financial intermediaries take Tobin's Q and its dynamics as given when choosing their investment portfolio. Lastly, drift and diffusion processes $\mu_{\Lambda,t}$ and $\sigma_{\Lambda,t}$ also do not depend on ϕ_t , but because financial intermediaries also take as given the consumption decisions of households.

The optimization problem in the HJB equation is thus the same for all of the financial intermediaries regardless of their individual net worth $n_{f,t}$. Optimal leverage multiple ϕ_t is also the same for all of the financial intermediaries, and so is value v_t . A representative financial intermediary in equilibrium therefore exists.

The optimization problem in the HJB equation is linear in ϕ_t . Its solution is

$$\phi_t \begin{cases} = \min \{ \lambda v_t, \Phi_t \} & \text{if } RR_{f,t} > 0 \\ \in [0, \min \{ \lambda v_t, \Phi_t \}] & \text{if } RR_{f,t} = 0 \\ = 0 & \text{if } RR_{f,t} < 0 \end{cases}, \quad (5)$$

with

$$RR_{f,t} \equiv \frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t) + (\sigma_{\Lambda,t} + \sigma_{v,t}) \sigma_{q,t}, \quad (6)$$

defined the same as in the paper but using a different notation.

I substitute optimal leverage multiple (5) in (2). I obtain

$$\widetilde{RR}_{f,t} + \frac{\gamma}{v_t} + \mu_{v,t} - \gamma + \sigma_{v,t}\sigma_{q,t} = 0, \quad (7)$$

with

$$\widetilde{RR}_{f,t} \equiv \left[\frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t) + (\sigma_{\Lambda,t} + \sigma_{v,t})\sigma_{q,t} \right] \phi_t, \quad (8)$$

and with ϕ_t being given by (5). These two expressions are also the same as in the paper (but here they are also expressed using a different notation). To derive $\widetilde{RR}_{f,t}$, I use that $\mu_{\Lambda,t} = -(i_t - \pi_t)$ —which follows from the portfolio problem of households, as I will show next. (This substitution is possible here because financial intermediaries take the behavior of households as given.) Note that $\widetilde{RR}_{f,t} = RR_{f,t}\phi_t$, as in the paper.

Formulae (5) to (8) verify the conjecture that individual financial intermediaries take v_t and its dynamics as given. Those same formulae, together with (3) and (4), imply that $\int_t^\infty \gamma e^{-\gamma s} \Lambda_s n_{f,s} ds$ is linear in $n_{f,t}$ —which, combined with (1), verifies that V_t is linear in $n_{f,t}$.

Households Value function U_t satisfies the usual HJB equation:

$$0 = \max_{c_t, l_t, \bar{k}_{h,t} \geq 0} \left\{ \ln c_t - \chi \frac{l_t^{1+\psi}}{1+\psi} + \frac{1}{dt} E_t [dU_t] - \rho U_t \right\}. \quad (9)$$

I conjecture that U_t is given by

$$U_t = U(n_{h,t}, J_t), \quad (10)$$

with $U : \mathbb{R}^2 \rightarrow \mathbb{R}$ being a twice continuously differentiable function, and with $J_t \in \mathbb{R}$ being a real-valued process that evolves over time stochastically according to a diffusion process with the same Brownian shock dZ_t as that in the law of motion of A_t . Process J_t could be interpreted as a summary statistic of the aggregate variables that are relevant for the portfolio problem of households. I conjecture that households take J_t and its drift and diffusion processes $\mu_{J,t}$ and $\sigma_{J,t}$ as given.

I apply Ito's Lemma to both sides of (10), then evaluate the resulting expression into

(9). I obtain

$$\rho U_t = \max_{c_t, l_t, \bar{k}_{h,t} \geq 0} \left\{ \begin{array}{l} \ln c_t - \chi \frac{l_t^{1+\psi}}{1+\psi} + \frac{\partial U_t}{\partial n_{h,t}} \mu_{n_{h,t}} n_{h,t} + \frac{\partial U_t}{\partial J_t} \mu_{J,t} J_t + \\ \frac{1}{2} \frac{\partial^2 U_t}{(\partial n_{h,t})^2} (\sigma_{n_{h,t}} n_{h,t})^2 + \frac{\partial^2 U_t}{\partial J_t \partial n_{h,t}} \sigma_{J,t} J_t \sigma_{n_{h,t}} n_{h,t} + \frac{1}{2} \frac{\partial^2 U_t}{(\partial J_t)^2} (\sigma_{J,t} J_t)^2 \end{array} \right\}, \quad (11)$$

with the optimization problem on the RHS being the portfolio problem of households—here expressed differently than in the paper.

