Online Appendix:
"Slow Debt, Deep Recessions"

Joachim Jungherr
University of Bonn

Immo Schott
Université de Montréal and
CIREQ

July 2020

This Online Appendix contains additional results and details regarding the empirical facts presented in Section I of the paper (Online Appendix A), additional details on the dynamic business cycle model of Section III (Online Appendix B), as well as a brief overview of the empirical literature on seniority structures and debt covenants (Online Appendix C).

A. Empirical Facts

In Appendix A.1, we provide additional results on the dynamic co-movement between output and firm credit, and output and firm assets. Details on data sources follow in Appendix A.2.

A.1. Additional Results

Figure A.1 repeats the exercise from Figure 2 in the main text using quarterly data. The bars to the left of Figure A.1 show pairwise correlations between total firm credit growth in quarter t and GDP growth in quarter $t + x$. The bars to the right show the corresponding correlations between corporate debt and corporate value added. The results confirm the slow-moving behavior of debt. The correlations peak at a lag of five to six quarters (firm credit vs. GDP growth) and seven quarters (corporate credit vs. corporate value added), respectively.

For completeness, we also calculate the dynamic co-movement between firm assets and output. Figure A.2 shows annual growth rates of real firm assets (book value, marked-to-market) together with annual growth of real GDP. In contrast to firm debt, firm assets do not lag output growth. This is confirmed by Figure A.3 which displays

pairwise correlations between asset growth (both marked-to-market and at historical cost) and output growth at various time lags.
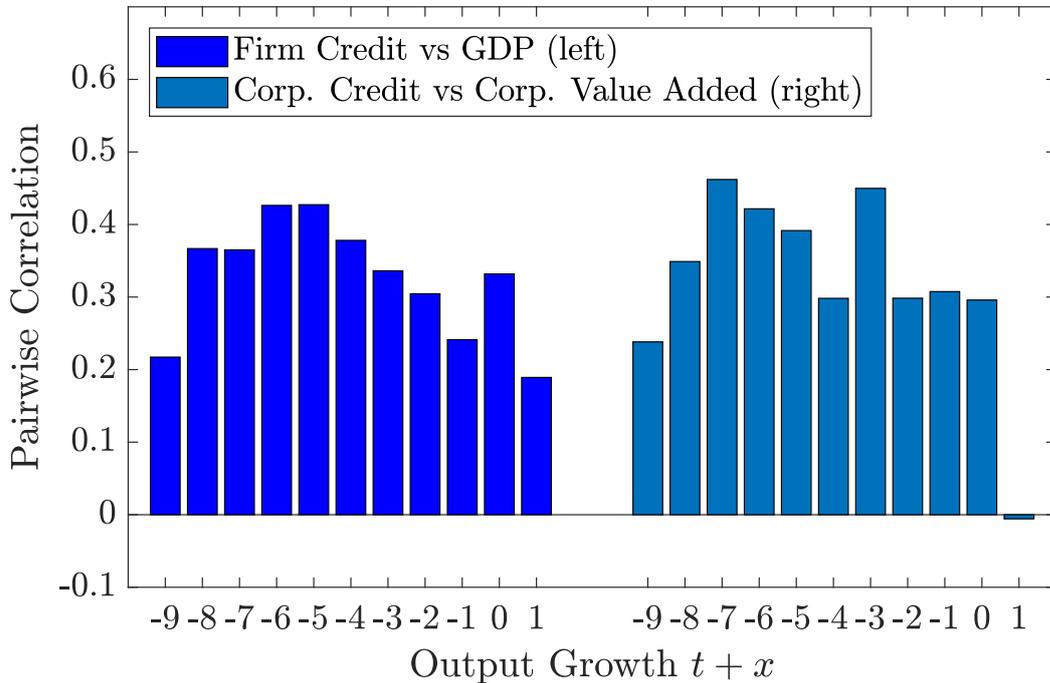

Figure A.1: Correlations Firm Credit Growth t with Output Growth $t + x$ (Flow of Funds)

Note: Bars show pairwise correlations. The left bars show correlations between quarterly growth of real total debt of non-financial firms at the end of period t and real GDP growth in period $t + x$. The right bars show correlations between quarterly growth of real total debt of non-financial corporate firms at the end of period t and real growth of non-financial corporate value added in period $t + x$. All variables are seasonally adjusted. Data is from the Flow of Funds 1984-2015.

A.2. Data Sources

The data used in Section I of the main text is from the Flow of Funds Accounts of the US Federal Reserve Board and from Compustat. Consumer price data comes from the US Bureau of Labor Statistics.

A.2.1. Flow of Funds Data

Annual data retrieved from the Flow of Funds: *GDP* is 'Gross domestic product' (Flow of Funds code FU086902005.A). *Firm Credit* is 'Nonfinancial business; debt securities and loans; liability' (FL144104005.A). *Leverage* is *Firm Credit* divided by the sum of 'Nonfinancial corporate business; total assets' (FL102000005.A) and 'Nonfinancial noncorporate business; total assets' (FL112000005.A). These variables measure assets
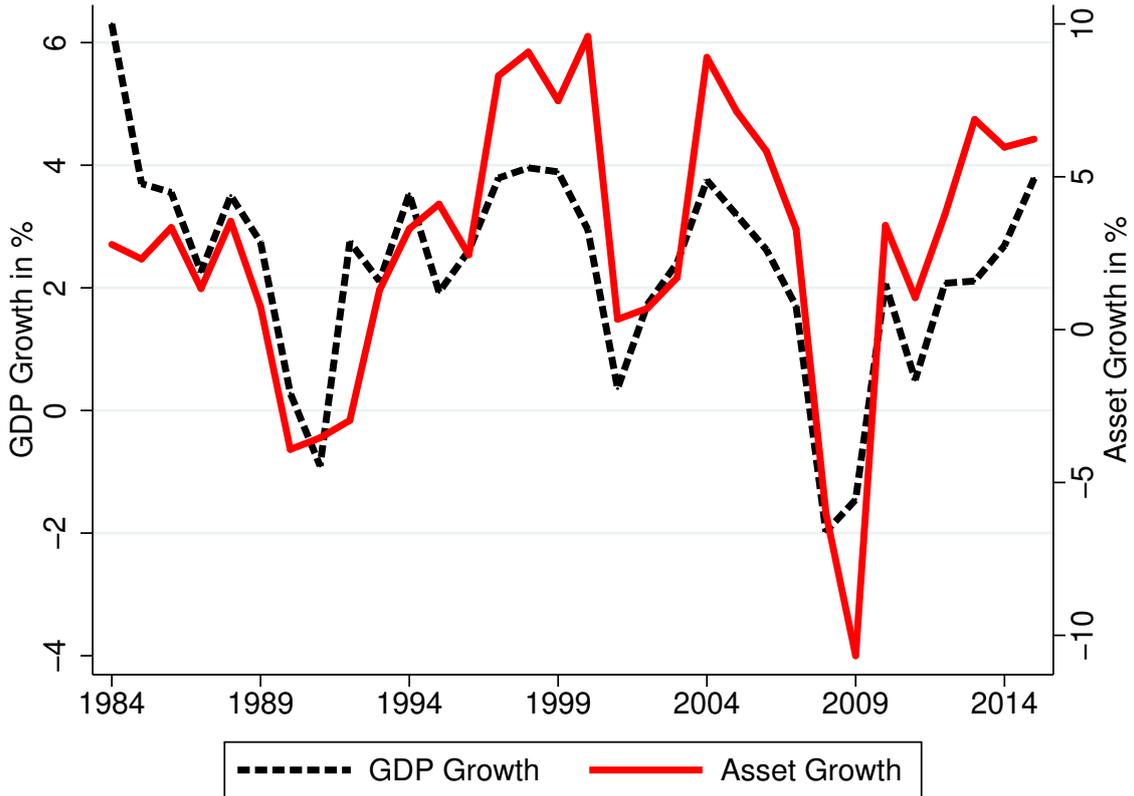
Figure A.2: Firm Asset Growth and GDP

