On-line Appendix
Anchored Inflation Expectations*

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Abstract

This document provides a detailed description of data and some technical details on the marginalized particle filter; the model; and estimation.

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A Data

We use different data sources, divided in three broad categories. All data are collected at a quarterly frequency and expressed in term of annualized percentage growth.

Inflation data:

Price growth for the US is measured by Consumer Price Inflation (CPI). Here we use the 'Consumer Price Index for All Urban Consumers: All Items in U.S. City Average', seasonally adjusted measure from US Bureau of Labor Statistics, downloaded from FRED database of the Federal Reserve Bank of St Louis (Mnemonic: CPIAUCSL) in 2016. The sample period is 1955Q1-2015Q4. This series is publicly available.

Bibliography reference: Bureau of Labor Statistics (1955-2015)

We also measure price inflation in eight other countries: France, Germany, Italy, Spain in the European Monetary Union and, additionally, Canada, Japan, Sweden and Switzerland. For each country, we use the national consumer price indexes. The non-seasonally adjusted variables have been re-downloaded from Haver Analytics in 2021 (for the purpose of this replication files). The series are then seasonally adjusted using the X-13ARIMA-SEATS filter (http://www.seasonal.website/). The sample period for the price level is 1955Q1-2015Q4. These data are proprietary and accessed from an account at the Deutsche Bundesbank. The data source is the Organization for Economic Cooperation and Development (OECD) and the mnemonic are: France (N132PC@OECDMEI), Germany (N134PC@OECDMEI), Italy (N136PC@OECDMEI), Spain (N184PC@OECDMEI), Canada (N156PC@OECDMEI), Japan (N158PC@OECDMEI), Sweden (N144PC@OECDMEI), Switzerland (N146PC@OECDMEI).

Bibliography references: Organization for Economic Cooperation and Development (1955-2015a), Organization for Economic Cooperation and Development (1955-2015b), Organization for Economic Cooperation and Development (1955-2015c), Organization for Economic Cooperation and Development (1955-2015h), Organization for Economic Cooperation and Development (1955-2015f), Organization for Economic Cooperation and Development (1955-2015g), Organization for Economic Cooperation and Development (1955-2015d) and Organization for Economic Cooperation and Development (1955-2015e).

Finally we use the price of crude oil as measured by the: 'Spot Crude Oil Price: West Texas Intermediate'. This series was created by the Federal Reserve Bank of St. Louis to expand the history of the monthly West Texas Intermediate oil price series in FRED

(mnemonic: WTISPLC) which combines 'https://fred.stlouisfed.org/series/OILPRICE' and 'https://fred.stlouisfed.org/series/MCOILWTICO'. The sample period is 1955Q1-2015Q4. The series was downloaded in 2016 from the FRED database and it is publicly available.

Bibliography reference: Federal Reserve Bank of St Louis (1955-2015)

Survey-Based Expectations for the US

We measure expectations of both professional forecasters and households. We use both short-term forecast (up to one year ahead) and long-term forecasts. Starting with professional forecasters we use multiple measures for their short-term forecasts. As for the inflation measures the data are assembled at quarterly frequency. We obtained CPI inflation forecasts ranging from one-quarter ahead to four-quarter-ahead from the *Survey of Professional Forecasters*. The data (name: CPI Inflation Rate (CPI: CPI3-CPI6)) include the mean forecast across individual forecasters and covers the sample 1981Q3-2015Q4. The data are publicly available and can be obtained from the Federal Reserve Bank of Philadelphia (https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/survey-of-professional-forecasters). They were downloaded in 2016.

Bibliography references: Federal Reserve Bank of Philadelphia (1981-2015b), Federal Reserve Bank of Philadelphia (1981-2015d), Federal Reserve Bank of Philadelphia (1981-2015c) and Federal Reserve Bank of Philadelphia (1981-2015a)

Our dataset also include short-term forecasts from the *Livingston Survey*. It includes three time series available at bi-annual frequency. The first two series measure the mean six-month-ahead forecasts of CPI inflation across forecast participants (files: 'Growth of Mean Forecast for the Levels of Survey Variables'). In the first series, covering the period 1955Q1-2015Q4, the inflation forecast is calculated relative to a base period (variable name: 'G-BP-To-6M'). In the second series the inflation forecast is calculated relative to a base forecast: the series is shorter and starts in 1992Q2 (variable name: 'G-ZM-To-6M'). The third series measure the median twelve-month forecast across forecasters, calculated relative to a base period (variable name: 'G-BP-To-12M'). The series is included in the files:'Growth of Median Forecast for the Levels of Survey Variables' and is available since 1955Q1. The data are publicly available and can be obtained from the Federal Reserve Bank of Philadelphia (https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/livingston-survey).

Bibliography references: Federal Reserve Bank of Philadelphia (1955-2015b), Federal Reserve Bank of Philadelphia (1992-2015) and Federal Reserve Bank of Philadelphia (1955-2015a)

Turning to long-term forecasts, we draw from many different data sources. In detail, from the *Survey of Professional Forecasters* we use the 10-Year CPI Inflation Rate (CPI10) forecast (mean across forecasters) available at quarterly for the sample 1991Q4-2015Q4; and the 5-Year Forward 5-Year Annual-Average CPI Inflation Rate (CPIF5) (mean across forecasters) available at quarterly frequency for the sample 2005Q3-2015Q4.

Bibliography references: Federal Reserve Bank of Philadelphia (1991-2015) and Federal Reserve Bank of Philadelphia (2005-2015)

We also use one-to-ten years ahead (CPI-10Y) mean forecast from the *Livingston Survey* available for the sample 1990Q2-2015Q4. These series were obtained in 2016 from the Federal Reserve Bank of Philadelphia (see above for details) and are publicly available.

Bibliography references: Federal Reserve Bank of Philadelphia (1990-2015)

In addition we use long-term forecasts from the *BlueChip Economic Indicators* and *BlueChip Financial Forecasts* surveys of professional forecasters. These series are proprietary data and were collected at the Federal Reserve Bank of New York in 2016. We measure five-to-ten year mean CPI inflation (consensus) forecasts from both surveys from the 'five year averages' column in the 'Long-Range Consensus U.S. Economic Projections' (Economic Indicators) and 'Long-Range Estimates' (Financial Forecasts) sections respectively. The data are available bi-annually. The forecasts from Economic Indicators cover the range 1984Q1-2015Q4, while the Financial Forecasts are available for the sample 1986Q1-2015Q4. We also use the one-to-ten-year consensus forecast for CPI inflation from BlueChip Economic Indicators that is publicly available from the Federal Reserve Bank of Philadelphia (https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/survey-of-professional-forecasters: 'Additional 10-Year-Ahead Inflation Forecasts from Other Sources'; name of the file: 'Additional-CPIE10'). These data, downloaded in 2016, are available at bi-annual frequency for the sample 1979Q4-1991Q1.