The law of motion of net worth $n_{h,t}$ laid out in the paper implies

$$\mu_{n_{f,t}} n_{h,t} = \left[a_h \frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t) \right] q_t \bar{k}_{h,t} + (i_t - \pi_t) n_{h,t} + w_t l_t + Tr_t - c_t, \quad (12)$$

$$\sigma_{n_{f,t}} n_{h,t} = \bar{k}_{h,t} \sigma_{q,t} q_t. \quad (13)$$

Households take processes $r_{k,t}$, w_t , $i_t - \pi_t$, q_t , $\mu_{q,t}$, $\sigma_{q,t}$, and Tr_t as given.

The first-order conditions (F.O.C.s) of the optimization problem are

$$\frac{1}{c_t} = \frac{\partial U_t}{\partial n_{h,t}}, \quad (14)$$

$$\chi l_t^\psi = w_t \frac{\partial U_t}{\partial n_{h,t}}, \quad (15)$$

$$\left[\left[a_h \frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t) \right] \frac{\partial U_t}{\partial n_{h,t}} + \sigma_{q,t} \frac{\partial^2 U_t}{(\partial n_{h,t})^2} \sigma_{n_{h,t}} n_{h,t} + \sigma_{q,t} \frac{\partial^2 U_t}{\partial J_t \partial n_{h,t}} \sigma_{J,t} J_t \right] k_t = \mathbf{16}$$

The intra-temporal condition between consumption and labor laid out in the paper follows from combining the first two F.O.C.s. The other two optimality conditions in the paper follow from applying the same methodology as in [Cox, Ingersoll and Ross \(1985\)](#). Specifically, first evaluate the F.O.C.s in (11); second, differentiate the resulting expression with respect to $n_{h,t}$; third, rearrange the resulting expression accordingly to obtain the inter-temporal condition between consumption and deposits, $-\mu_{\Lambda,t} = (i_t - \pi_t)$, and the optimal capital choice

$$\bar{k}_{h,t} \begin{cases} \in \mathbb{R}_+ & \text{if } RR_{h,t} = 0 \\ = 0 & \text{if } RR_{h,t} < 0 \end{cases}, \quad (17)$$

with

$$RR_{h,t} \equiv a_h \frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t) + \sigma_{\Lambda,t} \sigma_{q,t}, \quad (18)$$

defined the same as in the paper. (I do not consider $RR_{h,t} > 0$ because that case cannot

arise in equilibrium.)

1.2 Aggregate Production Function and Labor Wedge

To economize in notation, in what follows, I make no distinction between individual and aggregate variables.

The market clearing conditions for labor and capital services are

$$\int_{-\infty}^t \theta e^{-\theta(t-s)} l_{s,t} = l_t, \quad (19)$$

$$\int_{-\infty}^t \theta e^{-\theta(t-s)} k_{s,t} = a_h \bar{k}_{h,t} + \bar{k}_{f,t}, \quad (20)$$

with $l_{s,t}$ and $k_{s,t}$ being the quantities demanded for those inputs, respectively, of a firm that had the opportunity to reset their nominal price for the last time at $s \leq t$. Those quantities satisfy

$$l_{s,t} = \frac{1}{A_t} \left(\frac{\alpha}{1-\alpha} \frac{r_{k,t}}{w_t} \right)^{1-\alpha} y_{d,t}(p_{*,s}) = \frac{1}{A_t} \left(\frac{\alpha}{1-\alpha} \frac{r_{k,t}}{w_t} \right)^{1-\alpha} \left(\frac{p_{*,s}}{p_t} \right)^{-\varepsilon} y_t, \quad (21)$$

$$k_{s,t} = \frac{1}{A_t} \left(\frac{1-\alpha}{\alpha} \frac{w_t}{r_{k,t}} \right)^{\alpha} y_{d,t}(p_{*,s}) = \frac{1}{A_t} \left(\frac{1-\alpha}{\alpha} \frac{w_t}{r_{k,t}} \right)^{\alpha} \left(\frac{p_{*,s}}{p_t} \right)^{-\varepsilon} y_t, \quad (22)$$

with the first equality resulting from the cost minimization problem of firms, and the second resulting from substituting for $y_{d,t}(p_{j,t}) \equiv (p_{j,t}/p_t)^{-\varepsilon} y_t$.

I substitute (21) in (19) and (22) in (20). I obtain

$$\frac{1}{A_t} \left(\frac{\alpha}{1-\alpha} \frac{r_{k,t}}{w_t} \right)^{1-\alpha} \omega_t y_t = l_t, \quad (23)$$

$$\frac{1}{A_t} \left(\frac{1-\alpha}{\alpha} \frac{w_t}{r_{k,t}} \right)^{\alpha} \omega_t y_t = a_h \bar{k}_{h,t} + \bar{k}_{f,t}, \quad (24)$$

with $\omega_t = \int_{-\infty}^t \theta e^{-\theta(t-s)} \left(\frac{p_{*,s}}{p_t} \right)^{-\varepsilon} ds$, as in the paper. Combining these two equations, I obtain the aggregate production function:

$$y_t = \zeta_t A_t l_t^{\alpha} \bar{k}^{1-\alpha}, \quad \text{with } \zeta_t \equiv \frac{a_t^{1-\alpha}}{\omega_t} \leq 1, \quad (25)$$

with $a_t \equiv a_h \bar{k}_{h,t}/\bar{k} + \bar{k}_{f,t}/\bar{k}$ also being the same as in the paper. By first dividing (23) over