Note: Asset Growth (solid red line, right axis) is annual growth of end-of-year real total assets (book value, marked-to-market) of non-financial firms. *GDP Growth* (dashed black line, left axis) is annual growth of real GDP. Data comes from the Flow of Funds.

at book value and marked-to-market. For the book value at historical cost, we use 'Nonfinancial corporate business; total assets at historical cost' (FL102000115.A). *Corporate Credit* is 'Nonfinancial corporate business; debt securities and loans; liability' (FL104104005.A). *Corporate Value Added* is 'Nonfinancial corporate business; gross value added' (FU106902501.A).

Annual flow variables are deflated using the annual 'CPI-All Urban Consumers' from the Bureau of Labor Statistics. End-of-year stock variables are deflated using the seasonally adjusted December value of the 'CPI-All Urban Consumers'.

Figure A.1 uses seasonally adjusted quarterly Flow of Funds data. *GDP* is 'Gross domestic product' (FA086902005.Q). *Firm Credit* is 'Nonfinancial business; debt securities and loans; liability' (FL144104005.Q). *Corporate Credit* is 'Nonfinancial corporate business; debt securities and loans; liability' (LA104104005.Q). *Corporate Value Added* is 'Nonfinancial corporate business; gross value added' (FA106902501.Q).

Quarterly flow variables are deflated using the quarter average of the seasonally adjusted monthly 'CPI-All Urban Consumers'. End-of-quarter stock variables are deflated
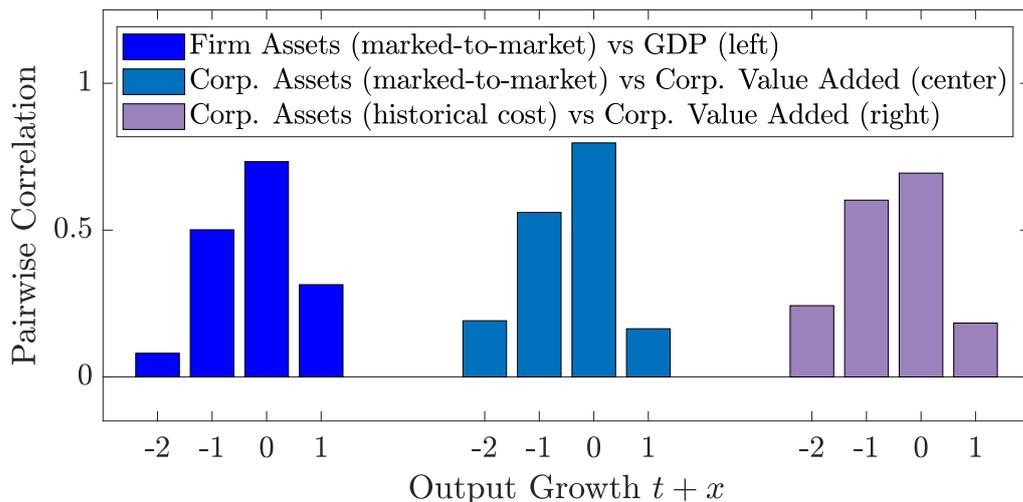
Figure A.3: Correlations Firm Asset Growth t with Output Growth $t+x$ (Flow of Funds)

Note: Bars show pairwise correlations. The left bars show correlations between annual growth of real total assets (book value, marked-to-market) of non-financial firms at the end of year t and real GDP growth in year $t+x$. The bars in the center show correlations between annual growth of real total assets (book value, marked-to-market) of non-financial corporate firms at the end of year t and real growth of non-financial corporate value added in year $t+x$. The bars to the right show the corresponding values for real total assets measured at historical cost instead of marked-to-market. Data comes from the Flow of Funds 1984-2015.

using the end-of-quarter value of the seasonally adjusted monthly 'CPI-All Urban Consumers'.

A.2.2. Compustat Data

We use firm-level data from Compustat 1984-2015. To facilitate comparison with the Flow of Funds data, we only include Compustat firm-year observations which are reported in December of a given year. We also exclude financial firms (SIC codes 6000-6999) as well as firm-year observations with an ISO Currency Code different from US Dollar. Furthermore, we exclude observations with negative *Firm Debt* (annual data item number 34 + data item 9) or *Sales* (data item 12), and those that do not report *Long-term Debt* (data item 9), *Firm Debt*, or *Sales*. The *Long-term Debt Share* is *Long-term Debt* (data item 9) divided by *Firm Debt* (data item 34 + data item 9). Total *Firm Debt* in our Compustat sample is on average about 90% of non-financial corporate debt from the Flow of Funds.

B. Business Cycle Model

In Appendix B, we provide details of our solution method for the business cycle model with long-term and short-term debt (B.1), define key model variables and describe the

construction of their empirical counterparts (B.2), provide results on parameter sensitivity (B.3), and lay out the setup of the frictionless model (*FL Model*, B.4) and the short-term debt model (*STD Model*, B.5) used in Figure 6 of the main text, as well as the constrained efficient allocation used in Figure 7 (*Constr. Eff.*, B.6).

B.1. Solution Method

This appendix presents a detailed description of the computational procedure that is used to find the equilibrium of the benchmark model with long-term and short-term debt laid out in Section III.F of the main text.

