Online Appendix: Distortions and the Structure of the World Economy

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A Derivation of the World's Input-Output Elasticities

In this Appendix, we derive the elasticity of the world's input-output structure with respect to changes in frictions and productivities. We start by deriving the input-output elasticities for the closed economy case with two sectors (equations (9) and (10) in the main text). The share of sector k in total intermediate consumption of sector j (where we have normalized wages to one) is given by

$$\gamma_{ij,ik} = \frac{A_{ik}^{\theta} \tau_{ij,ik}^{-\theta} P_{ik}^{-\theta(1-\beta_{ik})}}{(P_{ij})^{-\theta}}$$

where

$$P_{ij} = \left(A_{ij}^{\theta} P_{ij}^{-\theta(1-\beta_{ij})} + A_{ik}^{\theta} \tau_{ij,ik}^{-\theta} P_{ik}^{-\theta(1-\beta_{ik})} \right)^{-1/\theta}.$$

Totally differentiating the price equation and using the definition of the expenditure shares we get

$$-\theta dlog P_{ij} = \gamma_{ij,ij}(\theta dlog A_{ij} - \theta(1-\beta_{ij}) dlog P_{ij}) + \gamma_{ij,k}(\theta dlog A_{ik} - \theta dlog \tau_{ij,ik} - \theta(1-\beta_{ik}) dlog P_{ik}).$$

Let's define

$$\tilde{\gamma}_{ij,ij} = \frac{\gamma_{ij,ij}}{(1-\gamma_{ij,ij}(1-\beta_{ij}))}$$

$$\tilde{\gamma}_{ij,ik} = \frac{\gamma_{ij,ik}}{(1-\gamma_{ij,ij}(1-\beta_{ij}))},$$

then we have

$$dlog P_{ij} = -\tilde{\gamma}_{ij,ij} dlog A_{ij} - \tilde{\gamma}_{ij,ik} dlog A_{ik} + \tilde{\gamma}_{ij,ik} dlog \tau_{ij,ik} + \tilde{\gamma}_{ij,ik}(1-\beta_{ik}) dlog P_{ik}.$$

It is then trivial to show that

$$dlog P_{ik} = -\tilde{\gamma}_{ik,ik} dlog A_{ik} - \tilde{\gamma}_{ik,ij} dln A_{ij} + \tilde{\gamma}_{ik,ij} dlog \tau_{ik,ij} + \tilde{\gamma}_{ik,ij}(1-\beta_{ij}) dlog P_{ij}.$$

Solving for prices we get

$$dlogP_{ij} = -\tilde{\gamma}_{ij,ij} dlogA_{ij} - \tilde{\gamma}_{ij,ik} dlogA_{ik} + \tilde{\gamma}_{ij,ik} dlog\tau_{ij,ik} - \tilde{\gamma}_{ik,ik}\tilde{\gamma}_{ij,ik}(1-\beta_{ik}) dlogA_{ik}$$

$$-\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik}) dlogA_{ij} + \tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik}) dlog\tau_{ik,ij} + \tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}) dlogP_{ij}.$$

$$dlogP_{ij} = -\frac{\tilde{\gamma}_{ij,ij} + \tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} dlogA_{ij} - \frac{\tilde{\gamma}_{ij,ik} + \tilde{\gamma}_{ik,ik}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} dlogA_{ik}$$

$$+\frac{\tilde{\gamma}_{ij,ik}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} dlog\tau_{ij,ik} + \frac{\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} dlog\tau_{ik,ij}$$

Similarly

$$dlogP_{ik} = -\frac{\tilde{\gamma}_{ik,ik} + \tilde{\gamma}_{ij,ik}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} dlogA_{ik} - \frac{\tilde{\gamma}_{ik,ij} + \tilde{\gamma}_{ij,ij}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} dlogA_{ij}$$

$$+\frac{\tilde{\gamma}_{ik,ij}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} dlog\tau_{ik,ij} + \frac{\tilde{\gamma}_{ij,ik}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} dlog\tau_{ij,ik}$$

Totally differentiating the expenditure shares we have:

$$\gamma_{ij,ik} = \frac{A_{ik}^{\theta}\tau_{ij,ik}^{-\theta}P_{ik}^{-\theta(1-\beta_{ik})}}{(P_{ij})^{-\theta}}$$

$$dlog\gamma_{ij,ik} = \theta dlogA_{ik} - \theta dlog\tau_{ij,ik} - \theta(1-\beta_{ik}) dlogP_{ik} + \theta dlogP_{ij}$$

Plugging the total differential for prices we have that

$$\frac{dlog\gamma_{ij,ik}}{dlog\tau_{ij,ik}} = -\theta - \theta(1-\beta_{ik})\frac{\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} + \theta\frac{\tilde{\gamma}_{ij,ik}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}$$

$$\frac{dlog\gamma_{ij,ik}}{dlog\tau_{ij,ik}} = -\theta\left[\frac{1-\tilde{\gamma}_{ij,ik}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}\right].$$

Likewise,

$$\frac{dlog\gamma_{ij,ik}}{dlog\tau_{ik,ij}} = (1-\beta_{ik})\tilde{\gamma}_{ik,ij}\frac{dlog\gamma_{ij,ik}}{dlog\tau_{ij,ik}},$$

Similarly, we have

$$\frac{dlog\gamma_{ij,ik}}{dlogA_{ij}} = \theta(1-\beta_{ik})\frac{\tilde{\gamma}_{ik,ij}+\tilde{\gamma}_{ij,ij}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} - \theta\frac{\tilde{\gamma}_{ij,ij}+\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}$$

$$\frac{dlog\gamma_{ij,ik}}{dlogA_{ij}} = \theta\frac{(1-\beta_{ik})\tilde{\gamma}_{ik,ij}+(1-\beta_{ik})\tilde{\gamma}_{ij,ij}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})-\tilde{\gamma}_{ij,ij}-\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}$$

$$\frac{dlog\gamma_{ij,ik}}{dlogA_{ij}} = \theta\frac{(1-\beta_{ik})\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ij}\beta_{ij}+(1-\beta_{ik})\tilde{\gamma}_{ij,ij}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})-\tilde{\gamma}_{ij,ij}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}$$

$$\frac{dlog\gamma_{ij,ik}}{dlogA_{ij}} = \theta\frac{(1-\beta_{ik})\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ij}-\tilde{\gamma}_{ij,ij}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}$$

likewise

$$\frac{dlog\gamma_{ij,ik}}{dlogA_{ik}} = \theta\beta_{ij}\left(\frac{\tilde{\gamma}_{ij,ij}+(1-\beta_{ik})\tilde{\gamma}_{ik,ik}\tilde{\gamma}_{ij,ij}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}\right).$$

We now derive the elasticity of the world's input-output structure with respect to changes in frictions and productivities (equation (8) in the main text). Analogous to the two-sector case, totally differentiating the price equations we get:

$$d\log P_{ij} = \sum_{m=1}^{N}\sum_{h=1}^{J}\left[-\tilde{\gamma}_{ij,mh}d\log A_{mh}+\tilde{\gamma}_{ij,mh}d\log\tau_{ij,mh}+(1-\beta_{mh})\,\tilde{\gamma}_{ij,mh}d\log P_{mh}+\beta_m\tilde{\gamma}_{ij,mh}d\log w_m\right]$$

where

$$\tilde{\gamma}_{is,ip} \equiv \frac{\gamma_{is,ip}}{1-\gamma_{is,is}\,(1-\beta_{is})},$$

Let's define P be the vector that contains the log change in prices across all sectors and countries, that is

$$\underset{(NJ\times 1)}{P} = \begin{bmatrix} dlogP_{11} \\ . \\ . \\ dlogP_{1J} \\ . \\ . \\ dlogP_{NJ} \end{bmatrix}$$

Similarly, we define the vectors of log changes in TFPs and frictions as

$$\underset{(NJ\times 1)}{A} = \begin{bmatrix} dlogA_{11} \\ . \\ . \\ dlogA_{1J} \\ . \\ . \\ dlogA_{NJ} \end{bmatrix} \quad \underset{(NJNJ\times 1)}{\tau} = \begin{bmatrix} dlog\tau_{11,11} \\ . \\ . \\ dlog\tau_{11,1j} \\ . \\ . \\ dlog\tau_{NJ,11} \\ . \\ . \\ dlog\tau_{NJ,NJ} \end{bmatrix}$$