(24) and then combining the resulting expression with $\frac{1}{y_t}w_t = \chi l_t^\psi$, it follows that

$$r_{k,t} = \left(\frac{l_t}{l_E}\right)^{1+\psi} (1-\alpha) \frac{y_t}{k_t}, \quad (26)$$

with $l_E \equiv (\alpha/\chi)^{\frac{1}{1+\psi}}$ again the same as in the paper. And from combining (26) with the ratio of (23) to (24), it follows that

$$w_t = \left(\frac{l_t}{l_E}\right)^{1+\psi} \alpha \frac{y_t}{l_t}. \quad (27)$$

This last expression implies that $\left(\frac{l_E}{l_t}\right)^{1+\psi} = \frac{\alpha y_t/l_t}{w_t}$ is the labor wedge.

1.3 Aside—Inflation and Price Dispersion

Here, I prove the results stated in footnotes 6, 9, and 17 of the paper. These proofs are not required to solve the model.

1.3.1 Socially Optimal Inflation Rate in the Economy without Funding Frictions or Macro-Prudential Policy Intervention

In this economy, $\bar{k}_{f,t}/\bar{k} = \phi_t \eta_t = 1$ is efficient, and so is $a_t = a_h (1 - \phi_t \eta_t) + \phi_t \eta_t = 1$. The labor wedge $\left(\frac{l_E}{l_t}\right)^{1+\psi}$ and price dispersion ω_t , in general, however, are inefficient. Here, I show that if $\pi_t = \pi_{E,t} \equiv \frac{\theta}{\varepsilon-1} (1 - \omega_t^{\varepsilon-1})$ always, then $\left(\frac{l_E}{l_t}\right)^{1+\psi} = 1$ on impact and $\omega_t \rightarrow 1$ at the fastest possible pace regardless of the initial ω . This implies a trade-off during the convergence toward $\omega_t = 1$ between inflation stability and macroeconomic stability.

Labor Wedge In equilibrium, optimal real price $p_{*,t}/p_t$ satisfies

$$\frac{p_{*,t}}{p_t} = \frac{E_t \int_t^\infty \theta \exp \left\{ \int_t^s - [(\theta + \rho) - \varepsilon \pi_{\bar{s}}] d\bar{s} \right\} \frac{x_s(y_j)}{y_j} ds}{E_t \int_t^\infty \theta \exp \left\{ \int_t^s - [(\theta + \rho) - (\varepsilon - 1) \pi_{\bar{s}}] d\bar{s} \right\} ds}. \quad (28)$$

This follows from substituting $y_{d,s}(p_t) = (p_t/p_s)^{-\varepsilon} y_s$, $\Lambda_t = e^{-\rho t}/y_t$, and $p_t/p_s = \exp \left\{ - \int_t^s \pi_{\bar{s}} d\bar{s} \right\}$ into the F.O.C. of the problem of firms—expression (10) in the paper.

If $\pi_t = \pi_{E,t}$ always, because $\pi_t \equiv \frac{\theta}{\varepsilon-1} \left[1 - \left(\frac{p_{*,t}}{p_t} \right)^{-(\varepsilon-1)} \right]$, the LHS in (28) equals $\frac{1}{\omega_t}$.

And because $\mu_{\omega,t} \equiv \theta \left[\left(\frac{p_{*,t}}{p_t} \right)^{-\varepsilon} \frac{1}{\omega_t} - 1 \right] + \varepsilon \pi_t$, then $\mu_{\omega,t} = \pi_{E,t}$ always as well, which

combined with

$$\frac{x_s(y_j)}{y_j} = \left(\frac{l_s}{l_E}\right)^{1+\psi} \frac{1}{\omega_s} = \left(\frac{l_s}{l_E}\right)^{1+\psi} \frac{1}{\omega_t} \exp\left\{-\int_t^s \pi_{F,\tilde{s}} d\tilde{s}\right\} \quad (29)$$

implies that the RHS in that same expression equals

$$\frac{1}{\omega_t} \frac{E_t \int_t^\infty \theta \exp\left\{\int_t^s -[(\theta + \rho) - (\varepsilon - 1)\pi_{F,\tilde{s}}] d\tilde{s}\right\} \left(\frac{l_s}{l_E}\right)^{1+\psi} ds}{E_t \int_t^\infty \theta \exp\left\{\int_t^s -[(\theta + \rho) - (\varepsilon - 1)\pi_{F,\tilde{s}}] d\tilde{s}\right\} ds}. \quad (30)$$