We compute the equilibrium of a dynamic open economy business cycle model with a given international risk-free rate r and an endogenous wage w. Due to distortionary taxes, default, and lack of commitment, the equilibrium allocation is inefficient. One cannot directly compute a centralized solution but must solve the decentralized equilibrium allocation. All agents take the factor prices r and w as given. The aggregate state of the economy S consists of aggregate productivity z' and the aggregate stock of existing debt $B : S = (z', B)$. Given the current aggregate state S and a law of motion $S' = F(S)$, agents forecast current and future values of the wage $w(S)$. There is a constant unit mass of ex-ante identical firms. The endogenous state variable of an individual firm is b. In equilibrium, we therefore have: $B' = b' = (1 - \gamma)\tilde{b}^L$.

We find the global solution to the dynamic firm problem in (24) and the equilibrium defined in Section III.F by value function iteration with interpolation. The key difficulty consists in finding the equilibrium price of long-term debt p^L. Optimal firm behavior depends on p^L which itself depends on the expected future price of long-term debt which in turn depends on future firm behavior. We solve this fixed point problem by computing the equilibrium of a finite-horizon economy. Starting from a final date T, we iterate backward until all prices and quantities have converged. We then treat the first-period equilibrium allocation as the equilibrium of the infinite-horizon economy. Given that the continuation value of an individual firm $V((1 - \gamma)\tilde{b}^L, S')$ and the future price of long-term debt $g((1 - \gamma)\tilde{b}^L, S')$ are zero in the final period T, this is a suitable starting point for the iteration process.

The computational procedure is implemented in Matlab. To compute the solution of the firm problem (24), we create grids for the endogenous state of an individual firm b, the endogenous state of the aggregate economy B, and the exogenous state z'. For b, we use a linear grid with $\#_b$ grid points and support $[0, \bar{b}]$, where $\bar{b}$ is set sufficiently high such that $(1 - \gamma)\tilde{b}^L(b, S) < \bar{b}$ for all firm states (b, S). The grid for B is identical. The stochastic process of $\ln z'$ is approximated using a grid with $\#_z$ grid points and a transition matrix Π constructed following the Rouwenhorst method as in Kopecky and Suen (2010). The results presented in the paper are computed using $\#_b = 10$, $\bar{b} = 0.3$, and $\#_z = 5$. This yields a state space (b, S) with $10 \times 10 \times 5 = 500$ grid points. Convergence is typically achieved after about 300 periods. Thanks to interpolation, the computational procedure is robust to variations in $\#_b$ and $\#_z$. For instance, we have carried out computations using $\#_b = 8$, $\#_b = 20$, or $\#_z = 3$. In all cases, the results are highly similar.

The algorithm proceeds as follows:

1. Start at final date T. Set the value function $V_T(b_{iT}, S_T) = 0$ and the price function of long-term debt $g_T(b_{iT}, S_T) = 0$ for all (b_{iT}, S_T).

2. At the end of period $T - 1$, apply the following steps:

   a) Given the aggregate state $S_{T-1} = (z_T, B_{T-1})$, guess aggregate firm capital $K_T(S_{T-1})$.

   b) Given $K_T(S_{T-1})$, aggregate labor demand is found by using firms' first order condition for labor (19): $L_T^d = (\zeta(1-\psi)z_T K_T^{\psi\zeta}/w_T)^{1/(1-\zeta(1-\psi))}$. Aggregate labor supply follows from (26): $L_T^s = w_T^{1/\theta}$. Using labor market clearing, compute the equilibrium wage w_T.

   c) The guess for $K_T(S_{T-1})$ has provided us with an initial guess for $w_T(S_{T-1})$. Given this guess for w_T, solve the firm-level problem (24) for the firm state (b_{iT-1}, S_{T-1}) with $b_{iT-1} = B_{T-1}$.

      - In this final period, no new long-term debt is issued and all existing long-term debt matures at time T: $\tilde{b}_{iT}^L = b_{iT-1}$ with $\gamma = 1$. The firm problem at the end of period $T-1$ can be re-written in terms of only two choice variables: k_{iT} and $\tilde{b}_{iT}^S$. Given z_T, k_{iT}, and w_T, individual labor demand l_{iT} is given by (19). Using z_T, k_{iT}, l_{iT}, $\tilde{b}_{iT}^S$, $\tilde{b}_{iT}^L$, and w_T, we can compute firm output y_{iT}, the asset value in case of default $\underline{q}(\varepsilon_{iT})$, and the threshold value $\bar{\varepsilon}_{iT}$. This determines the default probability $\Phi(\bar{\varepsilon}_{iT}) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\bar{\varepsilon}_{iT}/(\sigma_\varepsilon\sqrt{2})\right)\right]$.

      - Using $\bar{\varepsilon}_{iT}$, $\Phi(\bar{\varepsilon}_{iT})$, $\tilde{b}_{iT}^S$, $\tilde{b}_{iT}^L$, and $\underline{q}_{iT}$, the price of short-term debt p_{iT-1}^S is given by (22). The fact that ε_{iT} is drawn from a continuous probability distribution implies that the threshold value $\bar{\varepsilon}_{iT}$ and the bond price p_{iT-1}^S are continuous as well.

      - Using these constraints, numerically solve for the combination of firm capital k_{iT} and short-term debt $\tilde{b}_{iT}^S$ that maximizes the firm objective in (24). None of the firm choices is restricted to lie on a grid. The dividend payout constraint $\tilde{\underline{e}}$ is set such that it is not binding in equilibrium. The exact value of $\tilde{\underline{e}}$ does not affect equilibrium variables.

        Note that the bond price p_{it}^S is pinned down by the firm's current policy. The equilibrium bond price and firm policy are computed in a single step. It is not necessary to compute bond prices for all possible firm actions in an 'outer loop' before computing optimal firm policy in a subsequent 'inner loop'. Avoiding this 'inner loop-outer loop' procedure reduces the number of necessary computations.

   d) Compare the solution of the firm problem for capital k_{iT} to the guess K_T. Because there is a constant unit mass of ex-ante identical firms, these two must be identical in equilibrium. In this case, aggregate labor supply L_T^s is

equal to aggregate labor demand $L_T^d = l_{iT}$ at the wage w_T. The labor market clears. If the absolute distance between k_{iT} and K_T is below a pre-defined tolerance level, continue to the next step, otherwise update K_T and return to step 2b.

e) Once we have found the equilibrium wage $w(S_{T-1})$ and the solution to the firm problem (24) for the firm state (b_{iT-1}, S_{T-1}) with $b_{iT-1} = B_{T-1}$, we compute the solution to (24) for all firm states (b_{iT-1}, S_{T-1}) with $b_{iT-1} \neq B_{T-1}$. The equilibrium wage $w(S_{T-1})$ is held constant during this step because it only depends on the aggregate state S_{T-1}.

f) Use these results to store the value function $V_{T-1}(b_{iT-1}, S_{T-1})$ and the price function of long-term debt $g_{T-1}(b_{iT-1}, S_{T-1})$ in all firm states (b_{iT-1}, S_{T-1}).