Finally we define the vectors of log changes in input-output shares and wages as

$$\underset{(NJNJ\times 1)}{\Gamma} = \begin{bmatrix} dlog\gamma_{11,11} \\ . \\ . \\ dlog\gamma_{11,1J} \\ . \\ dlog\gamma_{11,NJ} \\ dlog\gamma_{JN,11} \\ . \\ . \\ dlog\gamma_{JN,JN} \end{bmatrix} \quad \underset{(NJ\times 1)}{\omega} = \begin{bmatrix} dlogw_1 \\ dlogw_1 \\ . \\ . \\ dlogw_n \\ dlogw_n \end{bmatrix} \quad \underset{(NJNJ\times 1)}{\tilde{\omega}} = \begin{bmatrix} \omega \\ \omega \\ . \\ . \\ \omega \\ \omega \end{bmatrix}$$

Therefore, the log change in the expenditure share with respect to changes in frictions is given by

$$dlog\gamma_{ij,ik} = \theta dlogA_{ik} - \theta dlog\tau_{ij,ik} - \theta(1-\beta_{ik})dlogP_{ik} + \theta dlogP_{ij} - \theta\beta_{ik}dlogw_i$$

Plugging the expressions for sectoral prices, we can express therefore the change in expenditure shares across countries and sectors in the world, Γ, in matrices as a function of parameters, frictions, and TFP, namely

$$\Gamma = \theta\tilde{A} - \theta\tau + \Sigma P - \theta\beta\tilde{\omega}$$

where we define Σ_{ij} to be a $J \times NJ$ matrix that is the sum of a $J \times NJ$ matrix that contains the element θ in the column $(i-1)J + j$ and zeros elsewhere, and another $J \times NJ$ matrix that contains the element $-\theta(1-\beta_{mh})$ in the diagonal and zeros elsewhere.

Then we define,

$$\underset{(NJ\times NJ)}{\Sigma} = \begin{bmatrix} \Sigma_{11} \\ . \\ . \\ \mathit{\Sigma}_{1J} \\ . \\ . \\ \Sigma_{NJ} \end{bmatrix} \quad \text{and} \quad \underset{(NJNJ\times 1)}{\tilde{A}} = \begin{bmatrix} A \\ . \\ . \\ A \\ . \\ . \\ A \end{bmatrix}$$

To solve for the log change in prices, we first define the following matrices

$$\underset{NJ\times NJ}{Z} = \begin{bmatrix} 0 & \tilde{\gamma}_{11,12}(1-\beta_{12}) & . & . & \tilde{\gamma}_{11,1J}(1-\beta_{1J}) & . & . & \tilde{\gamma}_{11,NJ}(1-\beta_{NJ}) \\ \tilde{\gamma}_{12,11}(1-\beta_{11}) & 0 & . & . & \tilde{\gamma}_{12,1J}(1-\beta_{1J}) & . & . & \tilde{\gamma}_{12,NJ}(1-\beta_{NJ}) \\ . & . & 0 & . & . & . & . & . \\ \tilde{\gamma}_{1J,11}(1-\beta_{11}) & \tilde{\gamma}_{1J,12}(1-\beta_{12}) & . & . & 0 & . & . & \tilde{\gamma}_{1J,NJ}(1-\beta_{NJ}) \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \tilde{\gamma}_{NJ,11}(1-\beta_{11}) & \tilde{\gamma}_{NJ,12}(1-\beta_{12}) & . & . & \tilde{\gamma}_{NJ,1J}(1-\beta_{1J}) & . & . & 0 \end{bmatrix}$$

$$\underset{NJ\times NJ}{\tilde{Z}} = \begin{bmatrix} \tilde{\gamma}_{11,11} & 0 & & . & 0 & & . & 0 \\ 0 & \tilde{\gamma}_{11,12} & . & . & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & \tilde{\gamma}_{NJ,11} & . & 0 & \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 0 & . & . & \tilde{\gamma}_{NJ,NJ} \end{bmatrix}$$

and let $\Omega = I - Z$ and $\mathit{\Upsilon} = Z + \tilde{Z}$. Finally, we define the matrix

$$\underset{NJ\times NJNJ}{\Theta} = \begin{bmatrix} \tilde{\gamma}_{11,11} & . & \tilde{\gamma}_{11,1J} & . & . & \tilde{\gamma}_{11,NJ} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\gamma}_{12,11} & . & \tilde{\gamma}_{12,NJ} & 0 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\gamma}_{NJ,11} & . & \tilde{\gamma}_{NJ,NJ} \end{bmatrix}$$

$$\Omega P = -\Upsilon A + \Theta\tau + \beta\Upsilon\omega$$

and therefore

$$P = -\Omega^{-1}\Upsilon A + \Omega^{-1}\Theta\tau + \Omega^{-1}\beta\Upsilon\omega$$

Finally, we can obtain the change in input-output shares as

$$\Gamma = (\theta\tilde{A} - \Sigma\Omega^{-1}\Upsilon A) + (-\theta 1_{NJNJ\times 1} + \Sigma\Omega^{-1}\Theta)\tau + \Sigma\Omega^{-1}\beta\Upsilon\omega - \theta\beta\tilde{\omega}$$

where $1_{NJNJ\times 1}$ is a vector of ones of size $NJNJ \times 1$. This equation can be written more generally as

$$\Gamma = \mathcal{F}(\theta, \beta, \gamma)A + \mathcal{H}(\theta, \beta, \gamma)\tau + \mathcal{O}(\theta, \beta, \gamma)\omega$$

B Data appendix

This Appendix describes the list of sectors and countries we included in the empirical analysis. The list of sectors is: Agriculture, Hunting, Forestry and Fishing (NACE AtB); Mining and Quarrying (NACE C); Food, Beverages and Tobacco (NACE 15t16); Textiles and Textile Products(NACE 17t18); Leather, Leather and Footwear(NACE 19); Wood and Products of Wood and Cork (NACE 20); Pulp, Paper, Paper , Printing and Publishing (NACE 21t22); Coke, Refined Petroleum and Nuclear Fuel (NACE 23); Chemicals and Chemical Products (NACE 24); Rubber and Plastics (NACE 25); Other Non-Metallic Mineral (NACE 26); Basic Metals and Fabricated Metal (NACE 27t28); Machinery, Nec (NACE 29); Electrical and Optical Equipment (NACE 30t33); Transport Equipment (NACE 34t35); Manufacturing, Nec; Recycling (NACE 36t37); Electricity, Gas and Water Supply (NACE E); Construction (NACE F); Sale, Maintenance and Repair of Motor Vehicles Retail Sale of Fuel (NACE 50); Wholesale Trade and Commission Trade, Except of Motor Vehicles (NACE 51); Retail Trade, Except of Motor Vehicles ; Repair of Household Goods (NACE 52); Hotels and Restaurants (NACE H); Inland Transport (NACE 60); Water Transport (NACE 61); Air Transport (NACE 62); Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies (NACE 63); Post and Telecommunications (NACE 64); Financial Intermediation (NACE J); Real Estate Activities (NACE 70); Renting of M&Eq and Other Business Activities (NACE 71t74); Public Admin and Defense; Compulsory Social Security (NACE L); Education (NACE M); Health and Social Work (NACE N); Other Community, Social and Personal Services (NACE O); Private Households with Employed Persons (NACE P). We drop from the analysis the Private Households with Employed Persons as it presented generally incomplete data. The list of countries is: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech

Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovak Republic, Slovenia, South Korea, Spain, Sweden, Taiwan, Turkey, United Kingdom, United States, and the constructed Rest of the World.

C Sufficient Statistics to Identify Internal Frictions

In this appendix, we derive the sufficient statistic for internal frictions and TFP. We do so in the open-economy version of the model, but as we explain in the main text, the formula for closed economy is analogous by dropping the country indices. The share of the input of the sector k in the sector j is given by:

$$\gamma_{ij,ik} = \frac{A_{ik}^{\theta} \tau_{ij,ik}^{-\theta} P_{ik}^{-\theta(1-\beta_{ik})}}{\left(P_{ij}/w_i^{\beta_{ik}}\right)^{-\theta}}.$$

Then we have that

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = \tau_{ij,ik}^{-\theta} \left(\frac{P_{ij}}{P_{ik}}\right)^{\theta}$$

On the other hand, the consumer's problem yields the following consumption share in country i, sector j:

$$\alpha_{ij} = \chi_{ij} \left(\frac{P_{ij}}{P_i}\right)^{1-\sigma}$$

and therefore

$$\frac{\alpha_{ij}}{\alpha_{ik}} = \frac{\chi_{ij}}{\chi_{ik}} \left(\frac{P_{ij}}{P_{ik}}\right)^{1-\sigma}.$$