For expression (28) to hold, then, both conditional expectations in (30) must take the same value always. Let $I_{N,t}$ denote the conditional expectation in the numerator and $I_{D,t}$ denote that in the denominator. By first applying Ito's Lemma to $I_{N,t}$ and $I_{D,t}$ and then equalizing the resulting drift processes, it follows that

$$\left(\frac{l_t}{l_E}\right)^{1+\psi} \theta dt + [(\varepsilon - 1)\pi_{E,t} - (\theta + \rho)] I_{N,t} dt + E_t [dI_{N,t}] = \theta dt + [(\varepsilon - 1)\pi_{E,t} - (\theta + \rho)] I_{D,t} dt + E_t [dI_{D,t}]. \quad (31)$$

Because $E_t [dI_{N,t}] = E_t [dI_{D,t}]$, the labor wedge vanishes on impact, that is, $l_t = l_E$ always as well.

Price Dispersion Inflation rate $\pi_{E,t}$ is the rate that minimizes $\mu_{\omega,t}$. Put differently,

$$\pi_{E,t} \equiv \arg \min_{\pi_t} \mu_{\omega,t} = \arg \min_{\pi_t} \left\{ \theta \left[\left(1 - \frac{\varepsilon - 1}{\theta} \pi_t\right)^{\frac{\varepsilon}{\varepsilon - 1}} \frac{1}{\omega_t} - 1 \right] + \varepsilon \pi_t \right\}. \quad (32)$$

Also, inflation rate $\pi_{E,t}$ satisfies

$$\pi_{E,t} = \min_{\pi_t} \mu_{\omega,t} = \min_{\pi_t} \left\{ \theta \left[\left(1 - \frac{\varepsilon - 1}{\theta} \pi_t\right)^{\frac{\varepsilon}{\varepsilon - 1}} \frac{1}{\omega_t} - 1 \right] + \varepsilon \pi_t \right\}. \quad (33)$$

Because $\mu_{\omega,t} = \pi_{E,t} \leq 0$ with "equality" only if $\omega_t = 1$, then price dispersion ω_t vanishes at the fastest possible pace regardless of the initial ω if and only if $\pi_t = \pi_{E,t}$ always.

1.3.2 Inflation and Price Dispersion if the Employment Gap is a Constant

In this subsection, I restrict attention to the case with $l_t = l \in \mathbb{R}_+$. First, I consider stationary equilibria with constants ω , $p_{*,t}/p_t$, and π . I show that within that class of equilibria, if $\ln(l/l_E) \geq 0$, then the equilibrium exists and it is unique, and that if $\ln(l/l_E) <$

0, then either no, one, or two equilibria exist depending on whether $\ln(l/l_E)$ is far away from or close to zero. Afterward, I verify numerically that if $\ln(l/l_E) \geq 0$, the equilibrium is stable, and that if $\ln(l/l_E) < 0$, only one is—for the case in which equilibria exist. Lastly, I restrict attention to stable stationary equilibria, and I verify numerically that price dispersion is asymmetric around the equilibrium with $\pi = 0$.

Existence and Number From (28) it follows that, in a stationary equilibrium, optimal real price $p_{*,t}/p_t$ satisfies

$$\frac{p_{*,t}}{p_t} = \frac{\rho + \theta - (\varepsilon - 1)\pi}{\rho + \theta - \varepsilon\pi} \left(\frac{l}{l_E}\right)^{1+\psi} \frac{1}{\omega}. \quad (34)$$

And from $\mu_{\omega,t} = 0$ it follows that in that same equilibrium, price dispersion ω satisfies

$$\frac{1}{\omega} = \left(1 - \frac{\varepsilon}{\theta}\pi\right) \left(\frac{p_{*,t}}{p_t}\right)^\varepsilon. \quad (35)$$

These imply

$$\left(\frac{p_{*,t}}{p_t}\right)^{-(\varepsilon-1)} = \frac{\rho + \theta - (\varepsilon - 1)\pi}{\rho + \theta - \varepsilon\pi} \left(\frac{l}{l_E}\right)^{1+\psi} \left(1 - \frac{\varepsilon}{\theta}\pi\right). \quad (36)$$

By combining this last expression with the definition of inflation—equation (12) in the paper—it follows that

$$1 - \left(\frac{l}{l_E}\right)^{1+\psi} = \frac{\rho\pi}{-(\varepsilon - 1)\varepsilon\pi^2 + [(\rho + \theta)\varepsilon + \theta(\varepsilon - 1)]\pi - \theta(\rho + \theta)}, \quad (37)$$

which specifies an equilibrium correspondence between l/l_E and π .

