

Online Appendix: Sectoral Heterogeneity in Nominal Price Rigidity and the Origin of Aggregate Fluctuations

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A.1 Steady-State Solution and Log-linear System

A. Steady-State Solution

Without loss of generality, set $a_k = 0$. We show below conditions for the existence of a symmetric steady state across firms in which

$$W_k = W, Y_{jk} = Y, L_{jk} = L, Z_{jk} = Z, P_{jk} = P \text{ for all } j, k.$$

Symmetry in prices across all firms implies

$$P^c = P^k = P_k = P$$

such that, from equations (2), (3), (11), and (14) in the main body of the paper,

$$\begin{aligned} C_k &= \omega_{ck} C, \\ C_{jk} &= \frac{1}{n_k} C_k, \\ Z_{jk}(k') &= \omega_{kk'} Z, \\ Z_{jk}(j', k') &= \frac{1}{n_{k'}} Z_{jk}(k'). \end{aligned}$$

The vector $\Omega_c \equiv [\omega_{c1}, \dots, \omega_{cK}]'$ represents steady-state sectoral shares in value-added C , $\Omega = \{\omega_{kk'}\}_{k,k'=1}^K$ is the matrix of input-output linkages across sectors, and $\aleph \equiv [n_1, \dots, n_K]'$ is the vector of steady-state sectoral shares in gross output Y .

It also holds that

$$C = \sum_{k=1}^K \int_{\mathfrak{S}_k} C_{jk} dj,$$

$$Z_{jk} = \sum_{k'=1}^K \int_{\mathfrak{S}_{k'}} Z_{jk}(j', k') dj' = Z.$$

From Walras' law in equation (21) and symmetry across firms, it follows

$$Y = C + Z. \tag{A.1}$$

Walras' law also implies for all j, k

$$Y_{jk} = C_{jk} + \sum_{k'=1}^K \int_{\mathfrak{S}_{k'}} Z_{j'k'}(j, k) dj',$$

$$Y = \frac{\omega_{ck}}{n_k} C + \frac{1}{n_k} \left(\sum_{k'=1}^K n_{k'} \omega_{k'k} \right) Z,$$

so \aleph satisfies

$$n_k = (1 - \psi) \omega_{ck} + \psi \sum_{k'=1}^K n_{k'} \omega_{k'k},$$

$$\aleph = (1 - \psi) [I - \psi \Omega']^{-1} \Omega_c,$$

for $\psi \equiv \frac{Z}{Y}$. Note by construction $\aleph'_\iota = 1$, which must hold given the total measure of firms is 1.

Steady-state labor supply from equation (8) is

$$\frac{W_k}{P} = g_k L_k^\varphi C^\sigma.$$

In a symmetric steady state, $L_k = n_k L$, so this steady state exists if $g_k = n_k^{-\varphi}$ such that $W_k = W$ for all k . Thus, steady-state labor supply is given by

$$\frac{W}{P} = L^\varphi C^\sigma. \tag{A.2}$$

Households' budget constraint, firms' profits, production function, efficiency of production

(from equation (17)) and optimal prices in steady state are, respectively,

$$CP = WL + \Pi \quad (\text{A.3})$$

$$\Pi = PY - WL - PZ \quad (\text{A.4})$$

$$Y = L^{1-\delta} Z^\delta \quad (\text{A.5})$$

$$\delta WL = (1 - \delta) PZ \quad (\text{A.6})$$

$$sP = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^\delta \quad (\text{A.7})$$

for $\xi \equiv \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta}$.

Equation (A.7) solves

$$\frac{W}{P} = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{1-\delta}}. \quad (\text{A.8})$$

This latter result together with equations (A.5), (A.6), and (A.7) solves

$$\frac{\Pi}{P} = \frac{1}{\theta} Y.$$

Plugging the previous result in equation (A.4) and using equation (A.1) yields

$$\begin{aligned} C &= \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right] Y \\ Z &= \delta \left(\frac{\theta - 1}{\theta} \right) Y, \end{aligned} \quad (\text{A.9})$$

such that $\psi \equiv \delta \left(\frac{\theta - 1}{\theta} \right)$.

This result and equation (A.7) gives

$$L = \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\delta}{1-\delta}} Y,$$

where Y from before together with equations (A.2), (A.9) and (A.8) solves

$$Y = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{(1-\delta)(\sigma+\varphi)}} \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{\frac{\delta\varphi}{(1-\delta)(\sigma+\varphi)}} \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\sigma}{\sigma+\varphi}}.$$

B. Log-linear System

B.1 Aggregation

Aggregate and sectoral consumption which we interpret as real sales of final goods, given by equations (2) and (3), are

$$\begin{aligned} c_t &= \sum_{k=1}^K \omega_{ck} c_{kt}, \\ c_{kt} &= \frac{1}{n_k} \int_{\mathfrak{S}_k} c_{jkt} dj. \end{aligned} \tag{A.10}$$

Aggregate and sectoral production of intermediate inputs are

$$\begin{aligned} z_t &= \sum_{k=1}^K n_k z_{kt}, \\ z_{kt} &= \frac{1}{n_k} \int_{\mathfrak{S}_k} z_{jkt} dj, \end{aligned} \tag{A.11}$$

where equations (11) and (14) imply that $z_{jk} = \sum_{r=1}^K \omega_{kr} z_{jk}(r)$ and $z_{jk}(r) = \frac{1}{n_r} \int_{\mathfrak{S}_r} z_{jkr}(j', r) dj'$.