Therefore,

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = \tau_{ij,ik}^{-\theta} \left(\frac{\alpha_{ij}}{\alpha_{ik}} \frac{\chi_{ik}}{\chi_{ij}}\right)^{\frac{\theta}{1-\sigma}},$$

and solving for the internal friction we get

$$\tau_{ij,ik} = \left(\frac{\gamma_{ik,ik}}{\gamma_{ij,ik}}\right)^{1/\theta} \left(\frac{\alpha_{ij}}{\alpha_{ik}} \frac{\chi_{ik}}{\chi_{ij}}\right)^{\frac{1}{1-\sigma}},$$

and the relative change in friction is given by

$$\hat{\tau}_{ij,ik} = \left(\frac{\hat{\gamma}_{ik,ik}}{\hat{\gamma}_{ij,ik}}\right)^{1/\theta} \left(\frac{\hat{\alpha}_{ij}}{\hat{\alpha}_{ik}}\right)^{\frac{1}{1-\sigma}}.$$

As in the main text, we can express the changes in expenditure shares as a function of frictions and TFPs as

$$\hat{\tau}_{ij,ik}\frac{\left(\hat{A}_{ij}\right)^{1/\beta_{ij}}}{\left(\hat{A}_{ik}\right)^{1/\beta_{ik}}} = \left(\frac{\hat{\gamma}_{ij,ij}^{1/\beta_{ij}}}{\hat{\gamma}_{ik,ik}^{1/\beta_{ik}}}\right)^{\frac{1}{\theta}} \left(\frac{\hat{\gamma}_{ik,ik}}{\hat{\gamma}_{ij,ik}}\right)^{\frac{1}{\theta}}.$$

Using the expression for the changes in frictions we can recover the change in TFPs relative to a reference sector k:

$$\hat{A}_{ij} = \left(\frac{\hat{\alpha}_{ij}}{\hat{\alpha}_{ik}}\right)^{\frac{\beta_{ij}}{\sigma-1}} \left(\frac{\hat{\gamma}_{ij,ij}^{1/\beta_{ij}}}{\hat{\gamma}_{ik,ik}^{1/\beta_{ik}}}\right)^{\frac{\beta_{ij}}{\theta}}.$$

D Additional Theoretical Results

In this appendix, we derive two additional theoretical results. First, we derive the formula for the change in internal frictions in a nested CES framework in which domestic goods and foreign goods are aggregated with different elasticities of substitution. We also extend this nested CES framework and allow for an arbitrary number of nests in the production function. Second, we derive the formula to identify internal frictions in a model with sale taxes.

D.1 Frictions in a Nested CES Framework

As in the main text, goods from country i and sector j, Q_{ij}, are produced with a Cobb-Douglas production function,

$$Q_{ij} = A_{ij} L_{ij}^{\beta_{ij}} M_{ij}^{(1-\beta_{ij})}$$

where A_{ij} is the TFP of country i and sector j, and L_{ij} and M_{ij} are labor and materials used by sector j.

Different from the main text, we assume that M_{ij} is a CES aggregate of composite material goods from different countries, with an elasticity of substitution equal to ω, namely

$$M_{ij} = \left(\sum_n M_{in,j}^{\frac{\omega}{1+\omega}}\right)^{\frac{1+\omega}{\omega}},$$

and the composite material good from country n and sector j used to produce in country i is another CES aggregate of materials from different sectors, with a different elasticity of substitution θ, namely

$$M_{in,j} = \left(\sum_k \iota_{ij,nk} Q_{ij,nk}^{\frac{\theta}{1+\theta}} \right)^{\frac{1+\theta}{\theta}}.$$

Notice that when $\theta = \omega$ this nested CES production function maps to the one in Section 1.3.

The problem of the lower nest is

$$\min_{Q_{ij,nk}} \sum_k P_{ij,nk} Q_{ij,nk} \text{ s.t. } \left(M_{in,j} = \sum_k \iota_{ij,nk} Q_{ij,nk}^{\frac{\theta}{1+\theta}} \right)^{\frac{1+\theta}{\theta}}$$

and the solution is given by

$$P_{in,j} = \left(\sum_k \iota_{ij,nk}^{1+\theta} P_{ij,nk}^{-\theta} \right)^{-\frac{1}{\theta}}$$

$$Q_{ij,nk} = \left(\frac{P_{ij,nk}}{\iota_{ij,nk} P_{in,j}} \right)^{-(1+\theta)} M_{in,j}$$

Now the problem of the upper nest is,

$$\min_{M_{in,j}} \sum_n P_{in,j} M_{in,j} \text{ s.t.: } M_{ij} = \left(\sum_n M_{in,j}^{\frac{\omega}{1+\omega}} \right)^{\frac{1+\omega}{\omega}},$$

with the solution given by

$$P_{ij} = \left(\sum_n P_{in,j}^{-\omega} \right)^{-\frac{1}{\omega}}$$

$$M_{in,j} = \left(\frac{P_{in,j}}{P_{ij}} \right)^{-(1+\omega)} M_{ij}$$

Combining both we obtain the demand for home goods purchased from sector k by sector j,

$$Q_{ij,nk} = \left(\frac{P_{ij,nk}}{\iota_{ij,nk} P_{in,j}} \right)^{-(1+\theta)} \left(\frac{P_{in,j}}{P_{ij}} \right)^{-(1+\omega)} M_{ij}$$

Similarly, the total demand for sector j home goods is given by

$$M_{in,j} = \left(\frac{P_{in,j}}{P_{ij}} \right)^{-(1+\omega)} M_{ij}$$

As we will use later on, we denote by $\lambda_{ii,j}$ the share of total expenditure in sector j and country i spent in home materials. Hence, we have

$$\lambda_{ii,j} = \left(\frac{P_{ii,j}}{P_{ij}}\right)^{-\omega}$$

The share of total expenditure in sector j and country i spent on goods from sector k is given by

$$\begin{aligned}
\gamma_{ij,ik} &= \frac{P_{ij,ik}Q_{ij,ik}}{P_{ij}M_{ij}} \\
&= P_{ij,ik}\left(\frac{P_{ij,ik}}{\iota_{ij,ik}P_{ii,j}}\right)^{-(1+\theta)}\left(\frac{P_{ii,j}}{P_{ij}}\right)^{-(1+\omega)}\frac{1}{P_{ij}} \\
&= P_{ij,ik}\left(\frac{P_{ij,ik}}{\iota_{ij,ik}}\right)^{-(1+\theta)}\left(\frac{1}{P_{ii,j}}\right)^{-\theta}\left(\frac{P_{ii,j}}{P_{ij}}\right)^{-\omega} \\
&= (P_{ij,ik})^{-\theta}\left(\frac{1}{\iota_{ij,ik}}\right)^{-(1+\theta)}\left(\frac{1}{P_{ii,j}}\right)^{-\theta}\left(\frac{P_{ii,j}}{P_{ij}}\right)^{-\omega}
\end{aligned}$$

use that $P_{ij,ik} = \kappa_{ij,ik}c_{ik}$, hence

$$\gamma_{ij,ik} = \left(\kappa_{ij,ik}\left(\iota_{ij,ik}\right)^{-\frac{(1+\theta)}{\theta}}\right)^{-\theta}\left(\frac{c_{ik}}{P_{ii,j}}\right)^{-\theta}\left(\frac{P_{ii,j}}{P_{ij}}\right)^{-\omega}$$

As in the main text, we define $\tau_{ij,nk} = \kappa_{ij,nk}(\iota_{ij,nk})^{-(1+\theta)/\theta}$ as frictions that prevent the use of inputs from sector k and country n in the production of sector j and country i. Using this expression, we obtain that

$$\gamma_{ij,ik} = \left(\frac{\tau_{ij,ik}c_{ik}}{P_{ii,j}}\right)^{-\theta}\left(\frac{P_{ii,j}}{P_{ij}}\right)^{-\omega}.$$

We can write the following formula,

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = \frac{\left(\frac{\tau_{ij,ik}c_{ik}}{P_{ii,j}}\right)^{-\theta}\left(\frac{P_{ii,j}}{P_{ij}}\right)^{-\omega}}{\left(\frac{c_{ik}}{P_{ii,k}}\right)^{-\theta}\left(\frac{P_{ii,k}}{P_{ik}}\right)^{-\omega}}.$$

Hence

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = (\tau_{ij,ik})^{-\theta}\left(\frac{P_{ii,j}}{P_{ii,k}}\right)^{\theta-\omega}\left(\frac{P_{ij}}{P_{ik}}\right)^{\omega},$$

which can be expressed as,

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = (\tau_{ij,ik})^{-\theta}\,\frac{\lambda_{ii,j}}{\lambda_{ii,k}}\left(\frac{P_{ii,j}}{P_{ii,k}}\right)^{\theta}.$$