If $\ln(l/l_E) = 0$, the equilibrium is unique, and it is given by $\pi = 0$. If $\ln(l/l_E) > 0$, the equilibrium is also unique, but it is given by a positive $\pi > 0$. This is because the slope of the quadratic equation in the denominator on the RHS is negative and the roots of that equation are $\frac{\theta}{\varepsilon}$ and $\frac{\rho+\theta}{\varepsilon-1}$. (Note that the equilibrium restricts $\pi < \frac{\theta}{\varepsilon}$, as otherwise (36) would imply a nonpositive optimal real price $p_{*,t}/p_t$.) Lastly, if $\ln(l/l_E) < 0$, either no, one, or two equilibria exist, depending on whether $\ln(l/l_E)$ is far away from or close to zero. This is because in this case, inflation $\pi < 0$ has to be negative, and because in region $\pi \in (-\infty, 0)$ the RHS is strictly concave in π and attains its global maximum at $\pi_{\max} \equiv -\sqrt{\frac{\theta}{\varepsilon} \frac{\rho+\theta}{\varepsilon-1}} < 0$. If $\ln(l/l_E) \lesssim 0$ is sufficiently close to zero, then one equilibrium has low deflation $\pi \lesssim 0$, while the other instead has high deflation $\pi \ll 0$.

Stability Let $I_{1,t}$ denote the time integral in the numerator on the RHS of (28) and $I_{2,t}$ denote that in the denominator. I conjecture that these integrals can be represented as a function only of state ω_t . That is, $I_{1,t} = I_1(\omega_t)$ and $I_{2,t} = I_2(\omega_t)$ for some real-valued functions I_1 and I_2 . This implies that $I_{1,t}$ and $I_{2,t}$ do not depend on state η_t , which allows me to drop the conditional expectations in that numerator and denominator. Another key implication is that $I_{1,t}$ and $I_{2,t}$ evolve over time deterministically, according to

$$\dot{I}_{1,t} = -\theta \left(\frac{l}{l_E} \right)^{1+\psi} \frac{1}{\omega_t} + (\rho + \theta - \varepsilon \pi_t) I_{1,t}, \quad (38)$$

$$\dot{I}_{2,t} = -\theta + [\rho + \theta - (\varepsilon - 1) \pi_t] I_{2,t}, \quad (39)$$

with

$$\pi_t = \frac{\theta}{\varepsilon - 1} \left[1 - \left(\frac{I_{1,t}}{I_{2,t}} \right)^{-(\varepsilon-1)} \right]. \quad (40)$$

I further conjecture that I_1 and I_2 are continuously differentiable, which implies $\dot{I}_{1,t} = I_1'(\omega_t) \dot{\omega}_t$ and $\dot{I}_{2,t} = I_2'(\omega_t) \dot{\omega}_t$. All of these allows me to express dynamic system (38)-(40) as a first-order ODEs for $\{I_1, I_2\}$ in state ω . (Note that $\dot{\omega}_t = \mu_{\omega,t} \omega_t$.) To pin down a unique solution to the ODEs, I use that the RHSs of (38) and (39) equal zero at the stationary equilibria. This is equivalent to imposing $I_1(\omega_{ss}) = I_{1,ss}$ and $I_2(\omega_{ss}) = I_{2,ss}$, with subindex ss indicating values at the stationary equilibria. To verify whether the stationary equilibria is stable or unstable, I check whether $\mu_{\omega}'(\omega_{ss}) < 0$ or $\mu_{\omega}'(\omega_{ss}) > 0$. I solve the ODEs numerically using spectral methods and the parameter values in Table 1 in the paper. For $\ln(l/l_E) \geq 0$, I obtain that the stationary equilibrium is stable, and for $\ln(l/l_E) \lesssim 0$, I obtain that only the stationary equilibrium with the low deflation $\pi \lesssim 0$ is stable.

Asymmetry I restrict attention to stable stationary equilibria because in the paper I analyze equilibria in the invariant distribution. Figure 1 verifies numerically that price dispersion ω is asymmetric around the equilibrium with $\pi = 0$.

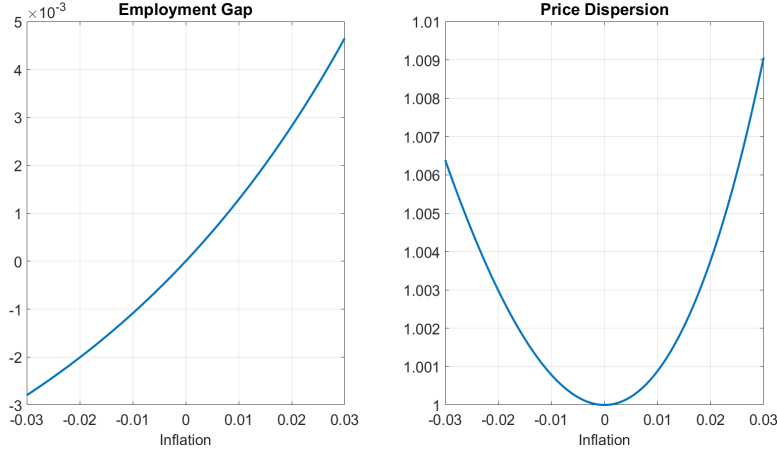


Figure 1: Assymetry of Price Dispersion around Equilibrium with Zero Inflation.