Bibliography references: Wolters Kluwer (1984-2015a), Wolters Kluwer (1986-2015) and Wolters Kluwer (1984-2015b)

Another publicly available source for long-run forecasts is Table 1 in Levin and Taylor (1998) which includes five-to-ten year ahead average inflation forecast from the households *Michigan Survey of Consumers* (sample 1975Q2-1977Q2); and one-to-ten years CPI inflation forecast from portfolio managers in the *Decision-Makers Poll* (sample 1978Q3-1980Q4). The data is available at irregular frequency.

Bibliography reference: Levin and Taylor (2010)

Finally, we measure five-to-ten-year ahead forecasts from the *Consensus Forecasts* survey published by Consensus Economics (https://www.consensuseconomics.com/). The data is proprietary and was collected at the Federal Reserve Bank of New York in 2016. Specifically, we use the 'Consensus 6-10 Year' long term forecast (mean across forecasters) available bi-annually for the sample 1990Q2-2015Q4.

Bibliography reference: Consensus Economics (1990-2015a)

For the US, we also use household inflation forecasts from the *Michigan Survey of Consumers* (https://data.sca.isr.umich.edu/). This data is also proprietary and has been collected at the Federal Reserve Bank of New York in 2016. The long-term forecasts (five-to-ten years ahead) are measured by the median: 'Expected Change in Prices During the Next 5 Years' (found in Table 33 in the survey). The short term forecasts are measured by the median: 'Expected Change in Prices During the Next Year' (found in Table 32 in the survey). The short-term forecasts are available at a quarterly frequency (with gaps at the beginning of the sample) for the sample 1968Q3-2015Q4. Long-term forecasts have the same format and cover the range: 1979Q1-2015Q4.

Bibliography references: University of Michigan (1968-2015) and University of Michigan (1979-2015)

Survey-Based Expectations for other countries:

We use the *Consensus Forecasts* survey of professional forecasters. The data is proprietary and was accessed at the Federal Reserve Bank of New York in 2016. To measure long-term forecasts, we use the 'Consensus 6-10 Year' long term CPI forecast (mean across forecasters) available bi-annually for the countries we study. While all variables are available up to 2015Q4, the beginning of the sample varies with each country: France (1990Q2), Germany (1990Q2), Italy (1990Q2), Spain (1995Q2), Canada (1989Q4), Japan (1990Q2),

Sweden (1995Q2), Switzerland (1998Q4). Short-term expectations are measured by using the consensus (mean) year-on-year CPI inflation forecast for current and following calendar year, for each country. We use forecasts for the current year and next year (year-over-year) taken in the middle month of each quarter. Unlike the long-term forecasts, for all countries the data set covers the sample 1990Q2-2015Q4.

Bibliography references: Consensus Economics (1990-2015a) and Consensus Economics (1990-2015b)

B Model summary

In the sequel we present the marginalized particle filter and smoother. For ease of notation note that we use φ_t for $\tilde{\varphi}_t$, and $(\varepsilon_t,\ \mu_t)$ for $(\tilde{\varepsilon}_t,\ \tilde{\mu}_t)$ in the main text. Recall the model is summarized by the following equations

$$\begin{aligned}
\pi_t &= (1-\gamma)\,\Gamma\bar{\pi}_t + \gamma\pi_{t-1} + \varphi_t + \mu_t \\
\bar{\pi}_t &= \bar{\pi}_{t-1} + \mathbf{k}_t^{-1} \times f_{t-1} \\
\mathbf{k}_t &= I(\bar{\pi}_{t-1}) \times (\mathbf{k}_{t-1} + 1) + (1 - I(\bar{\pi}_{t-1})) \times \bar{g}^{-1} \\
f_t &= (1-\gamma)\,(\Gamma - 1)\,\bar{\pi}_t + \mu_t + \epsilon_t \\
\varphi_t &= \rho_\varphi \varphi_{t-1} + \epsilon_t,
\end{aligned}$$

where the function $I(\bar{\pi}_t)$ is described as

$$I\left(\bar{\pi}\right) = \begin{cases} 1, \text{ if } \left|(1-\gamma)\,(\Gamma-1)\,\bar{\pi}\right| \leq \overline{\theta}\sigma^{\eta} \\ \\ 0, \text{ otherwise.} \end{cases}$$

The model can then be re-written as

$$\begin{aligned}
\mathbf{k}_t &= f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) \\
\bar{\pi}_t &= f_{\bar{\pi}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) + f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1})^{-1} \times \eta_{t-1} \\
\eta_t &= \mu_t + \epsilon_t \\
\varphi_t &= \rho_\varphi \varphi_{t-1} + \epsilon_t \\
\pi_t &= (1-\gamma_p)\,\Gamma f_{\bar{\pi}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) + (1-\gamma_p)\,\Gamma f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1})^{-1}\eta_{t-1} \\
&\quad +\gamma\pi_{t-1} + \rho_\varphi\varphi_{t-1} + \epsilon_t + \mu_t.
\end{aligned}$$

where

$$\begin{aligned}
f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) &= I(\bar{\pi}_{t-1}) \times (\mathbf{k}_{t-1} + 1) + (1 - I(\bar{\pi}_{t-1})) \times \bar{g}^{-1}, \\
f_{\bar{\pi}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) &= \left[1 - (1-\Gamma)\,(1-\gamma)\, f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1})^{-1}\right] \bar{\pi}_{t-1}.
\end{aligned}$$

We can also re-write the system in matrix notation. One way to write it is by separating linear and nonlinear states. For the linear variables we have:

$$\xi_t = f_\xi\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) + A_\xi\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right)\xi_{t-1} + S_\xi \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix},$$

where

$$\xi_t = \begin{bmatrix} \eta_t \\ s_t \\ \pi_t \end{bmatrix};$$

$$f_\xi\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) = \begin{bmatrix} \mathbf{0}_{2\times 1} \\ (1-\gamma)\,\Gamma f_{\bar{\pi}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) \end{bmatrix};$$

$$A_\xi\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_\varphi & 0 \\ (1-\gamma)\,\Gamma f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1})^{-1} & \rho_\varphi & \gamma \end{bmatrix};$$

$$S_\xi = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

For the nonlinear variables we can express

$$\mathbf{k}_t = f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1})$$

and

$$\bar{\pi}_t = f_{\bar{\pi}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) + A_{\bar{\pi}}\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right)\xi_{t-1}.$$

where

$$A_{\bar{\pi}}\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) = \begin{bmatrix} f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1})^{-1} \\ \mathbf{0}_{2\times 1} \end{bmatrix}'.$$