Using also the fact that

$$\frac{P_{ii,j}}{P_{ii,k}} = \frac{P_{ij}}{P_{ik}} \left(\frac{\lambda_{ii,j}}{\lambda_{ii,k}}\right)^{-1/\omega},$$

we get

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = (\tau_{ij,ik})^{-\theta} \left(\frac{\lambda_{ii,j}}{\lambda_{ii,k}}\right)^{\frac{\omega-\theta}{\omega}} \left(\frac{P_{ij}}{P_{ik}}\right)^{\theta}.$$

Finally, using as in the main text the expression $P_{ij}/P_{ik} = (\frac{\alpha_{ij}}{\alpha_{ik}} \frac{\chi_{ij}}{\chi_{ik}})^{\frac{1}{1-\sigma}}$ we obtain

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = (\tau_{ij,ik})^{-\theta} \left(\frac{\lambda_{ii,j}}{\lambda_{ii,k}}\right)^{\frac{\omega-\theta}{\omega}} \left(\frac{\alpha_{ij}}{\alpha_{ik}} \frac{\chi_{ij}}{\chi_{ik}}\right)^{\frac{\theta}{1-\sigma}}.$$

Therefore, we solve for the changes in internal frictions as

$$\hat{\tau}_{ij,ik} = \left(\frac{\hat{\gamma}_{ij,ik}}{\hat{\gamma}_{ik,ik}}\right)^{-\frac{1}{\theta}} \left(\frac{\hat{\lambda}_{ii,j}}{\hat{\lambda}_{ii,k}}\right)^{\frac{\omega-\theta}{\omega\theta}} \left(\frac{\hat{\alpha}_{ij}}{\hat{\alpha}_{ik}}\right)^{\frac{1}{1-\sigma}}$$

Notice that the formula maps to the one in Section 1.3 when $\omega = \theta$.

D.2 Frictions in a Nested CES Framework with an Arbitrary Number of Nests

As in the main text, goods from country i and sector j, Q_{ij}, are produced with a Cobb-Douglas production function,

$$Q_{ij} = A_{ij} L_{ij}^{\beta_{ij}} M_{ij}^{(1-\beta_{ij})}$$

where A_{ij} is the TFP of country i and sector j, and L_{ij} and M_{ij} are labor and materials used by sector j.

As in the previous section of this appendix, we assume that M_{ij} is a CES aggregate of composite material goods from different countries, with an elasticity of substitution equal to ω, namely

$$M_{ij} = \left(\sum_n M_{in,j}^{\frac{\omega}{1+\omega}}\right)^{\frac{1+\omega}{\omega}},$$

We now assume that $M_{in,j}$ is a CES aggregate of different $h \epsilon J$ bundles of composite material goods, where each of these h bundles aggregate material goods with different elasticities of substitution. In particular, we have that

$$M_{in,j} = \left(\sum_h \left(M_{in,j}^h\right)^{\frac{\theta}{1+\theta}}\right)^{\frac{1+\theta}{\theta}}$$

$$M^h_{in,j} = \left(\sum_{k \epsilon h} \iota^h_{ij,nk} \left(Q^h_{ij,nk} \right)^{\frac{\theta_h}{1+\theta_h}} \right)^{\frac{1+\theta_h}{\theta_h}}.$$

where the superscript h indicates that a given composite intermediate good belongs to the h-nest. Notice that when $\theta_h = \theta$ the nested CES production function maps to the one in the previous section of this appendix, and when $\theta_h = \theta = \omega$ this nested CES production function maps to the one in Section 1.3 in the main text.

The problem of the lower nest is

$$\min_{Q^h_{ij,nk}} \sum_{k \epsilon h} P^h_{ij,nk} Q^h_{ij,nk} \text{ s.t. } \left(M^h_{in,j} = \sum_{h \epsilon k} \iota^h_{ij,nk} \left(Q^h_{ij,nk} \right)^{\frac{\theta_h}{1+\theta_h}} \right)^{\frac{1+\theta_h}{\theta_h}}$$

and the solution is given by

$$P^h_{in,j} = \left(\sum_{k \epsilon h} \left(\iota^h_{ij,nk} \right)^{1+\theta_h} \left(P^h_{ij,nk} \right)^{-\theta_h} \right)^{-\frac{1}{\theta_h}}$$

$$Q^h_{ij,nk} = \left(\frac{P^h_{ij,nk}}{\iota^h_{ij,nk} P^h_{in,j}} \right)^{-(1+\theta_h)} M^h_{in,j}$$

The problem of the middle nest is

$$\min_{M^h_{in,j}} \sum_h P^h_{in,j} M^h_{in,j} \text{ s.t. } M_{in,j} = \left(\sum_h \left(M^h_{in,j} \right)^{\frac{\theta}{1+\theta}} \right)^{\frac{1+\theta}{\theta}}$$

and the solution is given by

$$P_{in,j} = \left(\sum_h \left(P^h_{in,j} \right)^{-\theta} \right)^{-\frac{1}{\theta}}$$

$$M^h_{in,j} = \left(\frac{P^h_{in,j}}{P_{in,j}} \right)^{-(1+\theta)} M_{in,j}$$

Now the problem of the upper nest is,

$$\min_{M_{in,j}} \sum_n P_{in,j} M_{in,j} \text{ s.t.: } M_{ij} = \left(\sum_n M_{in,j}^{\frac{\omega}{1+\omega}} \right)^{\frac{1+\omega}{\omega}},$$

with solution

$$P_{ij} = \left(\sum_n P_{in,j}^{-\omega} \right)^{-\frac{1}{\omega}}$$

$$M_{in,j} = \left(\frac{P_{in,j}}{P_{ij}} \right)^{-(1+\omega)} M_{ij}$$

Combining the demand in the three nests, we obtain the demand for home goods purchased from sector k by sector j,

$$Q_{ij,nk}^h = \left(\frac{P_{ij,nk}^h}{\iota_{ij,nk}^h P_{in,j}^h} \right)^{-(1+\theta_h)} \left(\frac{P_{in,j}^h}{P_{in,j}} \right)^{-(1+\theta)} \left(\frac{P_{in,j}}{P_{ij}} \right)^{-(1+\omega)} M_{ij}$$

As in the previous section of the appendix, we denote by $\lambda_{ii,j}$ the share of total expenditure in sector j and country i spent in home materials. Hence, we have

$$\lambda_{ii,j} = \left(\frac{P_{ii,j}}{P_{ij}} \right)^{-\omega}$$

Analogously, we denote by $\lambda_{ii,j}^h$ the share of total expenditure in sector j and country i spent in home materials in the set of h-nest goods, namely

$$\lambda_{ii,j}^h = \left(\frac{P_{ii,j}^h}{P_{ii,j}} \right)^{-\theta}$$

The share of total expenditure in sector j and country i spent on goods from sector k is given by

$$\begin{aligned}
\gamma_{ij,ik}^h &= \frac{P_{ij,ik}^h Q_{ij,ik}^h}{P_{ij} M_{ij}} \\
&= P_{ij,ik}^h \left(\frac{P_{ij,ik}^h}{\iota_{ij,ik}^h P_{ii,j}^h} \right)^{-(1+\theta_h)} \left(\frac{P_{ii,j}^h}{P_{ii,j}} \right)^{-(1+\theta)} \left(\frac{P_{ii,j}}{P_{ij}} \right)^{-(1+\omega)} \frac{1}{P_{ij}} \\
&= \left(P_{ij,ik}^h \right)^{-\theta_h} \left(\frac{1}{\iota_{ij,ik}^h} \right)^{-(1+\theta_h)} \left(\frac{1}{P_{ii,j}^h} \right)^{-\theta_h} \left(\frac{P_{ii,j}^h}{P_{ii,j}} \right)^{-\theta} \left(\frac{P_{ii,j}}{P_{ij}} \right)^{-\omega} .
\end{aligned}$$

Use that $P^h_{ij,ik} = \kappa^h_{ij,ik} c^h_{ik}$, namely

$$\gamma^h_{ij,ik} = \left(\frac{\kappa^h_{ij,ik} \left(\iota^h_{ij,ik}\right)^{\frac{-(1+\theta_h)}{\theta_h}} c^h_{ik}}{P^h_{ii,j}} \right)^{-\theta_h} \left(\frac{P^h_{ii,j}}{P_{ii,j}} \right)^{-\theta} \left(\frac{P_{ii,j}}{P_{ij}} \right)^{-\omega}.$$