3. In all periods $t < T-1$, apply the following steps. They closely follow the procedure from period $T-1$, with the addition of long-term debt $\tilde{b}_{it+1}^L$ as a new choice variable and the law of motion $S_{t+1} = F_t(S_t)$.

a) Given the aggregate state $S_t = (z_{t+1}, B_t)$, guess aggregate firm capital $K_{t+1}(S_t)$.

b) Given $K_{t+1}(S_t)$, compute the equilibrium wage w_{t+1} as in step 2b above.

c) Guess a value for the future aggregate state B_{t+1}. Together with the stochastic process of z_{t+1}, this yields a candidate law of motion for the aggregate state $S_{t+1} = F_t(S_t)$.

  i. Given the current guess for w_{t+1} and the candidate law of motion $S_{t+1} = F_t(S_t)$, solve the firm-level problem (24) for the firm state (b_{it}, S_t) with $b_{it} = B_t$.

   • The firm problem at the end of period t can be re-written in terms of three choice variables: capital k_{it+1}, short-term debt $\tilde{b}_{it+1}^S$, and long-term debt $\tilde{b}_{it+1}^L$. Compute individual labor demand l_{it+1}, firm output y_{it+1}, and the asset value in case of default $\underline{q}(\varepsilon_{it+1})$. The solution to the equilibrium of period $t+1$ (as computed previously) provides the value function $V_{t+1}((1-\gamma)\tilde{b}_{it+1}^L, S_{t+1})$. Use it together with $S_{t+1} = F_t(S_t)$ to compute the threshold value $\bar{\varepsilon}_{it+1}$ and the default probability $\Phi(\bar{\varepsilon}_{it+1})$. As above, none of the firm choices is restricted to lie on a grid. To compute the exact solution of $\bar{\varepsilon}_{it+1}$, off-grid values of $V_{t+1}((1-\gamma)\tilde{b}_{it+1}^L, S_{t+1})$ are approximated by cubic interpolation.

   • The price of short-term debt p_{it}^S is computed as above. The key equilibrium variable of the model is the price of long-term debt p_{it}^L as given by (23). It not only depends on the firm's current behavior but also on the future price of long-term debt which in turn depends on future firm behavior. The solution to the equilibrium of period $t+1$ (as computed previously) provides the future long-term debt price $g_{t+1}((1-\gamma)\tilde{b}_{it+1}^L, S_{t+1})$. Use it together with $S_{t+1} = F_t(S_t)$ to

7

compute p_{it}^L. To compute the exact solution of p_{it}^L, off-grid values of $g_{t+1}((1-\gamma)\tilde{b}_{it+1}^L, S_{t+1})$ are approximated by cubic interpolation. The fact that ε_{it+1} is drawn from a continuous probability distribution implies that p_{it}^S and p_{it}^L are continuous as well.

- Using these constraints, numerically solve for the combination of k_{it+1}, $\tilde{b}_{it+1}^S$, and $\tilde{b}_{it+1}^L$, that maximizes the firm objective in (24). As above, none of the firm choices is restricted to lie on a grid. The dividend payout constraint $\underline{\tilde{e}}$ is set such that it is not binding in equilibrium. The exact value of $\underline{\tilde{e}}$ does not affect equilibrium variables.

  Note that the equilibrium bond prices p_{it}^S and p_{it}^L are pinned down by the firm's current and future policy. Equilibrium bond prices and firm policy are computed in a single step. It is not necessary to compute bond prices for all possible firm actions in an 'outer loop' before computing optimal firm policy in a subsequent 'inner loop'. Avoiding this 'inner loop-outer loop' procedure reduces the number of necessary computations.

ii. Compare the solution of the firm problem for the future stock of existing debt $b_{it+1} = (1-\gamma)\tilde{b}_{it+1}^L$ to the guess B_{t+1}. Because there is a constant unit mass of ex-ante identical firms, these two must be identical in equilibrium. If the absolute distance between b_{it+1} and B_{t+1} is below a pre-defined level of tolerance, continue to step 3d, otherwise update the guess B_{t+1} and the candidate law of motion $S_{t+1} = F_t(S_t)$, and return to step 3(c)i.

d) Compare the solution of the firm problem for capital k_{it+1} to the guess K_{t+1}. Because there is a constant unit mass of ex-ante identical firms, these two must be identical in equilibrium. In this case, the labor market clears. If the absolute distance between k_{it+1} and K_{t+1} is below a pre-defined tolerance level, continue to the next step, otherwise update K_T and return to step 3c.

e) Once we have found the equilibrium wage $w(S_t)$ and the solution to the firm problem (24) for the firm state (b_{it}, S_t) with $b_{it} = B_t$, we compute the solution to (24) for all firm states (b_{it}, S_t) with $b_{it} \neq B_t$.

f) Use these results to store the value function $V_t(b_{it}, S_t)$ and the price function of long-term debt $g_t(b_{it}, S_t)$ in all firm states (b_{it}, S_t).

g) If the absolute distances between $V_t(b_{it}, S_t)$ and $V_{t+1}(b_{it+1}, S_{t+1})$, and between $g_t(b_{it}, S_t)$ and $g_{t+1}(b_{it+1}, S_{t+1})$ are above a pre-defined level of tolerance, continue with period $t-1$. If the absolute distances are below the tolerance level, the equilibrium allocation is found.

B.2. Model Moments and Empirical Moments

In this section, we define key model variables (B.2.1) and describe the construction of their empirical counterparts (B.2.2 and B.2.3).

Table B.1: Business Cycle Model - Variables

GDP	$y_t - f - \xi \int_{-\infty}^{\bar{\varepsilon}_t} \underline{q}_t \, \varphi(\varepsilon) \, d\varepsilon - H(\tilde{b}_{t+1}^S, \tilde{b}_{t+1}^L, b_{t+1})$
Total debt	$D \equiv \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\tilde{b}^L$
Leverage: Firm debt / Firm assets	D/k
Long-term debt share	$\frac{1}{D}\frac{\gamma+c}{\gamma+r}\frac{1-\gamma}{1+r}\tilde{b}^L$
Macaulay duration	$\frac{1+r}{\gamma+r}$
Default rate	$\Phi(\bar{\varepsilon})$
Short-term spread	$sp^S \equiv \frac{1+c}{p^S} - (1+r)$
Long-term spread	$sp^L \equiv \frac{\gamma+c}{p^L} + 1 - \gamma - (1+r)$
Average credit spread	$\frac{\tilde{b}^S}{\tilde{b}^S+\tilde{b}^L-b} \times sp^S + \frac{\tilde{b}^L-b}{\tilde{b}^S+\tilde{b}^L-b} \times sp^L$
Spread term structure	$sp^L - sp^S$

B.2.1. Model Moments

Table B.1 defines key model variables. Detailed derivations are provided below.