Notice that $\mathbf{k}_t$ does not depend on the linear state. In yet another formulation we can express the system in more compact notation:

$$\begin{aligned} \mathbf{k}_t &= f_{\mathbf{k}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) \\ \begin{bmatrix} \bar{\pi}_t \\ \xi_t \end{bmatrix} &= f\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) + A\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right)\xi_{t-1} + \begin{bmatrix} 0 \\ S_\xi \end{bmatrix}\begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix} \end{aligned}$$

where

$$\begin{aligned} f\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) &= \begin{bmatrix} f_{\bar{\pi}}(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}) \\ f_\xi\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) \end{bmatrix} \\ A\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) &= \begin{bmatrix} A_{\bar{\pi}}\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) \\ A_\xi\left(\bar{\pi}_{t-1}, \mathbf{k}_{t-1}\right) \end{bmatrix} \end{aligned}$$

and

$$\Sigma = E\left(\begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}\begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}'\right)$$

is the variance covariance of the innovations.

This notation is used below when computing the smoothed states. Finally, given the

data $Y_T = y_1 ... y_T$, the model observation equation is

$$y_t = h_{0,t} + h_{\bar{\pi},t}\bar{\pi}_t + H_t'\xi_t + R_t^{1/2}e_t^o$$

where the vectors and matrices h_0, $h_{\bar{\pi}}$, H_t' and R_t are defined to be consistent with the timing of available data, and e_t denotes observation errors.

C Algorithm for the Marginalized particle filter

This follows Schön, Gustafsson, and Nordlund (2005). For details of the particle filter we use Kitagawa (1996). We are looking for the following distributions:

$$p\left(\xi_t, \left[\bar{\pi}_t, \mathbf{k}_t\right] | Y_t\right) = p\left(\xi_t | \left[\bar{\pi}_t, \mathbf{k}_t\right], Y_t\right) \times p\left(\left[\bar{\pi}_t, \mathbf{k}_t\right] | Y_t\right).$$

The following describes the algorithm. Discussion and proofs are given below.

Algorithm:

1. Initialization. Choose $\bar{\pi}_{1|0}^{(i)}$, $\mathbf{k}_{1|0}^{(i)}$ from some distributions (drawing from normal for $\bar{\pi}$ and $\mathbf{k}_{0|0}^{(i)} = \bar{\mathbf{k}}_0$ (or draw from $U(0, \bar{g}^{-1})$), and $\xi_{1|0}^{(i)}$, $P_{1|0}^{(i)} = [\xi_0,\ P_{1|0}]$, where $P_{1|0}$ denotes the initial precision matrix in the linear Kalman filter.

2. For each $t = 1...T$, compute

$$\Omega_t = H_t' P_{t|t-1} H_t + R_t$$

and its inverse. For $i = 1, ..., N$, evaluate the importance weights

$$q_t^{(i)} = p\left(y_t | \bar{\pi}_{t|t-1}^{(i)}, \xi_{t|t-1}^{(i)}\right).$$

In order to do this, use

$$p\left(y_t | \bar{\pi}_{t|t-1}^{(i)}, \xi_{t|t-1}^{(i)}\right) = N\left(h_{0,t} + h_{\bar{\pi},t}\bar{\pi}_{t|t-1}^{(i)} + H_t'\xi_{t|t-1}^{(i)}, H_t' P_{t|t-1} H_t + R_t\right)$$

so that

$$q_t^{(i)} = w_{t-1}^{(i)} \times |\Omega_t|^{-1/2} \times$$

$$\exp\left\{-\frac{1}{2}\left(y_t - h_{0,t} - h_{\bar{\pi},t}\bar{\pi}_{t|t-1}^{(i)} - H_t'\xi_{t|t-1}^{(i)}\right)' \times \Omega_t^{-1} \times \left(y_t - h_{0,t} - h_{\bar{\pi},t}\bar{\pi}_{t|t-1}^{(i)} - H_t'\xi_{t|t-1}^{(i)}\right)\right\}.$$

where $w_{t-1}^{(i)}$ denotes the particle weight from the previous period. (In the expression above we eliminate the constant coefficient that is independent of (i) and the model parameters.)

3. Re-sampling.[1] Provided the number of effective particles (effective sample size), computed as

$$ESS = \frac{1}{\sum \left(w_t^{(j)}\right)^2},$$

falls below the threshold ($ESS < 0.75 * N$) we re-sample such that

$$p\left(\left[\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}\right] = \left[\bar{\pi}_{t|t-1}^{(j)}, \mathbf{k}_{t|t-1}^{(j)}\right]\right) = \frac{q_t^{(j)}}{\sum q_t^{(j)}}.$$

Here we use **systematic resampling**: see Kitagawa (1996), Hol, Schön, and Gustafsson (2006) for a discussion of resampling and different methods. The outcome of systematic resampling is a discrete distribution with particles $\left\{\bar{\pi}_{t|t}^{(k)}, \mathbf{k}_{t|t}^{(k)}\right\}_{k=1}^{N}$ and corresponding weights $w_t(i) = 1/N$ for $i = 1, ..., N$. In case of not resampling the weights are $w_t(i) = q_t^{(j)} / \sum q_t^{(j)}$.

4. Linear measurement equation: for $i = 1, ..., N$, evaluate

$$\begin{aligned}
\xi_{t|t}^{(i)} &= \xi_{t|t-1}^{(i)} + K_t\left(y_t - h_{0,t} - h_{\bar{\pi},t}\bar{\pi}_{t|t}^{(i)} - H_t'\xi_{t|t-1}^{(i)}\right) \\
K_t &= P_{t|t-1}H_t\Omega_t^{-1} \\
P_{t|t} &= P_{t|t-1} - K_t H_t' P_{t|t-1}
\end{aligned}$$

5. Particle filter prediction. For $i = 1, ..., N$, compute

$$\mathbf{k}_{t+1|t}^{(i)} = f_{\mathbf{k}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)})$$

and then draw $\bar{\pi}_{t+1|t}^{(i)}$ from distribution

$$\begin{aligned}
& p\left(\bar{\pi}_{t+1}|Y_t, \bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}\right) \\
= \; & N\left(f_{\bar{\pi}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) + f_{\mathbf{k}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)})^{-1} z_{t|t}^{(i)},\ f_{\mathbf{k}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)})^{-2} P_{t|t}^{[\eta,\eta]}\right)
\end{aligned}$$

[1] This means (roughly speaking) increasing the number of particles receiving high weight $\left(\frac{q_t^{(j)}}{\sum q_t^{(j)}}\right)$ and eliminating particles with very low weight, while keeping the number of particles equal to N.

where we use the notation: $P_{t|t}^{[x,z]} = P_{t|t}(x,z)$.