As in the main text, we define $\tau^h_{ij,nk} = \kappa^h_{ij,nk} (\iota^h_{ij,nk})^{-(1+\theta_h)/\theta_h}$ as frictions that prevent the use of inputs from sector k and country n in the production of sector j and country i. Using this expression, we obtain that

$$\gamma^h_{ij,ik} = \left(\frac{\tau^h_{ij,nk} c^h_{ik}}{P^h_{ii,j}} \right)^{-\theta_h} \left(\frac{P^h_{ii,j}}{P_{ii,j}} \right)^{-\theta} \left(\frac{P_{ii,j}}{P_{ij}} \right)^{-\omega},$$

and taking the cross-sectoral ratios we get

$$\begin{aligned} \frac{\gamma^h_{ij,ik}}{\gamma^h_{ik,ik}} &= \frac{\left(\frac{\tau^h_{ij,nk} c^h_{ik}}{P^h_{ii,j}} \right)^{-\theta_h} \left(\frac{P^h_{ii,j}}{P_{ii,j}} \right)^{-\theta} \left(\frac{P_{ii,j}}{P_{ij}} \right)^{-\omega}}{\left(\frac{c^h_{ik}}{P^h_{ii,k}} \right)^{-\theta_h} \left(\frac{P^h_{ii,k}}{P_{ii,k}} \right)^{-\theta} \left(\frac{P_{ii,k}}{P_{ik}} \right)^{-\omega}} \\ &= \left(\tau^h_{ij,nk}\right)^{-\theta_h} \left(\frac{P^h_{ii,j}}{P^h_{ii,k}} \right)^{\theta_h - \theta} \left(\frac{P_{ii,j}}{P_{ii,k}} \right)^{\theta - \omega} \left(\frac{P_{ij}}{P_{ik}} \right)^{\omega}. \end{aligned}$$

Using the expression $\lambda_{ii,j} = \left(\frac{P_{ii,j}}{P_{ij}}\right)^{-\omega}$, we get

$$\frac{\gamma^h_{ij,ik}}{\gamma^h_{ik,ik}} = \left(\tau^h_{ij,nk}\right)^{-\theta_h} \left(\frac{P^h_{ii,j}}{P^h_{ii,k}} \right)^{\theta_h - \theta} \left(\frac{P_{ii,j}}{P_{ii,k}} \right)^{\theta} \frac{\lambda_{ii,j}}{\lambda_{ii,k}},$$

and using $\lambda^h_{ii,j} = \left(\frac{P^h_{ii,j}}{P_{ii,j}}\right)^{-\theta}$, we obtain

$$\frac{\gamma^h_{ij,ik}}{\gamma^h_{ik,ik}} = \left(\tau^h_{ij,nk}\right)^{-\theta_h} \left(\frac{P^h_{ii,j}}{P^h_{ii,k}} \right)^{\theta_h} \frac{\lambda^h_{ii,j}}{\lambda^h_{ii,k}} \frac{\lambda_{ii,j}}{\lambda_{ii,k}}.$$

We now use again the expressions for $\lambda_{ii,j}$ and $\lambda^h_{ii,j}$ to obtain and expression for $P^h_{ii,j}/P^h_{ii,k}$, in particular

$$\frac{P^h_{ii,j}}{P^h_{ii,k}} = \left(\frac{\lambda^h_{ii,j}}{\lambda^h_{ii,k}} \right)^{-1/\theta} \left(\frac{\lambda_{ii,j}}{\lambda_{ii,k}} \right)^{-1/\omega} \frac{P_{ij}}{P_{ik}},$$

and therefore,

$$\frac{\gamma^h_{ij,ik}}{\gamma^h_{ik,ik}} = \left(\tau^h_{ij,nk}\right)^{-\theta_h} \left(\frac{\lambda^h_{ii,j}}{\lambda^h_{ii,k}}\right)^{\frac{\theta-\theta_h}{\theta}} \left(\frac{\lambda_{ii,j}}{\lambda_{ii,k}}\right)^{\frac{\omega-\theta_h}{\omega}} \left(\frac{P_{ij}}{P_{ik}}\right)^{\theta_h}.$$

Finally, using as in the main text the expression $P_{ij}/P_{ik} = (\frac{\alpha_{ij}}{\alpha_{ik}}\frac{\chi_{ij}}{\chi_{ik}})^{\frac{1}{1-\sigma}}$ we obtain

$$\frac{\gamma^h_{ij,ik}}{\gamma^h_{ik,ik}} = \left(\tau^h_{ij,nk}\right)^{-\theta_h} \left(\frac{\lambda^h_{ii,j}}{\lambda^h_{ii,k}}\right)^{\frac{\theta-\theta_h}{\theta}} \left(\frac{\lambda_{ii,j}}{\lambda_{ii,k}}\right)^{\frac{\omega-\theta_h}{\omega}} \left(\frac{\alpha_{ij}}{\alpha_{ik}}\frac{\chi_{ij}}{\chi_{ik}}\right)^{\frac{\theta_h}{1-\sigma}}.$$

Therefore, we solve for the changes in internal frictions as

$$\hat{\tau}^h_{ij,nk} = \left(\frac{\hat{\gamma}^h_{ij,ik}}{\hat{\gamma}^h_{ik,ik}}\right)^{-\frac{1}{\theta_h}} \left(\frac{\hat{\lambda}^h_{ii,j}}{\hat{\lambda}^h_{ii,k}}\right)^{\frac{\theta-\theta_h}{\theta\theta_h}} \left(\frac{\hat{\lambda}_{ii,j}}{\hat{\lambda}_{ii,k}}\right)^{\frac{\omega-\theta_h}{\omega\theta_h}} \left(\frac{\hat{\alpha}_{ij}}{\hat{\alpha}_{ik}}\right)^{\frac{1}{1-\sigma}}.$$

Notice that the formula maps to the one in Section 1.3 when $\omega = \theta = \theta_h$.

D.3 Frictions in a Model with Sale Taxes

We assume a sale tax τ_{ik} applies to purchases of sector k goods by all sectors in the economy. Hence, sourcing a good from sector k to sector j entails a cost $(1+\tau_{ik})\,\kappa_{ij,ik}c_{ik}$. Following the derivations in the main text, the bilateral expenditure shares $\gamma_{ij,ik}$ and $\gamma_{ik,ik}$ are give by,

$$\gamma_{ij,ik} = \frac{A^\theta_{ik}\,(1+\tau_{ik})^{-\theta}\,\tau^{-\theta}_{ij,ik}P^{-\theta(1-\beta_{ik})}_{ik}}{\left(P_{ij}/w_i^{\beta_{ik}}\right)^{-\theta}},$$

$$\gamma_{ik,ik} = \frac{A^\theta_{ik}\,(1+\tau_{ik})^{-\theta}\,P^{-\theta(1-\beta_{ik})}_{ik}}{\left(P_{ik}/w_i^{\beta_{ik}}\right)^{-\theta}}.$$

and taking the ratio, we get

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = \tau^{-\theta}_{ij,ik}\frac{(P_{ij})^\theta}{(P_{ik})^\theta},$$

or

$$\tau_{ij,ik} = \left(\frac{\gamma_{ik,ik}}{\gamma_{ij,ik}}\right)^{1/\theta}\frac{P_{ij}}{P_{ik}}.$$

Finally, using the consumption shares as in the main text and taking changes, we obtain

$$\hat{\tau}_{ij,ik} = \left(\frac{\hat{\gamma}_{ik,ik}}{\hat{\gamma}_{ij,ik}}\right)^{\frac{1}{\theta}} \left(\frac{\hat{\alpha}_j}{\hat{\alpha}_k}\right)^{\frac{1}{1-\sigma}},$$

Notice that the formula for the change in internal frictions is the same as the one derived in the main text. Of course, the computation of the elasticities of world's GDP to changes in internal frictions require to include the tariff revenues, namely income in country i is now given by

$$I_i = w_i L_i + \sum_{k,j} \tau_{ik} \gamma_{ij,ik} X_{ij}.$$

D.4 Skill Biased Technical Change

We now include the derivation of our sufficient statistic under skill-biased technical change (SBTC). Assume a production function that allows for SBTC, in particular we let

$$Q_j = A_j \left[(\mu_j)^{1/\rho} \left(A_j^s L_j^s\right)^{(\rho-1)/\rho} + (1-\mu_j)^{1/\rho} \left(L_j^u\right)^{(\rho-1)/\rho} \right]^{\beta_j \rho/(\rho-1)} M_j^{(1-\beta_j)},$$

where A_j^s is the skill-biased technological shifter, and we index by s, u the skilled and unskilled labor. To facilitate the exposition of the derivation, let's denote by L_j the composite of skilled and unskilled labor in the production functions, namely:

$$L_j = \left[(\mu_j)^{1/\rho} \left(A_j^s L_j^s\right)^{(\rho-1)/\rho} + (1-\mu_j)^{1/\rho} \left(L_j^u\right)^{(\rho-1)/\rho} \right]^{\rho/(\rho-1)}.$$