1.4 ODEs and Invariant Distribution

First, I derive the ODEs for the economy with flexible prices, then I do so for the economy with sticky prices. For both economies, first, I consider the case without macro-prudential policy interventions. Then, I derive the equation that analytically characterizes the invariant distribution.

1.4.1 Flexible Price Economy

If $\Phi = +\infty$, under the conjectures made in the paper, the equilibrium pricing condition for physical capital is

$$\begin{cases} RR_h \equiv a_h \frac{r_k}{q} + \mu_q + \mu_\Lambda + \sigma_\Lambda \sigma_q = 0 & \text{if } \eta < \bar{\eta} \\ RR_f \equiv \frac{r_k}{q} + \mu_q + \mu_\Lambda + (\sigma_\Lambda + \sigma_v) \sigma_q = 0 & \text{if } \eta \geq \bar{\eta} \end{cases} \quad (41)$$

with $\bar{\eta} \in (0, 1)$ being such that $\lambda v(\bar{\eta})\bar{\eta} = 1$. The equilibrium pricing condition for value v is

$$RR_f \phi + \frac{\gamma}{v} + \mu_v - \gamma + \sigma_v \sigma_q = 0, \quad (42)$$

with $\phi = \lambda v$ if $\eta < \bar{\eta}$ and $\phi = 1/\eta$ if $\eta \geq \bar{\eta}$.

Ito's Lemma implies

$$\mu_q = \mu_A + \varepsilon_q \mu_\eta + \frac{1}{2} \varepsilon_{q\eta} \varepsilon_q \sigma_\eta^2 + \varepsilon_q \sigma_A \sigma_\eta, \quad (43)$$

$$\sigma_q = \sigma_A + \varepsilon_q \sigma_\eta, \quad (44)$$

$$\mu_v = \varepsilon_v \mu_\eta + \frac{1}{2} \varepsilon_{v\eta} \varepsilon_v \sigma_\eta^2, \quad (45)$$

$$\sigma_v = \varepsilon_v \sigma_\eta, \quad (46)$$

$$\mu_y = \mu_A + \varepsilon_y \mu_\eta + \frac{1}{2} \varepsilon_{y\eta} \varepsilon_y \sigma_\eta^2 + \varepsilon_y \sigma_A \sigma_\eta, \quad (47)$$

$$\sigma_y = \sigma_A + \varepsilon_y \sigma_\eta, \quad (48)$$

$$\mu_\Lambda = -(\rho + \mu_y - \sigma_y^2), \quad (49)$$

$$\sigma_\Lambda = -\sigma_y, \quad (50)$$

with $\varepsilon_\vartheta \equiv \frac{\partial \vartheta}{\partial \eta}$ being the elasticity of mapping ϑ with respect to state η , and $\vartheta_\eta \equiv \frac{\partial \vartheta}{\partial \eta}$ the first-order derivative of ϑ with respect to that same state.

The law of motion of η implies

$$\mu_\eta = \frac{r_k}{q} \phi + (\mu_q + \mu_\Lambda - \sigma_q^2) (\phi - 1) - \gamma + \frac{\kappa}{\eta}, \quad (51)$$

$$\sigma_\eta = (\phi - 1) \sigma_q, \quad (52)$$

with $r_k = (1 - \alpha) \frac{y}{a\bar{k}}$, $y = \zeta A I_E^\alpha \bar{k}^{1-\alpha}$, $\zeta = a^{1-\alpha}$, and $a = a_h (1 - \phi\eta) + \phi\eta$.

I substitute (44) into (52) to express σ_η as a function of the parameters in the model, state η , mapping q , and the first- and second-order derivatives of q . I obtain

$$\sigma_\eta = \frac{\phi - 1}{1 - (\phi - 1) \varepsilon_q} \sigma_A, \quad (53)$$

which allows me to express σ_q , σ_v , σ_y , and σ_Λ as a function of those same objects and of v and the derivatives of v . I now substitute (43), (47), and (49) into (51) to express μ_η as a function of those same objects. I obtain

$$\begin{aligned} \mu_\eta = & \frac{1}{1 - (\varepsilon_q - \varepsilon_y) (\phi - 1)} \times \\ & \left[\frac{r_k}{q} \phi + \left[\mu_A + \frac{1}{2} \varepsilon_{q\eta} \varepsilon_q \sigma_\eta^2 + \varepsilon_q \sigma_A \sigma_\eta - \left(\rho + \mu_A + \frac{1}{2} \varepsilon_{y\eta} \varepsilon_y \sigma_\eta^2 + \varepsilon_y \sigma_A \sigma_\eta - \sigma_y^2 \right) - \sigma_q^2 \right] (\phi - 1) - \gamma + \frac{\kappa}{\eta} \right], \end{aligned} \quad (54)$$

which allows me to express μ_q , μ_v , μ_y , and μ_Λ also as a function of those objects.