The total amount of firm debt is the present value of future debt payments discounted at the riskless rate r:

$$D = \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{1+r}\tilde{b}^L + (1-\gamma)\frac{\gamma+c}{(1+r)^2}\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\tilde{b}^L + ...$$

$$= \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{1+r}\tilde{b}^L \sum_{j=0}^{\infty}\left(\frac{1-\gamma}{1+r}\right)^j = \frac{1+c}{1+r}\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\tilde{b}^L \tag{A1}$$

The long-term debt share of a given firm is the present value of debt payments due more than one year from today divided by the total amount of firm debt D:

$$\frac{1}{D}\left((1-\gamma)\frac{\gamma+c}{(1+r)^2}\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\tilde{b}^L + ...\right)$$

$$= \frac{1}{D}\frac{\gamma+c}{\gamma+r}\frac{1-\gamma}{1+r}\tilde{b}^L \tag{A2}$$

The Macaulay duration is the weighted average term to maturity of the cash flows from a bond divided by the price:

$$\mu = \frac{1}{p_r^L}\sum_{j=1}^{\infty}j(1-\gamma)^{j-1}\frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{p_r^L}\frac{1+r}{(\gamma+r)^2} \tag{A3}$$

where p_r^L is the price of a riskless long-term bond:

$$p_r^L = \sum_{j=1}^{\infty}(1-\gamma)^{j-1}\frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{r+\gamma} \tag{A4}$$

9

It follows for the Macaulay duration:

$$\mu = \frac{1+r}{\gamma+r} \tag{A5}$$

The short-term spread compares the gross return (in the absence of default) from buying a short-term bond with the riskless rate:

$$\frac{1+c}{p^S} - (1+r) \tag{A6}$$

The long-term spread compares the gross return (in the absence of default and assuming p^L is constant) from buying a long-term bond with the riskless rate:

$$\frac{\gamma+c+(1-\gamma)p^L}{p^L} - (1+r) = \frac{\gamma+c}{p^L} + 1 - \gamma - (1+r) \tag{A7}$$

B.2.2. Empirical Moments Table 2

Leverage and the long-term debt share are from Compustat for the years 1984-2015. We exclude financial firms (SIC codes 6000-6999) and utilities (SIC 4900-4949) as well as firm-year observations with an ISO Currency Code different from US Dollar.

Leverage is the average of the aggregate book value of total debt (annual data item 34 + data item 9) over the aggregate book value of total firm assets (at historical cost, data item 6). The long-term debt share is the average of aggregate debt with remaining term to maturity of more than one year (data item 9) over aggregate total firm debt (data item 34 + data item 9).

The average credit spread is from Adrian et al. (2013), Table 2, who use micro data on new debt issuances of various maturities by US corporations 1998-2010. We target the issuance amount weighted average spread on all loan and bond issuances. The model counterpart is the issuance weighted average of the credit spread on short-term debt and long-term debt as defined in Table B.1.

B.2.3. Empirical Moments Table 3

Annual data on GDP and total debt in Table 3 of the main text is from the Flow of Funds 1984-2015. This is the same data as used in Figure 1 and Figure 2 of the main text (see Appendix A.2 for details). The moments for total debt are calculated using data for all non-financial firms. Results are highly similar if we restrict ourselves to the corporate sector.

Leverage, b/k, and the long-term debt share are calculated using annual Compustat data 1984-2015. To facilitate comparison with the Flow of Funds data, we only include Compustat firm-year observations which are reported in December of a given year. We exclude financial firms (SIC codes 6000-6999) and utilities (SIC 4900-4949) as well as firm-year observations with an ISO Currency Code different from US Dollar. Furthermore, we exclude observations which report negative *Firm Debt* (data item 34 + data

item 9) or *Sales* (data item 12), as well as those which do not report information on *Long-term Debt* (data item 9), *Firm Debt*, or *Sales*.

Leverage is the average of the aggregate book value of total *Firm Debt* over the aggregate book value of total firm assets (at historical cost, data item 6). b/k at the end of year t is the CPI-deflated stock of aggregate debt at the end of year $t-1$ with remaining term to maturity of more than one year (data item 9) divided by CPI-deflated total assets at the end of year t (data item 6). The long-term debt share is the average of aggregate debt at the end of year t with remaining term to maturity of more than one year (data item 9) over aggregate total *Firm Debt* at the end of year t (data item 34 + item 9).

The default rate is from from Giesecke et al. (2014). It denotes the total defaulted value of US corporate debt over total par value at annual frequency (1984-2012).

Data on credit spreads is from the FRED database of the St. Louis Fed 1997-2015. We use this data source because it provides time series on credit spreads broken down by different maturities. *Average Credit Spread* is the ICE BofAML US Corporate Master Option-Adjusted Spread. This is a market capitalization-weighted average of option-adjusted spreads of US investment grade corporate bonds (remaining maturity above one year, minimum amount outstanding of 250 million USD, currently not in default) relative to a spot Treasury curve. The model counterpart is the issuance weighted average of the credit spread on short-term debt and long-term debt.

Spread Term Structure is the difference between the ICE BofAML US Corporate 7-10 Year Option-Adjusted Spread (remaining term to maturity between seven and ten years) and the ICE BofAML US Corporate 1-3 Year Option-Adjusted Spread (remaining term to maturity less than three years). A maturity between seven and ten years roughly matches the average maturity of a long-term bond in our model with $\gamma = 0.1284$. The maturity of less than three years is the shortest maturity available in FRED.

B.3. Parameter Sensitivity

In this section, we provide results on the sensitivity of key model moments with respect to parameter values. Table B.2 and Figure B.4 present results from the benchmark model with long-term and short-term debt (Section III.F) for different sets of parameter values. *Benchmark* corresponds to the parameter values calibrated to US data given in Table 1. In addition, results are shown for four different sets of parameter values. In each case, only the indicated parameter value differs from the values given in Table 1.