Using the FOCs with respect to each labor type from the cost minimization problem, we have

$$\frac{w^u}{w^s} = \left(\frac{(1-\mu_j)^{1/\rho}}{(\mu_j)^{1/\rho} \left(A_j^s\right)^{(\rho-1)/\rho}} \right) \left(\frac{L_j^u}{L_j^s}\right)^{-1/\rho},$$

or

$$\left(\frac{L_j^u}{L_j^s}\right) = \left(\frac{w^s}{w^u}\right)^{\rho} \frac{1-\mu_j}{\mu_j} \left(A_j^s\right)^{(1-\rho)} \tag{12}$$

Plugging this equation in the composite labor equation we get

$$L_j = \left[(\mu_j)^{1/\rho} \left(A_j^s L_j^s \right)^{(\rho-1)/\rho} + (1-\mu_j)^{1/\rho} \left(\left(\frac{w^s}{w^u} \right)^{\rho} \frac{(1-\mu_j) L_j^s}{(\mu_j) \left(A_j^s \right)^{(\rho-1)}} \right)^{(\rho-1)/\rho} \right]^{\rho/(\rho-1)}$$

or

$$L_j = \left(A_j^s \right)^{(1-\rho)} (w^s)^{\rho} L_j^s \left[(\mu_j)^{1/\rho} (w^s)^{1-\rho} \left(A_j^s \right)^{(\rho-1)} + (1-\mu_j) (w^u)^{1-\rho} (\mu_j)^{(1-\rho)/\rho} \right]^{\rho/(\rho-1)}$$

or

$$L_j = \left(A_j^s \right)^{(1-\rho)} (\mu_j)^{-1} (w^s)^{\rho} L_j^s \left[\mu_j \left(w^s / A_j^s \right)^{1-\rho} + (1-\mu_j) (w^u)^{1-\rho} \right]^{\rho/(\rho-1)}. \tag{13}$$

Define total labor payment as $wL_j = w^s L_j^s + w^u L_j^u$ and using (13) and (12) we get

$$wL_j = w^s L_j^s + w^u L_j^u$$

$$w \left(A_j^s \right)^{(1-\rho)} (\mu_j)^{-1} (w^s)^{\rho} \left[\mu_j \left(w^s / A_j^s \right)^{1-\rho} + (1-\mu_j) (w^u)^{1-\rho} \right]^{\rho/(\rho-1)} = w^s + w^u \frac{L_j^u}{L_j^s}$$

$$w \left[\mu_j \left(w^s / A_j^s \right)^{1-\rho} + (1-\mu_j) (w^u)^{1-\rho} \right]^{\rho/(\rho-1)} = \left(A_j^s \right)^{(\rho-1)} \mu_j (w^s)^{1-\rho} + \left(A_j^s \right)^{(\rho-1)} \mu_j (w^s)^{-\rho} w^u \frac{L_j^u}{L_j^s}$$

$$w \left[\mu_j \left(w^s / A_j^s \right)^{1-\rho} + (1-\mu_j) (w^u)^{1-\rho} \right]^{\rho/(\rho-1)} = \mu_j \left(w^s / A_j^s \right)^{1-\rho} + (w^u)^{1-\rho} (1-\mu_j),$$

hence

$$w = \left[\mu_j \left(w^s / A_j^s \right)^{1-\rho} + (w^u)^{1-\rho} (1-\mu_j) \right]^{1/(1-\rho)}.$$

As in the main text, the unit price of good Q_j is given by:

$$c_j = \frac{1}{A_j} w^{\beta_j} P_j^{(1-\beta_j)},$$

where notice that β_j is the share of value added in gross output, and the share of skill and unskilled labor are endogenous because of the CES structure.

From here, we can proceed as in the paper to identify internal frictions. The share of the input of the sector k in the sector j is:

$$\gamma_{jk} = \frac{(\tau_{jk} c_k)^{-\theta}}{\sum_{h=1}^{J} (\tau_{jh} c_h)^{-\theta}},$$

which in turn can be written as:

$$\gamma_{jk} = \frac{A_k^{\theta} \tau_{jk}^{-\theta} P_k^{-\theta(1-\beta_k)}}{(P_j / w^{\beta_k})^{-\theta}},$$

and taking the ratio, we get

$$\frac{\gamma_{jk}}{\gamma_{kk}} = \tau_{jk}^{-\theta} \frac{(P_j)^{\theta}}{(P_k)^{\theta}},$$

or

$$\tau_{jk} = \left(\frac{\gamma_{kk}}{\gamma_{jk}} \right)^{1/\theta} \frac{P_j}{P_k}.$$

Finally, using the consumption shares as in the main text and taking changes, we obtain

$$\hat{\tau}_{jk} = \left(\frac{\hat{\gamma}_{kk}}{\hat{\gamma}_{jk}} \right)^{\frac{1}{\theta}} \left(\frac{\hat{\alpha}_j}{\hat{\alpha}_k} \right)^{\frac{1}{1-\sigma}},$$

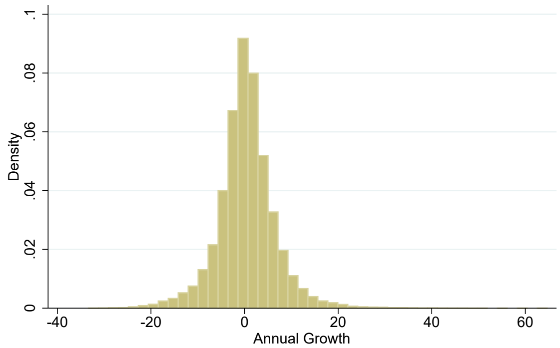
E Additional Empirical Results

In this appendix we present additional results. We first show the distribution of changes in internal frictions with different values of the elasticities σ and θ.

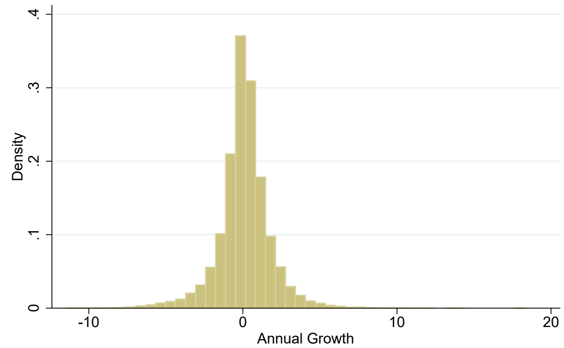
E.1 Distribution of Growth are in Internal Frictions with Different Elasticities σ and θ

Figure E.1 presents the histograms of the changes in internal frictions using values for the elasticities σ and θ ranging between 2 and 6.

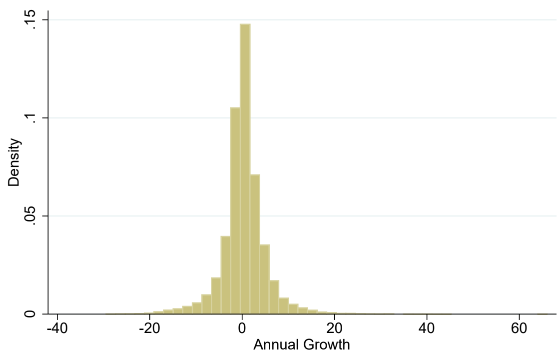
Figure E.1: Annual Growth Rate in Global Internal Frictions



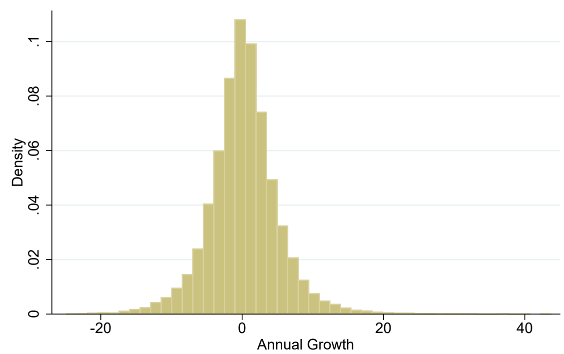
(a) $\sigma = \theta = 2$



(b) $\sigma = \theta = 6$



(c) $\theta = 2$, $\sigma = 4$



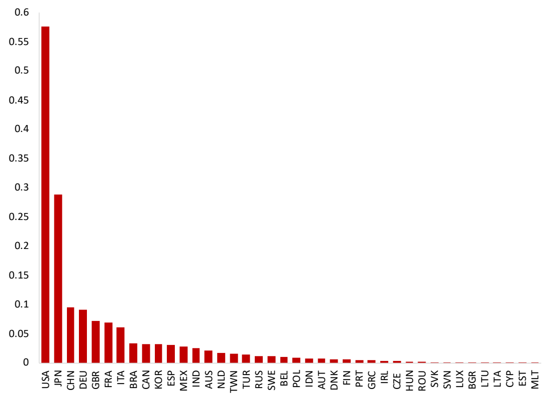
(d) $\theta = 4, \sigma = 2$

E.2 World's GDP elasticity to internal versus external frictions

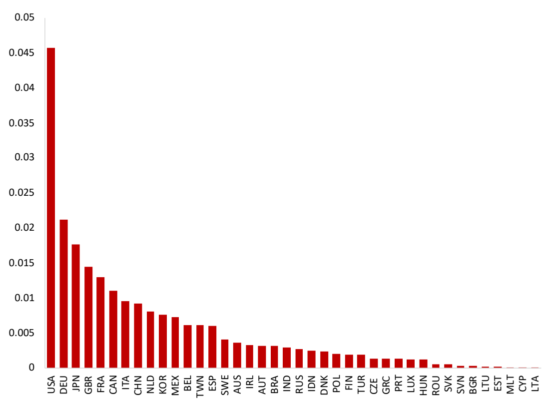
This appendix presents the world's real GDP elasticities (and the normalized elasticities) with respect to internal and external frictions for the years 2000 and 2005. Figure E.2 shows the elasticities for the year 2000 and Figure E.3 presents the elasticities for the year 2005.