Conditions (41)-(50), together with (53) and (54), determine an implicit second-order ODEs for $\{\frac{1}{A}q, v\}$ in η . To pin down a unique solution to the ODEs, I impose boundary conditions similar to those in [Maggiore \(2017\)](#)—who develops a banking autarky economy in which the portfolio problem of financial intermediaries is the same as in this model except for the specifics of the IC constraint. Specifically, I impose

$$\lim_{\eta \rightarrow 1} \sigma_q = \sigma_A, \quad \lim_{\eta \rightarrow 1} \frac{\partial \sigma_q}{\partial \eta} = 0, \quad \lim_{\eta \rightarrow 1} \sigma_v = 0, \quad \lim_{\eta \rightarrow 1} \frac{\partial \sigma_v}{\partial \eta} = 0, \quad (55)$$

which could be interpreted as amplification factors in σ_q and σ_v vanishing smoothly as financial intermediaries as a whole accumulate all of the total wealth in the economy.

Financially Regulated Economy I restrict attention to limits $\Phi < +\infty$ that are binding only when $\eta \in (\eta_L, \eta_H)$, with $0 < \eta_L < \bar{\eta} < \eta_H < 1$. Also, I consider only Φ that are strictly decreasing in interval (η_L, η_H) , with $\Phi(\eta_L) = \lambda v(\eta_L)$ and $\Phi(\eta_H)\eta_H = 1$. The equilibrium pricing condition for physical capital is

$$\begin{cases} RR_h \equiv a_h \frac{r_k}{q} + \mu_q + \mu_\Lambda + \sigma_\Lambda \sigma_q = 0 & \text{if } \eta < \eta_H \\ RR_f \equiv \frac{r_k}{q} + \mu_q + \mu_\Lambda + (\sigma_\Lambda + \sigma_v) \sigma_q = 0 & \text{if } \eta \geq \eta_H \end{cases}. \quad (56)$$

The rest of the derivation is the same as in the laissez-faire economy, but with $\phi = \lambda v$ if $\eta \leq \eta_L$; $\phi = \Phi < \min\{\lambda v, 1/\eta\}$ if $\eta \in (\eta_L, \eta_H)$; and $\phi = 1/\eta$ if $\eta \geq \eta_H$.

1.4.2 Sticky Price Economy

I restrict attention to mappings l and Φ that do not depend on ω . I conjecture that neither do $\frac{\omega}{A}q$ or v . Ito's Lemma then implies that $\mu_q + \mu_\Lambda$ also does not depend on ω . This is because $y = \zeta A l^\alpha \bar{k}^{1-\alpha}$, with $\zeta \equiv \frac{a^{1-\alpha}}{\omega}$, and because households have logarithmic preferences for consumption. Ito's Lemma also implies that neither do σ_q or σ_Λ depend on ω . And this is because ω evolves over time deterministically according to $d\omega/\omega = \mu_\omega dt + 0dZ$. All of these results combined imply that neither do expressions (41)-(54) depend on ω —which ensures that $\{\frac{\omega}{A}q, v\}$ is the solution to the same ODEs in η as in the laissez-faire economy but with $y = a^{1-\alpha} A l^\alpha \bar{k}^{1-\alpha}$ and $r_k = \left(\frac{l}{l_E}\right)^{1+\psi} (1-\alpha) \frac{y}{a\bar{k}}$. The same logic applies if, instead, $\Phi < +\infty$ is occasionally binding in the manner described above.

1.4.3 Invariant Distribution

Invariant density function g is the solution to a Kolmogorov-Chapman forward equation. In the flexible price economy, the equation is

$$-\frac{\partial}{\partial \eta} [\mu_\eta \eta g] + \frac{\partial^2}{\partial \eta^2} [(\sigma_\eta \eta)^2 g] = 0. \quad (57)$$

This implies that mapping g satisfies

$$g(\eta) \propto \frac{1}{(\sigma_\eta \eta)^2} \exp \left\{ 2 \int_0^\eta \frac{\mu_{\tilde{\eta}} \tilde{\eta}}{(\sigma_{\tilde{\eta}} \tilde{\eta})^2} d\tilde{\eta} \right\} \text{ with } \int_0^1 g(\eta) d\eta = 1. \quad (58)$$

In the sticky price economy, the equation is

$$-\frac{\partial}{\partial \omega} [\mu_\omega \omega g] - \frac{\partial}{\partial \eta} [\mu_\eta \eta g] + \frac{\partial^2}{\partial \eta^2} [(\sigma_\eta \eta)^2 g] = 0. \quad (59)$$