- $\gamma = 0.4$: The main benefit of borrowing at long maturities is that fewer bonds need to be issued each period to maintain a given amount of leverage. Using long-term debt therefore saves issuance costs. Increasing the repayment rate of long-term debt from 0.1284 (average Macaulay duration 6.5 years) to 0.4 (average duration 2.4 years) implies that firms need to roll-over their long-term debt at higher frequency which reduces the benefit of borrowing at long maturities. The equilibrium share of long-term debt falls. The lower stock of outstanding long-term debt induces firms to reduce average leverage relative to the *Benchmark* case. The

Table B.2: Parameter Sensitivity - Model Moments

	Benchmark	$\gamma = 0.4$	$\sigma_\varepsilon = 0.75$	$\xi = 0.45$	$\eta = 0$
Leverage: Debt / Assets	29.3%	21.4%	24.1%	33.1%	21.5%
Long-term debt share	75.4%	58.2%	79.0%	66.0%	26.8%
Average credit spread	2.3%	1.8%	3.3%	2.5%	1.7%
Default rate	2.6%	1.9%	3.3%	3.3%	1.8%
GDP volatility	3.1%	2.8%	3.0%	3.2%	2.7%

Note: Each column corresponds to a distinct set of parameter values. *Benchmark* is the equilibrium of the benchmark model with long-term and short-term debt (Section III.F) using the parameter values given in Table 1. All other columns use the same set of parameter values with the exception of the indicated model parameter. *Average credit spread* is the issuance weighted average of the credit spread on short-term debt and long-term debt as defined in Table B.1. *GDP volatility* is the standard deviation of linearly detrended annual ln GDP.
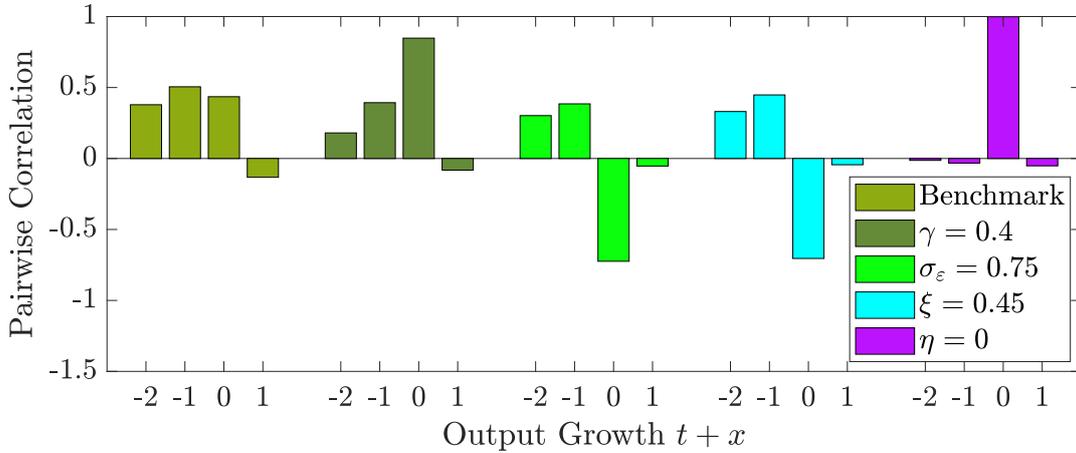

Figure B.4: Parameter Sensitivity - Firm Credit Growth t and Output Growth $t + x$

Note: Bars show pairwise correlations between annual growth of total firm debt at the end of year t and GDP growth in year $t + x$. Each group of bars corresponds to a distinct set of parameter values. *Benchmark* is the equilibrium of the benchmark model with long-term and short-term debt (Section III.F) using the parameter values given in Table 1. All other four-bar groups use the same set of parameter values with the exception of the indicated model parameter.

average default rate and credit spreads fall. As shown in Figure B.4, total firm debt co-moves more strongly with contemporaneous output at the higher value of γ. This reduces the volatility of leverage and credit spreads and thereby lowers GDP volatility from 3.1% to 2.8%.[1]

- $\sigma_\varepsilon = 0.75$: An increase in the standard deviation of the firm-specific capital quality shock ε from 0.6519 to 0.75 leads to higher default risk. Firms respond by reducing average leverage. However, higher default risk increases the sensitivity of the long-term bond price p^L with respect to firm behavior. Firms' incentive to increase leverage and default risk during a downturn at the expense of existing creditors is reinforced. As shown in Figure B.4, the increase in leverage during a downturn can be strong enough for debt to *rise* when output falls. The contemporaneous correlation between debt and output becomes negative.

- $\xi = 0.45$: A reduction in default costs from $\xi = 0.669$ to $\xi = 0.45$ shifts the trade-off between the tax advantage of debt and expected default costs in favor of higher average leverage and default risk. As explained above, higher default risk increases the sensitivity of the long-term bond price p^L with respect to firm behavior. This amplifies the counter-cyclical behavior of leverage and credit spreads and translates into higher GDP volatility.

- $\eta = 0$: A reduction in the debt issuance cost from 0.0077 down to zero implies that debt roll-over is now costless. This reduces the disadvantage of borrowing at short maturities. The equilibrium share of long-term debt falls. The lower stock of outstanding long-term debt leads to reduced average leverage, default risk, and credit spreads. As shown in Figure B.4, the lag in total debt with respect to output disappears. Without 'slow debt', GDP volatility falls to 2.7%.

  As shown in Table B.2, even in the absence of roll-over costs firms issue small positive amounts of long-term debt in this model. In Jungherr and Schott (2020) we show that, ceteris paribus, firms prefer owing a given stock of debt in the form of long-term rather than short-term bonds. The reason is that the positive probability of future default lowers the expected repayment of long-term debt from the firm to existing creditors. Because of default risk, firms discount the future at a higher rate than creditors.

B.4. Business Cycle Model - Frictionless

Figure 6 of the main text displays impulse response functions of a frictionless open economy business cycle model without default costs, taxes, or debt issuance costs. The Modigliani-Miller irrelevance result holds in this environment.

---

[1]For comparison, GDP volatility is 2.6% in the constrained efficient allocation, the frictionless model, and the short-term debt model.

B.4.1. Setup

There is a unit mass of ex-ante identical firms. The production technology is the same as in the benchmark model with long-term debt. Firm earnings are given as

$$z_t \left(k_{it}^{\psi} l_{it}^{1-\psi} \right)^{\zeta} + \varepsilon_{it} k_{it} - w l_{it} - \delta k_{it} - f \tag{A8}$$

The firm-specific idiosyncratic earnings shock ε_{it} is i.i.d. and follows a probability distribution $\varphi(\varepsilon)$ with zero mean. As in the long-term debt model, productivity evolves according to: $\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t$, where ϵ_t is white noise with standard deviation σ_z. Capital k_{it} and labor l_{it} are chosen at the end of period $t-1$ after z_t is realized.