Figure E.2: World's real GDP elasticity to internal versus external frictions (2000)

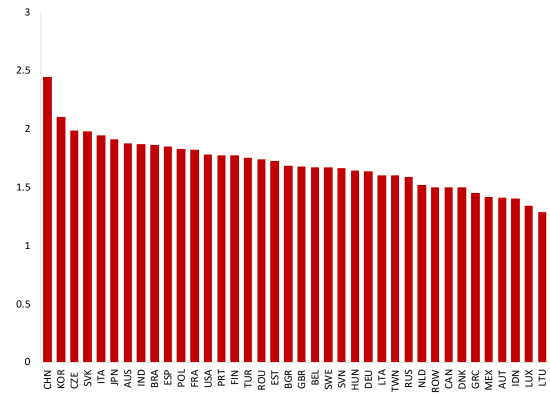
(a) Elasticity to internal frictions



(b) Elasticity to external frictions



(c) Normalized elasticity to internal frictions



(d) Normalized elasticity to external frictions

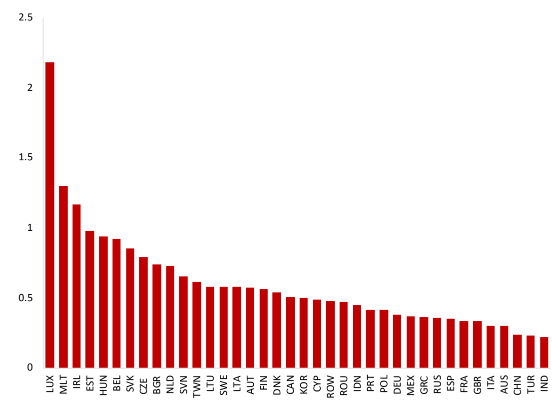
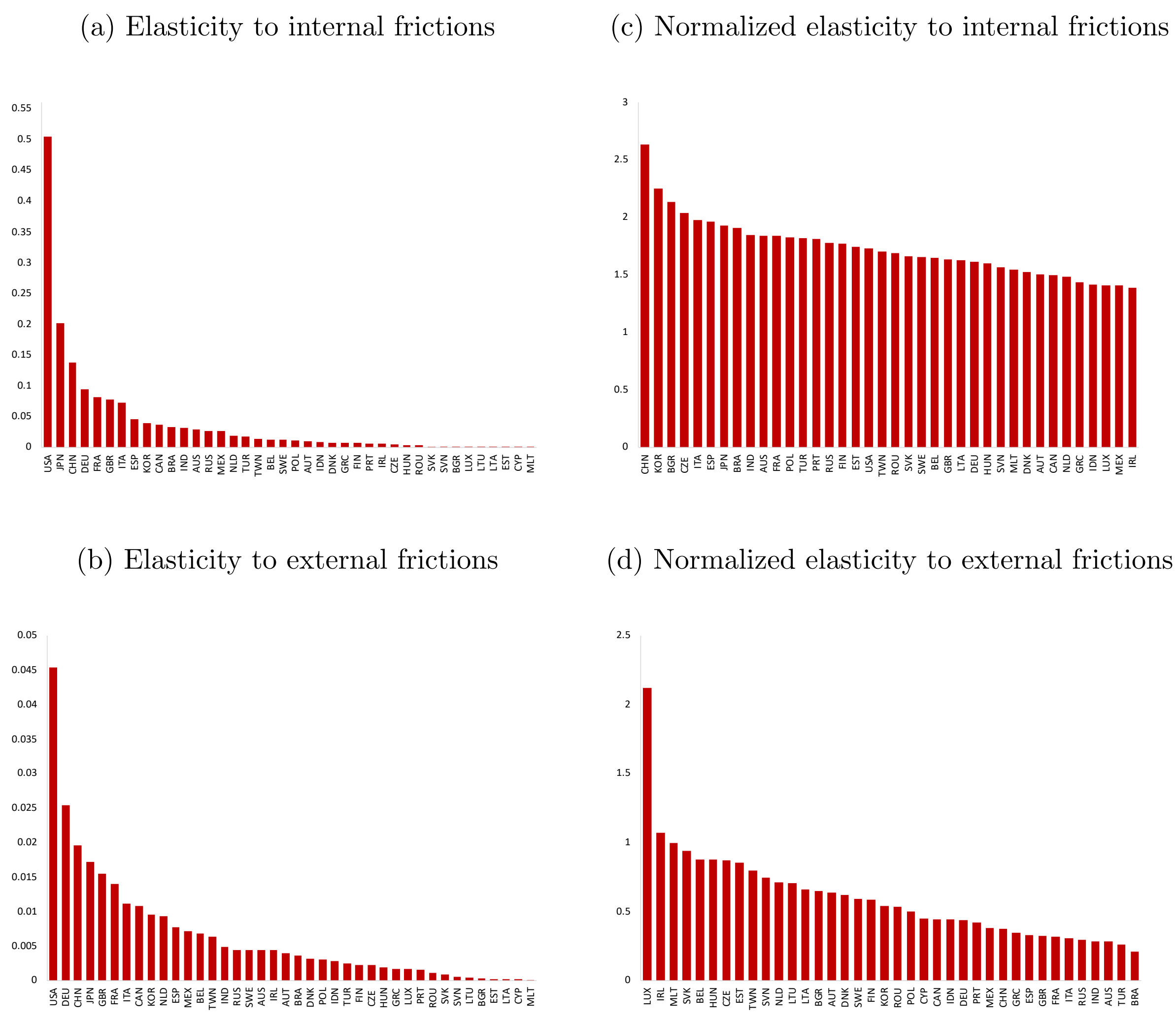


Figure E.3: World's real GDP elasticity to internal versus external frictions (2005)



(a) Elasticity to internal frictions

(b) Elasticity to external frictions

(c) Normalized elasticity to internal frictions

(d) Normalized elasticity to external frictions

E.3 Decomposition of the World's GDP Elasticity to Internal Frictions

This section of the appendix shows the ranking of the contribution of the own sector, the domestic network and the international network to the computed elasticities of the world's GDP to changes in internal frictions. The formula used to decompose the world's GDP elasticity to changes in internal frictions is given by

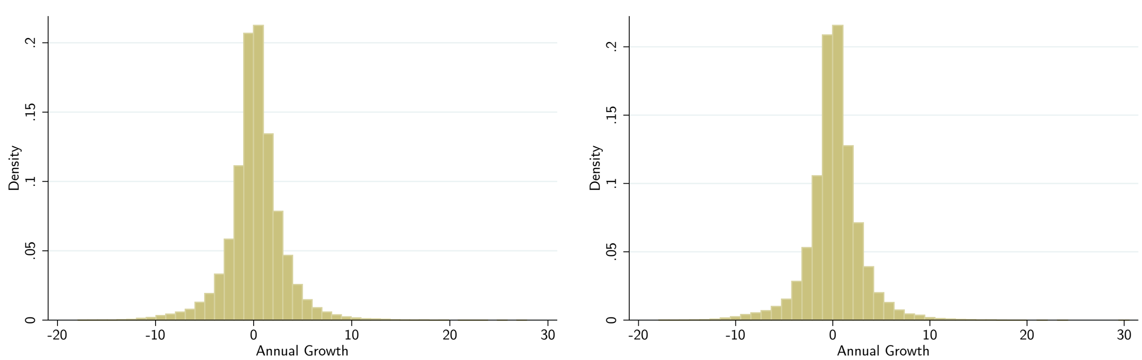
$$\zeta_{ij} = \omega_i \triangle GDP_i \frac{\omega_{ij} \triangle w_i / P_{ij}}{\sum_k \omega_{ik} \triangle w_i / P_{ik}} + \omega_i \triangle GDP_i \frac{\sum_{k \neq j} \omega_{ik} \triangle w_i / P_{ik}}{\sum_k \omega_{ik} \triangle w_i / P_{ik}} + \sum_{n \neq i} \omega_n \triangle GDP_n$$

where $\omega_i = \frac{VA_i}{\sum_n VA_n}$ and $\omega_{ij} = \frac{VA_{ij}}{\sum_k VA_{ik}}$. The term ζ_{ij} measures the change in world's real GDP due to changes in frictions in a given country/sector. The terms $\triangle GDP_i$ and VA_i refers to the change in real GDP (real wage) and value added in country i, respectively. The first term on the right-hand side of this decomposition describes the contribution of the change in the own sector real GDP. The second term shows the contribution of the local network (changes in real GDP in other sectors in the domestic economy), and the third term describes the contribution from the changes in real GDP in other sectors in foreign countries. Figure E.5 presents the results for each country in our sample.