1.5 Forward-Looking Equation and PDEs for Inflation

The forward-looking equation follows from substituting $\pi = \frac{\theta}{\varepsilon - 1} \left[1 - \left(\frac{p_*}{p} \right)^{-(\varepsilon - 1)} \right]$ into the LHS in (28) and from noting that ω evolves in accord with $\mu_\omega = \theta \left[\left(\frac{p_*}{p} \right)^{-\varepsilon} \frac{1}{\omega} - 1 \right] + \varepsilon \pi$. The PDEs is a variant of the forward-looking equation. Specifically, let N_t denote the numerator in (28) and let M_t denote the denominator. These two processes are Ito integrals; therefore, by Ito's Lemma:

$$\theta \frac{1}{\omega_t} \left(\frac{l_t}{l_E} \right)^{1+\psi} \frac{1}{N_t} dt + [\varepsilon \pi_t - (\theta + \rho)] dt + E_t \left[\frac{dN_t}{N_t} \right] = 0, \quad (60)$$

$$\theta \frac{1}{M_t} dt + [(\varepsilon - 1) \pi_t - (\theta + \rho)] dt + E_t \left[\frac{dM_t}{M_t} \right] = 0. \quad (61)$$

In the Markov equilibrium, neither do N nor M depend on state A . Drift processes $\mu_{N,t} dt \equiv E_t [dN_t/N_t]$ and $\mu_{M,t} dt \equiv E [dM_t/M_t]$ thus satisfy

$$\mu_N = \tilde{\varepsilon}_N \mu_\omega + \varepsilon_N \mu_\eta + \frac{1}{2} \varepsilon_{N\eta} \varepsilon_N \sigma_\eta^2, \quad (62)$$

$$\mu_M = \tilde{\varepsilon}_M \mu_\omega + \varepsilon_M \mu_\eta + \frac{1}{2} \varepsilon_{M\eta} \varepsilon_M \sigma_\eta^2, \quad (63)$$

with $\tilde{\varepsilon}_\vartheta \equiv \frac{\partial \vartheta}{\partial \omega} \omega$ being the elasticity of mapping ϑ with respect to state ω . From substituting (62) and (63) into (60) and (61), a PDEs for $\{N, M\}$ in (ω, η) follows. (Recall that $p_*/p = N/M$.) This PDEs analytically characterizes inflation, $\pi = \frac{\theta}{\varepsilon-1} \left[1 - \left(\frac{N}{M}\right)^{-(\varepsilon-1)} \right]$.

2 Rationalization for Productivity Edge

In this simple model, neither financial intermediaries nor households own physical capital; rather, a continuum of competitive producers own it. These producers are all identical, transform physical capital into capital services at a linear rate ξ , and rent these services to firms short term, from $(t, t + dt)$. Each producer owns a single unit of the aggregate capital stock \bar{k} , which in this simple model is not tradable. Each pays out all of her profit flows on the spot as dividends of her outstanding equity shares. Equity shares are traded in fully liquid markets at a real price q_t (which in the model in the paper represents the real price of physical capital). They can be owned only by financial intermediaries or households.

Productivity rate $\xi \in \{0, \xi_H\}$ is idiosyncratic and stochastic. In particular, ξ can take only two values: zero $\xi = 0$ or $\xi = \xi_H > 1$ a positive value above 1. By exerting effort, producers increase the probability of $\xi_H > 1$ from $p_H > 0$ to $P_H > p_H$. But exerting effort is costly for them, as it entails forgone private benefits $\beta > 0$ per unit of capital services rented out. I consider $P_H \xi_H - \frac{P_H}{P_H - p_H} \beta < 0$, which rules out contracting as a solution to the moral hazard problem. However, I incorporate the possibility of monitoring, which eliminates the private benefit of producers in full. If done by households, monitoring is costly, in particular, it reduces ξ_H to $a_h \xi_H$, with $a_h < 1$. If done by financial intermediaries, monitoring is costless. For monitoring to play a role, I assume that financial intermediaries cannot monitor on behalf of households and that monitoring is efficient. The latter assumption requires $P_H \xi_H a_h - \beta > 0$. I further assume $P_H \xi_H a_h > p_H$ so that households also prefer to monitor. Financial intermediaries and households can each monitor a continuum of producers.

In equilibrium, both financial intermediaries and households monitor the entire pool of producers they own equity shares of. But while financial intermediaries obtain a dividend return of $P_H \xi_H$ on those shares, households obtain a dividend return of $P_H \xi_H a_h < P_H \xi_H$. I normalize $P_H \xi_H$ to 1.

This simple model, then, rationalizes the productivity edge in the paper, $1 - a_h$, as resulting from differences in monitoring technologies between financial intermediaries and households, with monitoring's being useful for mitigating moral hazard problems between producers and their shareholders.

References

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