Just as before, there is a representative household with GHH preferences over consumption C_t and labor L_t:

$$u \left(C_t - \frac{L_t^{1+\theta}}{1+\theta} \right), \tag{A9}$$

with $u(\cdot)$ being strictly increasing and concave, and $\theta > 0$.

B.4.2. Optimal Firm Behavior

Conditional on k_{it} and the realization of z_t, an individual firm chooses labor to maximize static profits:

$$l_{it} = \left(\frac{(1-\psi)\zeta z_t k_{it}^{\psi\zeta}}{w_t} \right)^{\frac{1}{1-(1-\psi)\cdot\zeta}} \tag{A10}$$

Optimal capital demand solves:

$$\max_{k_{it}} - k_{it} + \frac{1}{1+r} \int_{-\infty}^{\infty} \left[(1-\delta)k_{it} + z_t \left(k_{it}^{\psi} l_{it}^{1-\psi} \right)^{\zeta} + \varepsilon k_{it} - w l_{it} - f \right] \varphi(\varepsilon) d\varepsilon \tag{A11}$$

subject to (A10). We define the profitability term A_t:

$$A_t = z_t^{\frac{1}{1-(1-\psi)\cdot\zeta}} \cdot \left(\frac{(1-\psi)\cdot\zeta}{w_t} \right)^{\frac{(1-\psi)\cdot\zeta}{1-(1-\psi)\cdot\zeta}} - w_t \left(\frac{(1-\psi)\cdot\zeta}{w_t} \right)^{\frac{1}{1-(1-\psi)\cdot\zeta}} \tag{A12}$$

This implies for optimal capital demand:

$$k_{it} = \left(\frac{A_t \psi \zeta}{(r+\delta)[1-(1-\psi)\zeta]} \right)^{\frac{1-(1-\psi)\zeta}{1-\zeta}} \tag{A13}$$

B.4.3. Equilibrium

Definition: Competitive Equilibrium. Given a realization of z_t, a competitive equilibrium consists of *(i)* quantities of capital k_{it} and labor l_{it}, and *(ii)* a wage rate w_t, such that:

1. k_{it} and labor l_{it} satisfy (A13) and (A10)

2. The labor market clears:
$$w_t^{\frac{1}{\theta}} = l_{it}$$

The parameters r, ζ, ψ, δ, θ, σ_z, and ρ_z are left unchanged with respect to the benchmark model.

B.5. Business Cycle Model - Short-term Debt

Figure 5, Figure 6, and Table 3 in the main text report results for a business cycle model of production, firm financing, and costly default in which firms use only short-term debt. This short-term debt model shares most of the setup with the long-term debt model described in Section III of the main text. A key difference is that firms cannot issue long-term debt now.

B.5.1. Optimal Firm Behavior

Given a realization of aggregate productivity z', an individual firm chooses a policy vector $\{k, l, \tilde{e}, \tilde{b}^S, \bar{\varepsilon}\}$ which solves

$$\max_{\left\{k,l,\tilde{e},\tilde{b}^S,\bar{\varepsilon}\right\}} -\tilde{e} + \frac{1}{1+r} \int_{\bar{\varepsilon}}^{\infty} \left[k - \tilde{b}^S + (1-\tau) \left[y + \varepsilon k - w(z')l - \delta k - f - c\tilde{b}^S\right]\right] \varphi(\varepsilon)d\varepsilon$$

(A14)

s.t.: $\quad y = z' \left(k^\psi l^{1-\psi}\right)^\zeta$

$\quad l = \left(\dfrac{\zeta(1-\psi)z'k^{\psi\zeta}}{w(z')}\right)^{\frac{1}{1-\zeta(1-\psi)}}$

$\quad \bar{\varepsilon}: \quad k - \tilde{b}^S + (1-\tau) \left[y + \bar{\varepsilon}k - w(z')l - \delta k - f - c\tilde{b}^S\right] = 0$

$\quad k = \tilde{e} + p^S \tilde{b}^S$

$\quad p^S = \dfrac{1}{1+r} \left[[1 - \Phi(\bar{\varepsilon})](1+c) + \dfrac{(1-\xi)}{\tilde{b}^S} \int_{-\infty}^{\bar{\varepsilon}} \underline{q} \; \varphi(\varepsilon)d\varepsilon\right]$

B.5.2. Equilibrium

Definition: Competitive Equilibrium. Given a realization of z', a competitive equilibrium consists of *(i)* a firm policy $\{k, l, \tilde{e}, \tilde{b}^S, \bar{\varepsilon}\}$, and *(ii)* a wage rate $w(z')$, such that:

1. $\{k, l, \tilde{e}, \tilde{b}^S, \bar{\varepsilon}\}$ solve the firm problem A14

2. The labor market clears:
$$w(z')^{\frac{1}{\theta}} = l$$

15

Table B.3: Short-term Debt Model - Parametrization

Parameter	Description	Value
f	fixed cost	0.1653
σ_ε	st. dev. idiosyncratic shock	0.67
ξ	default cost	0.194

Most parameters are left unchanged with respect to the benchmark model. We adjust the values of σ_ε and ξ in order to match the same average leverage ratio (29.3%) and the same average credit spread (2.3%) as in the benchmark model. We also change the value of the fixed cost of operation f in order to maintain zero firm profits on average. Table B.3 summarizes all parameter changes with respect to Table 1.

B.6. Business Cycle Model - Constrained Efficiency

The only difference between the recursive competitive equilibrium described in Section III.F of the main text and the equilibrium for the constrained efficient case lies in the nature of the firm problem. The value function $V(b,S)$ in (24) is replaced by the value $W(b,S)$ which solves:

$$W(b,S) = \max_{\phi(b,S)=\left\{k,\tilde{e},\tilde{b}^S,\tilde{b}^L,\bar{\varepsilon}\right\}} p^L b - T(b,S) - \tilde{e}$$

$$+ \frac{1}{1+r}\mathbb{E}_{S'|S}\left\{\int_{\bar{\varepsilon}}^{\infty}\left[q' + W\left((1-\gamma)\tilde{b}^L,S'\right)\right]\varphi(\varepsilon)d\varepsilon\right\} \quad (A15)$$

$$\text{s.t.:} \quad q' = k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)\left[y + \varepsilon k - w(S)l - \delta k - f - c(\tilde{b}^S + \tilde{b}^L)\right]$$

$$y = z'\left(k^\psi l^{1-\psi}\right)^\zeta$$

$$l = \left(\frac{\zeta(1-\psi)z'k^{\psi\zeta}}{w(S)}\right)^{\frac{1}{1-\zeta(1-\psi)}}$$

$$\bar{\varepsilon}: \quad q' + W\left((1-\gamma)\tilde{b}^L,S'\right) = 0$$

$$k = \tilde{e} + p^S\tilde{b}^S + p^L(\tilde{b}^L - b) - H(\tilde{b}^S,\tilde{b}^L,b)$$

$$p^S = \frac{1}{1+r}\mathbb{E}_{S'|S}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}}\underline{q}\,\varphi(\varepsilon)d\varepsilon\right]$$

$$p^L = g(b,S) = \frac{1}{1+r}\mathbb{E}_{z'|z}\left[[1-\Phi(\bar{\varepsilon})]\left[\gamma + c + (1-\gamma)g\left((1-\gamma)\tilde{b}^L,S'\right)\right]\right.$$

$$\left. + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}}\underline{q}\,\varphi(\varepsilon)d\varepsilon\right]$$

The state-contingent tax $T(b,S)$ in (A15) is specified such that in equilibrium: $T(b,S) = p^L b$. This makes sure that $W(b,S)$ differs from the value $V(b,S)$ in the decentralized

model only because of different firm behavior.