E.4 Distribution of growth are in internal frictions 1996-2011 and 1995-2010

This section of the appendix presents the distribution of the growth rate of internal frictions over the periods 1996-2011, and 1995-2010. Figure E.4 presents the histograms of the annual growth in internal frictions for these two sample periods.

Figure E.4: Annual Growth Rate in Global Internal frictions



(a) Period 1995-2010

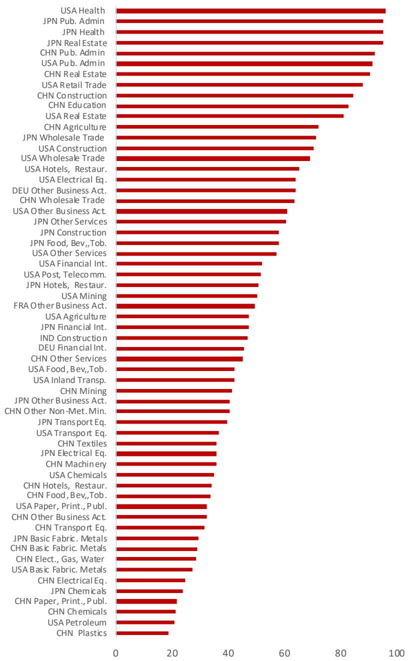
(b) Period 1996-2011

E.5 The Role of Internal Frictions on the Great Recession across Countries

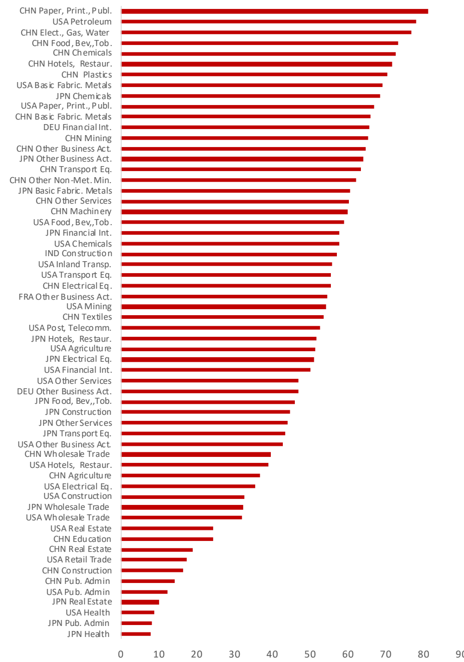
In this section of the appendix, we show the contribution of each country in our sample to the change in internal and external trade during the Great Recession. Table E.1 shows the observed change in internal and external trade in each country, and Table E.2 presents the percent contribution to this observed change in internal and external trade.

Figure E.5: Decomposition of the World's real GDP elasticity to internal friction)

(a) Contribution of own sector (percent)



(b) Contribution of domestic network (percent)



(c) Contribution of external network (percent)

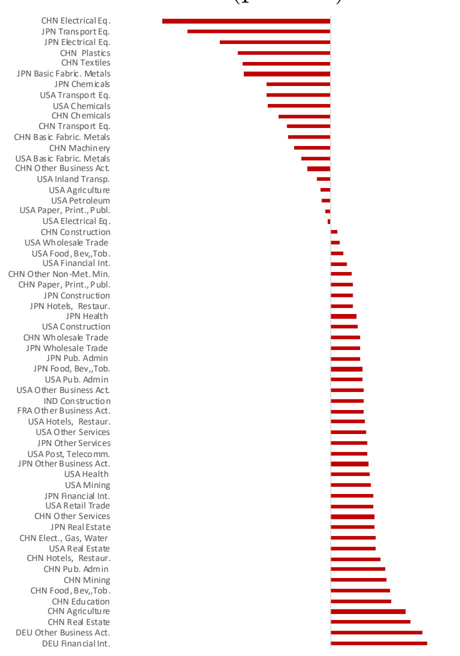


Table E.1: Trade change during the Great Recession (percent)

	External trade observed change 2007-2009	Internal trade observed change 2007-2009	Within-Sector trade observed change 2007-2009
AUS	-0.47%	8.41%	14.05%
AUT	-14.07%	8.07%	20.25%
BEL	-9.33%	-2.27%	6.69%
BGR	-17.52%	34.51%	31.25%
BRA	2.98%	13.48%	7.61%
CAN	-16.37%	-4.85%	5.10%
CHN	-0.24%	46.29%	43.31%
CYP	-2.45%	20.19%	36.77%
CZE	-12.65%	1.34%	-4.66%
DEU	-18.63%	-4.44%	-4.43%
DNK	-13.04%	-2.42%	6.69%
ESP	-18.56%	5.91%	8.03%
EST	-23.13%	-13.97%	-9.57%
FIN	-19.60%	0.57%	1.14%
FRA	-12.24%	0.13%	3.99%
GBR	-19.59%	-22.61%	-24.09%
GRC	-11.35%	-1.08%	2.70%
HUN	-19.64%	-3.67%	4.81%
IDN	5.23%	33.50%	30.21%
IND	-12.86%	12.02%	11.54%
IRL	-2.19%	-22.77%	-12.32%
ITA	-21.24%	-6.39%	-6.80%
JPN	-14.66%	10.83%	7.02%
KOR	-7.77%	-14.10%	-9.45%
LTU	-2.88%	-9.04%	-12.65%
LUX	1.89%	26.29%	-27.82%
LTA	-21.47%	-7.93%	-7.72%
MEX	-25.06%	-11.67%	-12.42%
MLT	-10.08%	12.91%	17.65%
NLD	-0.47%	2.77%	9.21%
POL	-12.01%	-2.59%	-4.62%
PRT	-14.38%	4.34%	4.17%
ROU	-10.65%	-0.66%	6.18%
RUS	-16.59%	1.09%	-6.12%
ROW	-12.41%	31.61%	28.19%
SVK	-10.51%	19.05%	33.86%
SVN	-20.77%	3.86%	4.36%
SWE	-18.57%	-12.73%	-16.66%
TUR	-23.93%	-2.38%	1.94%
TWN	-18.60%	-7.38%	-19.61%
USA	-14.57%	-8.02%	-6.15%

Table E.2: Contribution of internal frictions during the Great Recession (percent)

	External Trade Contribution of internal frictions	Internal Trade Contribution of internal frictions
AUS	195.57%	-24.52%
AUT	13.04%	-24.10%
BEL	11.41%	54.48%
BGR	15.17%	0.31%
BRA	-57.10%	-1.20%
CAN	-1.17%	15.46%
CHN	188.64%	7.91%
CYP	64.58%	-13.35%
CZE	14.33%	-69.13%
DEU	9.64%	8.09%
DNK	11.50%	65.26%
ESP	20.37%	-35.20%
EST	-1.31%	17.52%
FIN	12.09%	-562.96%
FRA	19.68%	41.93%
GBR	23.56%	-2.93%
GRC	52.01%	-170.85%
HUN	12.76%	24.87%
IDN	-52.16%	26.57%
IND	22.11%	20.64%
IRL	112.28%	20.05%
ITA	-0.50%	15.01%
JPN	26.22%	-25.83%
KOR	49.68%	6.83%
LTU	-68.74%	68.45%
LUX	-52.43%	1.56%
LTA	2.64%	18.19%
MEX	-4.44%	-9.54%
MLT	40.87%	14.01%
NLD	525.62%	80.46%
POL	9.33%	40.75%
PRT	19.36%	-12.02%
ROU	14.82%	-264.65%
RUS	5.80%	-29.42%
ROW	32.65%	17.29%
SVK	16.85%	-21.38%
SVN	13.05%	-78.78%
SWE	11.81%	5.38%
TUR	1.47%	-76.27%
TWN	17.21%	36.48%
USA	4.48%	22.36%