6. Linear model prediction

$$\begin{aligned}
\tilde{\xi}_{t|t}^{(i)} &= \xi_{t|t}^{(i)} + \tilde{K}_t^{(i)} \left(\bar{\pi}_{t+1|t}^{(i)} - f_{\bar{\pi}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) - f_{\mathbf{k}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)})^{-1} z_{t|t}^{(i)} \right) \\
\xi_{t+1|t}^{(i)} &= f_{\xi}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) + A_{\xi}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) \tilde{\xi}_{t|t}^{(i)} \\
P_{t+1|t} &= Q_{\xi} + \tilde{P}_{t|t}; \quad Q_{\xi} = S_{\xi} \Sigma S_{\xi}';
\end{aligned}$$

where

$$\begin{aligned}
\tilde{K}_t^{(i)} &= P_{t|t} A_{\bar{\pi}}' \left(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)} \right) \left(A_{\bar{\pi}} \left(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)} \right) P_{t|t} A_{\bar{\pi}}' \left(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)} \right) \right)^{-1} \\
&= f_{\mathbf{k}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) \begin{bmatrix} 1 \\ P_{t|t}^{[\eta,\varphi]} / P_{t|t}^{[\eta,\eta]} \\ P_{t|t}^{[\eta,\pi]} / P_{t|t}^{[\eta,\eta]} \end{bmatrix};
\end{aligned}$$

and

$$A_l(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) \tilde{K}_t^{(i)} = f_{\mathbf{k}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) \begin{bmatrix} 0 \\ \frac{P_{t|t}^{[\eta,s]}}{P_{t|t}^{[\eta,\eta]}} \rho_{\varphi} \\ \frac{P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} \gamma + \frac{P_{t|t}^{[\eta,s]}}{P_{t|t}^{[\eta,\eta]}} \rho_{\varphi} + f_{\mathbf{k}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)})^{-1} (1-\gamma) \Gamma \end{bmatrix}$$

$$\tilde{P}_{t|t} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \iota_1 & \iota_2 + \iota_1 \\ 0 & \iota_2 + \iota_1 & 2\iota_2 + \iota_1 + \left(P_{t|t}^{[\pi,\pi]} - \frac{\left(P_{t|t}^{[\eta,\pi]} \right)^2}{P_{t|t}^{[\eta,\eta]}} \right) \gamma^2 \end{bmatrix}$$

where

$$\begin{aligned}
\iota_1 &= \left(P_{t|t}^{[\varphi,\varphi]} - \frac{\left(P_{t|t}^{[\eta,\varphi]} \right)^2}{P_{t|t}^{[\eta,\eta]}} \right) \rho_{\varphi}^2; \\
\iota_2 &= \left(-\frac{P_{t|t}^{[\eta,\varphi]} P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} + P_{t|t}^{[s,\pi]} \right) \rho_{\varphi} \gamma.
\end{aligned}$$

Notice that, importantly, $P_{t+1|t}$ is independent of particles. This is key for a fast evaluation of the likelihood.

Finally, the log-Likelihood is approximated by

$$L(\cdot) = \sum_{t=1}^{T} \log p\left(y_t | Y_{t-1}\right)$$

where

$$\begin{aligned} p\left(y_t|Y_{t-1}\right) &= p\left(y_t|\xi_t, [\bar{\pi}_t, \mathbf{k}_t]\right) p\left(\xi_t, [\bar{\pi}_t, \mathbf{k}_t]\,|Y_{t-1}\right) \\ &= p\left(y_t|\xi_t, [\bar{\pi}_t, \mathbf{k}_t]\right) p\left(\xi_t|\,[\bar{\pi}_t, \mathbf{k}_t]\,, Y_{t-1}\right) p\left([\bar{\pi}_t, \mathbf{k}_t]\,|Y_{t-1}\right), \\ &\rightarrow \\ L(\cdot) &\simeq \sum_{t=1}^{T} \log\left(\sum_{i=1}^{N} q_t^{(i)}\right). \end{aligned}$$

Algorithm ends.

In the estimation performed in this paper we set the number of particles $N = 2500$; To avoid injecting randomness in the calculation of the likelihood, the "chatter" of changing random numbers, we keep the simulated (standardized) innovations constant as we evaluate different parameters—see the discussion in Fernandez-Villaverde and Rubio-Ramirez (2007). In detail, we fix the following innovations: random initial conditions for the nonlinear state variables; random draws to compute shocks in the nonlinear prediction step; and random draws in the resampling step.

D Marginalized Smoother

We follow Lindsten and Schön (2013) using the 'joint backward simulation' Rao-Blackwellised particle smoother. See also Godsill, Doucet, and West (2004). We compute a smoothed path for the states, conditional on a parameter draw, for the sample $t = 1...T$. The algorithm, in conjunction with the forward filter above, allows producing the full distribution of state and parameters using Carter and Khon (1994).

The objective is to draw $j = 1...M$ trajectories of the model variables $\left\{\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\mathbf{k}}_{t|T}^{(j)}, \tilde{\xi}_{t|T}^{(j)}\right\}_{t=1}^{T}$. The forward filter allows drawing $\tilde{\bar{\pi}}_{T|T}^{(j)}$ $\tilde{\mathbf{k}}_{T|T}^{(j)}$ from the empirical distribution of $\left\{\bar{\pi}_{t|t}^{(k)}, \mathbf{k}_{t|t}^{(k)}\right\}_{k=1}^{N}$ where each particle has weight $w_t^{(k)}$. Moreover, conditional on the draw $\tilde{\bar{\pi}}_{t|T}^{(j)}$, $\tilde{\mathbf{k}}_{T|T}^{(j)}$, it allows drawing the linear state $\tilde{\xi}_{T|T}^{(j)}$ from the normal distribution $N\left(\xi_{T|T}^{(j)},\ P_{T|T}\right)$. Given this we

compute M paths as follows.