C. Empirical Literature on Seniority and Covenants

Market participants try to mitigate the commitment problem generated by risky long-term debt through various contracting features such as seniority structures or debt covenants. While a formal analysis of these instruments is beyond the scope of this paper, in the following we provide a brief overview of the empirical literature on this topic.

Seniority: The majority of U.S. corporate bonds consists of senior unsecured bonds (68%). Subordinated debt makes up for only 5% in value (Gomes et al., 2016), and less than 25% of the number of bond issues (Billett et al., 2007). Secured debt is an alternative way to grant priority to certain debt claims. Secured debt is less than 20% of the number of bond issues (Billett et al., 2007), and less than 20% of the value of issuance (Benmelech et al., 2020). In the cross-section of firms, the share of secured debt is higher for firms with higher default risk (Benmelech et al., 2020).

Covenants: Firms exert a negative externality on existing creditors if they increase default risk by issuing additional debt and by reducing equity injections or increasing dividend payout. The empirical literature finds that less than 25% of U.S. investment grade corporate bonds include covenants which restrict the issuance of additional debt, and less than 20% feature restrictions of firms' dividend policy. Nash et al. (2003) document that 15.66% of 364 investment grade bond issues in 1989 and in 1996 feature restrictions on additional debt. 8.24% include restrictions of the firm's dividend policy.[2] In a sample of 100 bond issues between 1999-2000, Begley and Freedman (2004), Table 2, p. 24, report that 9% contain additional borrowing restrictions. The percentage for dividend restrictions is identical (9%). Billett et al. (2007), Table III, p. 707, calculate that 22.8% of 15,504 investment grade bond issued between 1960 and 2003 had a covenant which restricts future borrowing of identical (or lower) seniority. 17.1% had a covenant which restricts dividend policy.[3] Reisel (2014), Table 4, p. 259, finds in a sample of 4,267 bond issues from 1989 - 2006 that 5.9% of investment grade bonds feature covenants which restrict additional borrowing or the firm's dividend policy. In the cross-section of firms, these covenants are more common for junk bonds than for investment grade bonds (Billett et al., 2007; Green, 2018).

While debt covenants are relatively infrequent for investment grade corporate bonds,

---

[2]Of the 496 bonds considered in their Compustat sample, 120 feature additional debt restrictions (Table 3, p. 218). Of those, 57 bonds are investment grade (Table 4, p.220). It follows that out of a total of 364 investment grade bonds (Table 2, p.216), 15.66% feature additional debt restrictions. Out of the full sample, 99 bonds include restrictions of the firm's dividend policy (Table 3, p. 218). Of those, 30 bonds are investment grade (Table 4, p.220). It follows that 8.24% of the investment grade bonds in the sample feature dividend restrictions.

[3]Future borrowing of identical (or lower) seniority is restricted by funded debt restrictions (4.5%), subordinate debt restrictions (0.8%), and total leverage tests (17.5%). Issuance of secured debt (with effective priority over existing debt) is more frequently restricted than unsecured debt. Dividend policy is restricted by dividend payment restrictions (12.1%) and share repurchase restrictions (5.0%).

they are widely used in bank lending. Roberts and Sufi (2009) document that covenant violations are frequent and that they impact firm behavior. However, the mere existence of a restrictive covenant may not be sufficient to grant protection to lenders. Covenants are frequently weakened by "fine print" clauses (Ivashina and Vallee, 2020), and in about two thirds of all covenant violations creditors take no action and there are no consequences for the borrowing firm (Roberts and Sufi, 2009).

References

ADRIAN, T., P. COLLA, AND H. S. SHIN (2013): "Which financial frictions? Parsing the evidence from the financial crisis of 2007 to 2009," *NBER Macroeconomics Annual*, 27, 159–214.

BEGLEY, J. AND R. FREEDMAN (2004): "The changing role of accounting numbers in public lending agreements," *Accounting Horizons*, 18, 81–96.

BENMELECH, E., N. KUMAR, AND R. RAJAN (2020): "The decline of secured debt," NBER Working Paper No. 26637.

BILLETT, M. T., T.-H. D. KING, AND D. C. MAUER (2007): "Growth opportunities and the choice of leverage, debt maturity, and covenants," *Journal of Finance*, 62, 697–730.

GIESECKE, K., F. A. LONGSTAFF, S. SCHAEFER, AND I. A. STREBULAEV (2014): "Macroeconomic effects of corporate default crisis: A long-term perspective," *Journal of Financial Economics*, 111, 297–310.

GOMES, J., U. JERMANN, AND L. SCHMID (2016): "Sticky leverage," *American Economic Review*, 106, 3800–3828.

GREEN, D. (2018): "Corporate refinancing, covenants, and the agency cost of debt," SSRN Working Paper 3424053.

IVASHINA, V. AND B. VALLEE (2020): "Weak credit covenants," NBER Working Paper No. 27316.

JUNGHERR, J. AND I. SCHOTT (2020): "Optimal debt maturity and firm investment," Https://sites.google.com/site/immoschott/research. Accessed 2020-01-05.

KOPECKY, K. A. AND R. M. SUEN (2010): "Finite state Markov-chain approximations to highly persistent processes," *Review of Economic Dynamics*, 13, 701–714.

NASH, R. C., J. M. NETTER, AND A. B. POULSEN (2003): "Determinants of contractual relations between shareholders and bondholders: Investment opportunities and restrictive covenants," *Journal of Corporate Finance*, 9, 201–232.

REISEL, N. (2014): "On the value of restrictive covenants: Empirical investigation of public bond issues," *Journal of Corporate Finance*, 27, 251–268.

ROBERTS, M. R. AND A. SUFI (2009): "Control rights and capital structure: An empirical investigation," *Journal of Finance*, 64, 1657–1695.