Algorithm:

For $t = T-1:-1:1$

For each $j = 1...M$

For each $i = 1...N$, compute

$$w^{j}_{t|t+1}(i) = \frac{w_t(i) p\left(\tilde{\bar{\pi}}^{(j)}_{t+1|t+1}, \tilde{\mathbf{k}}^{(j)}_{t+1|t+1}, \tilde{\xi}^{(j)}_{t+1|t+1} | \bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}, Y_t\right)}{\sum_{k}^{N} w_t(k) p\left(\tilde{\bar{\pi}}^{(j)}_{t+1|t+1}, \tilde{\mathbf{k}}^{(j)}_{t+1|t+1}, \tilde{\xi}^{(j)}_{t+1|t+1} | \bar{\pi}^{(k)}_{t|t}, \mathbf{k}^{(k)}_{t|t}, Y_t\right)}$$

where the last line makes use of the fact that $w_t(i) = 1/N$ because of resampling in the forward filter. The probability distribution above can be expressed as

$$p\left(\bar{\pi}_{t+1|T}, \mathbf{k}_{t+1|T}, \xi_{t+1|T} | \bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}, Y_t\right) =$$

$$p\left(\bar{\pi}_{t+1|T}, \xi_{t+1|T} | \bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}, Y_t\right) \times p\left(\mathbf{k}_{t+1|T} | \bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}, Y_t\right),$$

which uses $\mathbf{k}_{t+1|t} = f_{\mathbf{k}}(\bar{\pi}_t, \mathbf{k}_t)$. We can then evaluate

$$p\left(\tilde{\bar{\pi}}^{(j)}_{t+1|T}, \tilde{\xi}^{(j)}_{t+1|T} | \bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}, Y_t\right) \times 1_{\tilde{\mathbf{k}}^{(j)}_{t+1} = f_{\mathbf{k}}\left(\bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}\right)} =$$

$$\begin{cases} p^{j,i}_{t|t+1}, \text{ if } \tilde{\mathbf{k}}^{(j)}_{t+1|T} = f_{\mathbf{k}}\left(\bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}\right) \\ \\ \mathbf{0}, \text{ otherwise} \end{cases}$$

where

$$\begin{aligned}
p^{j,i}_{t|t+1} &= \propto \exp\left\{-\frac{1}{2}\ln\left(\left|\tilde{\Omega}^{(i)}_t\right|\right) - \frac{1}{2}\left(\eta^{j,i}_{t+1}\right)' \times \left(\tilde{\Omega}^{(i)}_t\right)^{-1}\left(\eta^{j,i}_{t+1}\right)\right\} \\
\eta^{j,i}_{t+1} &= \begin{bmatrix} \tilde{\bar{\pi}}^{(j)}_{t+1|t+1} \\ \tilde{\xi}^{(j)}_{t+1|t+1} \end{bmatrix} - \left[f\left(\bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}\right) + A\left(\bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}\right)\xi^{(i)}_{t|t}\right]. \\
\tilde{\Omega}^{(i)}_t &= Q + A\left(\bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}\right) P_{t|t} A\left(\bar{\pi}^{(i)}_{t|t}, \mathbf{k}^{(i)}_{t|t}\right)' \\
Q &= \begin{bmatrix} 0 & 0 \\ 0 & S_\xi \Sigma S'_\xi \end{bmatrix}.
\end{aligned}$$

and where we can use

$$1_{\tilde{\mathbf{k}}_{t+1}^{(j)}=f_{\mathbf{k}}\left(\bar{\pi}_{t|t}^{(i)},\mathbf{k}_{t|t}^{(i)}\right)} = 1_{\tilde{\mathbf{k}}_{t+1}^{(j)}=\bar{g}} \times 1_{\left|(1-\gamma)(\Gamma-1)\bar{\pi}_{t|t}^{(i)}\right|>\bar{\theta}\sigma^{\eta}} + \\ + \left(1-1_{\tilde{\mathbf{k}}_{t+1}^{(j)}=\bar{g}}\right) \times \left(1_{\left|(1-\gamma)(\Gamma-1)\bar{\pi}_{t|t}^{(i)}\right|\leq\bar{\theta}\sigma^{\eta}}\right) \times 1_{\mathbf{k}_{t|t}^{(i)}=\mathbf{k}_{t+1|t}^{(j)}-1}.$$

Moving to the linear variables ξ_t, for each $j = 1...M$, draw the nonlinear variables $\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\mathbf{k}}_{t|T}^{(j)}$, from $\left\{\bar{\pi}_{t|t}^{(k)}, \mathbf{k}_{t|t}^{(k)}\right\}_{k=1}^{N}$ using the new set of weights $\left\{w_{t|t+1}^{j}(k)\right\}_{k=1}^{N}$. Conditional on the draw, sample from

$$p\left(\xi_t|\bar{\pi}_{1:t}^{j}, \mathbf{k}_{1:t}^{j}, \xi_{t+1}, \bar{\pi}_{t+1}, Y_t\right).$$

In particular we draw $\tilde{\xi}_{t|T}^{(j)}$ from the distribution

$$N\left(\xi_{t|t}^{(j)} + \Delta_t^{(j)}\left(\left[\tilde{\bar{\pi}}_{t+1|T}^{(j)}, \tilde{\xi}_{t+1|T}^{(j)\prime}\right]' - f\left(\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\xi}_{t|T}^{(j)}\right) - A\left(\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\xi}_{t|T}^{(j)}\right)\xi_{t|t}^{(j)}\right), \Lambda_{t|t}^{(j)}\right)$$

where $\xi_{t|t}^{(j)}$ is the element in $\left\{\bar{\pi}_{t|t}^{(k)}, \mathbf{k}_{t|t}^{(k)}\right\}_{k=1}^{N}$ that corresponds to the same draw j from which the particles $\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\mathbf{k}}_{t|T}^{(j)}$ are obtained, and where

$$\Delta_t^{(j)} = P_{t|t}A\left(\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\mathbf{k}}_{t|T}^{(j)}\right)'\left(Q + A\left(\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\mathbf{k}}_{t|T}^{(j)}\right)P_{t|t}A\left(\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\mathbf{k}}_{t|T}^{(j)}\right)'\right)^{-1}$$

$$\Lambda_{t|t}^{(j)} = P_{t|t} - \Delta_t^{(j)}A\left(\tilde{\bar{\pi}}_{t|T}^{(j)}, \tilde{\mathbf{k}}_{t|T}^{(j)}\right)P_{t|t}.$$

Algorithm ends.

E Estimation in other countries

Observation equation. The Consensus forecasts can be expressed as

$$E_t^{Cons}\pi_{Y1,Q2} = \sum_{j=1}^{4} w(j)\,\pi_{t-j} + \hat{E}_t\sum_{i=0}^{2} w(5+i)\,\pi_{t+i}$$

$$E_t^{Cons}\pi_{Y1,Q1} = \sum_{j=1}^{3} w(j)\,\pi_{t-j} + \hat{E}_t\sum_{i=0}^{3} w(4+i)\,\pi_{t+i}$$

where the vector

$$w = \left(\frac{1}{16} \ \frac{2}{16} \ \frac{3}{16} \ \frac{4}{16} \ \frac{3}{16} \ \frac{2}{16} \ \frac{1}{16} \right)$$

defines the appropriate weights, and the notation $\pi_{i,j}$ denotes the inflation forecast of year i inflation, taken in the current year in quarter j. The remaining four forecasts concern expectations for inflation in the next calendar year, taken in each quarter of the current year. Similarly they can be expressed as

$$\begin{aligned}
E_t^{Cons}\pi_{Y2,Q4} &= \sum_{j=1}^{2} w(j)\,\pi_{t-j} + \hat{E}_t \sum_{i=0}^{4} w(3+i)\,\pi_{t+i} \\
E_t^{Cons}\pi_{Y2,Q3} &= \sum_{j=1}^{1} w(j)\,\pi_{t-j} + \hat{E}_t \sum_{i=0}^{5} w(2+i)\,\pi_{t+i} \\
E_t^{Cons}\pi_{Y2,Q2} &= \hat{E}_t \sum_{i=0}^{6} w(1+j)\,\pi_{t+i} \\
E_t^{Cons}\pi_{Y2,Q1} &= \hat{E}_t \sum_{i=1}^{7} w(j)\,\pi_{t+i}
\end{aligned}$$

where the last two forecasts are purely forward looking. The observation equation can then be written

$$\begin{bmatrix}
\pi_t \\
\hat{E}_t \sum_{i=1}^{2} w(5+i)\,\pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{3} w(4+i)\,\pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{4} w(3+i)\,\pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{5} w(2+i)\,\pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{6} w(1+i)\,\pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{7} w(i)\,\pi_{t+i}
\end{bmatrix} = \pi^{*,F} + H_t'\xi_t + R_t o_t^{Cons},$$

where $\pi^{*,F}$ denotes the country-specific sample mean of inflation.

As discussed in the main text, the aim is to evaluate model predictions under the posterior distribution obtained with US data on inflation and forecasts by professional forecasters. However, there are few parameters that we choose to estimate independently. In particular, we estimate the inflation mean and the standard deviation of measurement error on survey-forecasts. These parameters are necessarily country-specific and can impact significantly the model's predictions.

For the US, the Metropolis-Hasting algorithm is used to simulate the posterior distribu-

tion

$$P\left(\Theta^{US}|Y_t^{US}\right) = L(Y_t^{US}|\Theta^{US})P(\Theta^{US})$$

where $L(Y_t^{US}|\Theta^{US})$ the model likelihood. For the other countries we use the US posterior distribution as prior for the common parameters. We can then simulate the posterior distribution

$$P^F\left(\Theta^F|Y_t^F, Y_t^{US}, \Theta^{US}\right) = \tilde{L}(Y_t^F|\Theta^{US}, \Theta^F)^{\lambda^F} L(Y_t^{US}|\Theta^{US})p(\Theta^{US})p(\Theta^F)$$

where the parameter λ^F is the weight that is given to the likelihood of the model for other countries Notice that the case of $\lambda^F = 0$ corresponds to evaluating the parameters that are common to the US, Θ^{US}, at the posterior distribution for the US. The remaining parameters, Θ^F, are instead evaluated at their prior. In our estimation we set $\lambda^F = 0.05$ implying a very low weight on the foreign model likelihood, $\tilde{L}(Y_t^F|\Theta^{US}, \Theta^F)$. As a result, the posterior distribution of the common parameters with the US is essentially the same as for the US, while the likelihood informs about the country-specific parameters. Tables in the additional technical appendix give the parameter estimates for all other countries. They are obtained using 200000 draws from the simulated posterior distribution.

F Marginal Likelihood

To compute the marginal likelihood for the US baseline model we use the Geweke harmonic mean estimator. For each draw Θ_i we compute

$$p(y) = \left\{\frac{1}{D}\sum_{i=1}^{D}\frac{f\left(\Theta_i\right)}{p\left(y|\Theta_i\right)p\left(\Theta_i\right)}\right\}^{-1}$$

where the function $f\left(.\right)$ is the density of a Normal distribution with mean and variance corresponding to the mean and variance of the posterior draws sample. Moreover the distribution is truncated so that

$$\begin{aligned} f\left(\Theta_i\right) &= \tau^{-1}\left(2\pi\right)^{-d/2}\left|V_\Theta\right| exp\left[-0.5\left(\Theta_i-\bar{\Theta}\right)'V_\Theta^{-1}\left(\Theta_i-\bar{\Theta}\right)\right] \\ &\quad\times\left\{\Theta_i:\ \left(\Theta_i-\bar{\Theta}\right)'V_\Theta^{-1}\left(\Theta_i-\bar{\Theta}\right)<\chi^2_{\tau,d}\right\} \end{aligned}$$

where $\chi^2_{\tau,d}$ is the $(1-\tau)$ quantile of the χ^2_d distribution and d is the dimension of the parameters' vector. In order to compute the marginal likelihood we set $\tau = 0.5$.

G Additional Material

G.1 Derivation of Γ.

Substituting for the marginal cost into the aggregate supply equation gives

$$\begin{aligned}
\pi_t - \gamma\pi_{t-1} &= \mu_t + \xi_p s_t + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\alpha\beta\xi_p s_{T+1} + (1-\alpha)\,\beta\,(\pi_{T+1} - \gamma\pi_T)\right] \\
&= \tilde{\mu}_t + \tilde{\kappa}\varphi_t + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\alpha\beta\tilde{\kappa}\left[\varphi_{T+1} - (\pi_{T+1} - \bar{\pi}_t) + \gamma\,(\pi_T - \bar{\pi}_t)\right]\right] \\
&\quad + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[(1-\alpha)\,\tilde{\beta}\,(\pi_{T+1} - \gamma\pi_T)\right]
\end{aligned}$$

where

$$\tilde{\kappa} = \left(1 + \frac{\xi_p}{\lambda_x\phi}\right)^{-1} \frac{\xi_p}{\lambda_x\phi} = \frac{\xi_p}{\xi_p + \lambda_x\phi} \quad \text{and} \quad \tilde{\beta} = \frac{\beta\lambda_x\phi}{\xi_p + \lambda_x\phi} = \frac{\beta}{1 + \lambda_x^{-1}\phi^{-1}}.$$

Rearranging gives

$$\begin{aligned}
\left(1 + (1-\alpha)\,\tilde{\beta} - \alpha\beta\tilde{\kappa}\right)\pi_t - \gamma\pi_{t-1} &= \tilde{\mu}_t + \tilde{\kappa}\varphi_t + \\
&\quad + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\alpha\beta\tilde{\kappa}\varphi_{T+1} - (1-\alpha\beta\gamma)\left(\alpha\beta\tilde{\kappa} - (1-\alpha)\,\tilde{\beta}\right)\pi_{T+1}\right] \\
&\quad + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\alpha\beta\tilde{\kappa}\,(1-\gamma)\,\bar{\pi}_t\right]
\end{aligned}$$

The rational expectations equilibrium is computed from

$$\pi_t - \gamma\pi_{t-1} = \tilde{\mu}_t + \tilde{\kappa} E_t \sum_{T=t}^{\infty} \tilde{\beta}^{T-t}\varphi_T$$

giving

$$\pi_t - \gamma\pi_{t-1} = \tilde{\mu}_t + \omega_\varphi\varphi_t$$

where

$$\omega_\varphi = \frac{\tilde{\kappa}}{1-\tilde{\beta}\rho}.$$

It can be further simplified as

$$\begin{aligned}\omega_\varphi &= \frac{\frac{\xi_p}{\xi_p+\lambda_x\phi}}{1-\frac{\beta\lambda_x\phi}{\xi_p+\lambda_x\phi}\rho} \\ &= \frac{1}{1+(1-\beta\rho)\,\lambda_x\xi_p^{-1}\phi}\end{aligned}$$

Substituting the discounted forecast for inflation

$$E_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\,\pi_{T+1} = \left(\frac{1}{1-\alpha\beta}-\frac{\gamma}{1-\alpha\beta\gamma}\right)\bar{\pi}_t+\frac{\gamma}{1-\alpha\beta\gamma}\pi_t+\frac{\omega_\varphi\rho}{(1-\alpha\beta\rho)\,(1-\alpha\beta\gamma)}\varphi_t.$$

into the aggregate supply curve gives

$$\begin{aligned}\pi_t-\gamma\pi_{t-1} &= \tilde{\mu}_t+\left[\tilde{\kappa}+\frac{\alpha\beta\tilde{\kappa}\rho}{1-\alpha\beta\rho}-\frac{(1-\alpha\beta\gamma)\left(\alpha\beta\tilde{\kappa}-(1-\alpha)\,\tilde{\beta}\right)}{(1-\alpha\beta\gamma)\,(1-\alpha\beta\rho)}\omega_\varphi\rho\right]\varphi_t+ \\ &\quad+\left[\alpha\beta\tilde{\kappa}\frac{1-\gamma}{1-\alpha\beta}+\left((1-\alpha)\,\tilde{\beta}-\alpha\beta\tilde{\kappa}\right)\left(\frac{1-\alpha\beta\gamma}{1-\alpha\beta}-\frac{(1-\alpha\beta\gamma)\,\gamma}{1-\alpha\beta\gamma}\right)\right]\bar{\pi}_t.\end{aligned}$$

Using rational expectations about transitional dynamics and the fact that

$$\tilde{\kappa}+\frac{\alpha\beta\tilde{\kappa}\rho}{1-\alpha\beta\rho}-\frac{(1-\alpha\beta\gamma)\left(\alpha\beta\tilde{\kappa}-(1-\alpha)\,\tilde{\beta}\right)}{(1-\alpha\beta\gamma)\,(1-\alpha\beta\rho)}\omega_\varphi\rho=\omega_\varphi$$

permits

$$\begin{aligned}\pi_t-\gamma\pi_{t-1} &= \tilde{\mu}_t+\omega_\varphi\varphi_t+ \\ &\quad+\left[\alpha\beta\tilde{\kappa}\frac{1-\gamma}{1-\alpha\beta}+\left((1-\alpha)\,\tilde{\beta}-\alpha\beta\tilde{\kappa}\right)\left(\frac{1-\alpha\beta\gamma}{1-\alpha\beta}-\frac{(1-\alpha\beta\gamma)\,\gamma}{1-\alpha\beta\gamma}\right)\right]\bar{\pi}_t\end{aligned}$$

Simplifying then provides

$$\begin{aligned}\pi_t - \gamma\pi_{t-1} &= \tilde{\mu}_t + \omega_\varphi \varphi_t + \\ &+ \frac{1}{\xi_p + \lambda_x \phi}\left[\alpha\beta\xi_p \frac{1-\gamma}{1-\alpha\beta} + \left((1-\alpha)\beta\lambda_x\phi - \alpha\beta\xi_p\right)\left(\frac{1-\alpha\beta\gamma}{1-\alpha\beta} - \frac{(1-\alpha\beta\gamma)\gamma}{1-\alpha\beta\gamma}\right)\right]\bar{\pi}_t\end{aligned}$$

or

$$\begin{aligned}\pi_t - \gamma\pi_{t-1} &= \tilde{\mu}_t + \omega_\varphi \varphi_t + \\ &+ \frac{1}{\xi_p + \lambda_x \phi}\left[\alpha\beta\xi_p \left(\frac{1-\gamma}{1-\alpha\beta} - \frac{1-\alpha\beta\gamma}{1-\alpha\beta}\right)\right]\bar{\pi}_t \\ &+ \frac{1}{\xi_p + \lambda_x \phi}\left[(1-\alpha)\beta\lambda_x\phi\left(\frac{1-\alpha\beta\gamma}{1-\alpha\beta} - \frac{(1-\alpha\beta\gamma)\gamma}{1-\alpha\beta\gamma}\right) + \alpha\beta\xi_p\gamma\right]\bar{\pi}_t\end{aligned}$$

or

$$\pi_t - \gamma\pi_{t-1} = \tilde{\mu}_t + \omega_\varphi \varphi_{t-1} + (1-\gamma)\Gamma\bar{\pi}_t + \omega_\varphi \varepsilon_t$$

where

$$\Gamma = \frac{1}{1 + \lambda_x^{-1}\phi^{-1}\xi_p}\frac{(1-\alpha)\beta}{1-\alpha\beta}.$$

Using $\phi = 1$ we have

$$\Gamma = \frac{1}{1 + \xi_p\lambda_x^{-1}}\frac{(1-\alpha)\beta}{1-\alpha\beta}$$

as in the main text.

G.1.1 Proof In Appendix C, the crucial step is the derivation of the prediction for the linear state (step 6). To dervied this, notice first that given the link between $\bar{\pi}_t$ and the linear state we can use

$$\begin{bmatrix} \xi_t | Y_t, \bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)} \\ \bar{\pi}_{t+1|t} - f_{\bar{\pi}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) | Y_t, \bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)} \end{bmatrix} \sim N\left(\begin{bmatrix} \xi_{t|t}^{(i)} \\ A_{\bar{\pi},t}^{(i)}\xi_{t|t}^{(i)} \end{bmatrix}, \begin{bmatrix} P_{t|t}^{(i)} & A_{\bar{\pi},t}^{(i)}P_{t|t}^{(i)} \\ P_{t|t}^{(i)}A_{\bar{\pi}}^{(i)\prime} & A_{\bar{\pi},t}^{(i)}P_{t|t}^{(i)}A_{\bar{\pi}}^{(i)\prime} \end{bmatrix}\right).$$

Using properties of the normal distribution, we can now get the conditional distribution

$$\xi_{t|t}^{*(i)} = \xi_{t|t}^{(i)} + P_{t|t}^{(i)} A_{\bar{\pi},t}^{(i)\prime} \left(A_{\bar{\pi},t}^{(i)} P_{t|t}^{(i)} A_{\bar{\pi},t}^{(i)\prime} \right)^{-1} \left(\bar{\pi}_{t+1|t}^{(i)} - f_{\bar{\pi}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}) - f_{\mathbf{k},t}^{(i)-1} z_{t|t}^{(i)} \right)$$

$$P_{t|t}^{*(i)} = P_{t|t}^{(i)} - P_{t|t}^{(i)} A_{\bar{\pi},t}^{(i)\prime} \left(A_{\bar{\pi},t}^{(i)} P_{t|t}^{(i)} A_{\bar{\pi},t}^{(i)\prime} \right)^{-1} A_{\bar{\pi},t}^{(i)} P_{t|t}^{(i)}$$

where

$$A_{\bar{\pi},t}^{(i)} = A_{\bar{\pi}} \left(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)} \right);$$

$$f_{\mathbf{k},t}^{(i)-1} = f_{\mathbf{k}}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)})^{-1}.$$

The predictions for the linear state are then

$$\xi_{t+1|t}^{(i)} = f_{\xi,t}^{(i)} + A_{\xi,t}^{(i)} \xi_{t|t}^{*(i)}$$

where

$$f_{\xi,t}^{(i)} = f_{\xi}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)});$$

$$A_{\xi,t}^{(i)} = A_{\xi}(\bar{\pi}_{t|t}^{(i)}, \mathbf{k}_{t|t}^{(i)}),$$

and

$$P_{t+1|t}^{(i)} = A_{\xi,t}^{(i)} P_{t|t} A_{\xi,t}^{(i)\prime} +$$

$$- A_{\xi,t}^{(i)} \left[P_{t|t} A_{\bar{\pi},t}^{(i)\prime} \left(A_{\bar{\pi},t}^{(i)} P_{t|t} A_{\bar{\pi},t}^{(i)\prime} \right)^{-1} A_{\bar{\pi},t}^{(i)} P_{t|t} \right] A_{\xi,t}^{(i)\prime} + Q_{\xi}.$$

Here we show that $P_{t+1|t}^{(i)} = P_{t+1|t}$ for every (i). For given initial $P_{t|t}$, it is straightforward to show that

$$\left(A_{\bar{\pi},t}^{(i)} P_{t|t} A_{\bar{\pi},t}^{(i)\prime} \right)^{-1} = \frac{1}{f_{\mathbf{k},t}^{(i)-2} P_{t|t}^{[\eta,\eta]}}.$$

Then a little algebra leads to the following:

$$\begin{aligned}
\bar{P}_{t|t}^{(i)} &= P_{t|t}A_{\bar{\pi},t}^{(i)\prime}\left(\frac{1}{f_{\mathbf{k},t}^{(i)-2}P_{t|t}^{[\eta,\eta]}}\right)A_{\bar{\pi},t}^{(i)}P_{t|t} \\
&= \begin{bmatrix} P_{t|t}^{[\eta,\eta]} & P_{t|t}^{[\eta,\varphi]} & P_{t|t}^{[\eta,\pi]} \\ P_{t|t}^{[\eta,\varphi]} & \frac{\left(P_{t|t}^{[\eta,\varphi]}\right)^2}{P_{t|t}^{[\eta,\eta]}} & P_{t|t}^{[\eta,\varphi]}\frac{P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} \\ P_{t|t}^{[\eta,\pi]} & P_{t|t}^{[\eta,\varphi]}\frac{P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} & \frac{\left(P_{t|t}^{[\eta,\pi]}\right)^2}{P_{t|t}^{[\eta,\eta]}} \end{bmatrix} \\
&= \bar{P}_{t|t}.
\end{aligned}$$

Next, evaluate

$$\tilde{P}_{t|t} = A_{\xi,t}^{(i)}P_{t|t}A_{\xi,t}^{(i)\prime} - A_{\xi,t}^{(i)}\bar{P}_{t|t}A_{\xi,t}^{(i)\prime} = A_{\xi,t}^{(i)}\left(P_{t|t} - \bar{P}_{t|t}\right)A_{\xi,t}^{(i)\prime}$$

where

$$\left(P_{t|t} - \bar{P}_{t|t}\right) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & P_{t|t}^{[\varphi,\varphi]} - \frac{\left(P_{t|t}^{[\eta,\varphi]}\right)^2}{P_{t|t}^{[\eta,\eta]}} & P_{t|t}^{[\varphi,\pi]} - \frac{\left(P_{t|t}^{[\eta,\varphi]}\right)^2 P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} \\ 0 & P_{t|t}^{[\eta,\pi]} - \frac{\left(P_{t|t}^{[\eta,\varphi]}\right)^2 P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} & P_{t|t}^{[\pi,\pi]} - \frac{\left(P_{t|t}^{[\eta,\pi]}\right)^2}{P_{t|t}^{[\eta,\eta]}} \end{bmatrix}.$$

Finally,

$$\tilde{P}_{t|t} = A_{\xi,t}^{(i)}\left(P_{t|t} - \bar{P}_{t|t}\right)A_{\xi,t}^{(i)\prime} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \iota_1 & \iota_2 + \iota_1 \\ 0 & \iota_2 + \iota_1 & 2\iota_2 + \iota_1 + \left(P_{t|t}^{[\pi,\pi]} - \frac{\left(P_{t|t}^{[\eta,\pi]}\right)^2}{P_{t|t}^{[\eta,\eta]}}\right)\gamma^2 \end{bmatrix}$$

where

$$\begin{aligned}
\iota_1 &= \rho_\varphi^2\left(P_{t|t}^{[\varphi,\varphi]} - \frac{\left(P_{t|t}^{[\eta,\varphi]}\right)^2}{P_{t|t}^{[\eta,\eta]}}\right) \\
\iota_2 &= \left(-\frac{P_{t|t}^{[\eta,\varphi]}P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} + P_{t|t}^{[\varphi,\pi]}\right)\rho_\varphi\gamma.
\end{aligned}$$

So we can express

$$P^{(i)}_{t+1|t} = P_{t+1|t} = Q_\xi + \tilde{P}_{t|t}.$$

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