Sectoral and aggregate prices are (equations (5), (7), and (13)),

$$\begin{aligned} p_{kt} &= \int_{\mathfrak{S}_k} p_{jk} dj \text{ for } k = 1, \dots, K \\ p_t^c &= \sum_{k=1}^K \omega_{ck} p_{kt}, \\ p_t^k &= \sum_{k'=1}^K \omega_{kk'} p_{k't}. \end{aligned}$$

Aggregation of labor is

$$\begin{aligned} l_t &= \sum_{k=1}^K l_{kt}, \\ l_{kt} &= \int_{\mathfrak{S}_k} l_{jkt} dj. \end{aligned} \tag{A.12}$$

B.2 Demand

Households' demands for goods in equations (4) and (6) for all $k = 1, \dots, K$ become

$$\begin{aligned} c_{kt} - c_t &= \eta(p_t^c - p_{kt}), \\ c_{jkt} - c_{kt} &= \theta(p_{kt} - p_{jkt}). \end{aligned} \tag{A.13}$$

In turn, firm jk 's demands for goods in equation (12) and (15) for all $k, r = 1, \dots, K$,

$$\begin{aligned} z_{jkt}(k') - z_{jkt} &= \eta(p_t^k - p_{k't}), \\ z_{jkt}(j', k') - z_{jkt}(k') &= \theta(p_{k't} - p_{j'k't}). \end{aligned} \tag{A.14}$$

Firms' gross output satisfies Walras' law,

$$y_{jkt} = (1 - \psi) c_{jkt} + \psi \sum_{k'=1}^K \int_{\mathfrak{S}_{k'}} z_{j'k't}(j, k) dj'. \tag{A.15}$$

Total gross output follows from the aggregation of equations (21),

$$y_t = (1 - \psi) c_t + \psi z_t. \tag{A.16}$$

B.3 IS and Labor Supply

The household Euler equation in equation (9) becomes

$$c_t = \mathbb{E}_t [c_{t+1}] - \sigma^{-1} \{i_t - (\mathbb{E}_t [p_{t+1}^c] - p_t)\}.$$

The labor supply condition in equation (8) is

$$w_{kt} - p_t^c = \varphi l_{kt} + \sigma c_t. \tag{A.17}$$

B.4 Firms

Production function:

$$y_{jkt} = a_{kt} + (1 - \delta) l_{jkt} + \delta z_{jkt} \tag{A.18}$$

Efficiency condition:

$$w_{kt} - p_t^k = z_{jkt} - l_{jkt} \quad (\text{A.19})$$

Marginal costs:

$$mc_{kt} = (1 - \delta) w_{kt} + \delta p_t^k - a_{kt} \quad (\text{A.20})$$

Optimal reset price:

$$p_{kt}^* = (1 - \alpha_k \beta) mc_{kt} + \alpha_k \beta \mathbb{E}_t [p_{kt+1}^*]$$

Sectoral prices:

$$p_{kt} = (1 - \alpha_k) p_{kt}^* + \alpha_k p_{kt-1}$$

B.5 Taylor Rule:

$$i_t = \phi_\pi (p_t^c - p_{t-1}^c) + \phi_c c_t$$

A.2 Solution of Key Equations in Section III

A. Solution of Equation (26)

Setting $\sigma = 1$ and $\varphi = 0$ in equation (A.17) yields

$$w_{kt} = c_t + p_t^c = 0, \quad (\text{A.21})$$

where the equality follows from the assumed monetary policy rule, so equation (A.20) becomes

$$mc_{kt} = \delta p_t^k - a_{kt}. \quad (\text{A.22})$$

Here, sectoral prices for all $k = 1, \dots, K$ are governed by

$$\begin{aligned} p_{kt} &= (1 - \lambda_k) mc_{kt} \\ &= \delta (1 - \lambda_k) p_t^k - (1 - \lambda_k) a_{kt}, \end{aligned}$$

which in matrix form solves

$$p_t = - [\mathbb{I} - \delta (\mathbb{I} - \Lambda) \Omega]^{-1} (\mathbb{I} - \Lambda) a_t. \quad (\text{A.23})$$

$p_t \equiv [p_{1t}, \dots, p_{Kt}]'$ is the vector of sectoral prices, Λ is a diagonal matrix with the vector $[\lambda_1, \dots, \lambda_K]'$ on its diagonal, Ω is the matrix of input-output linkages, and $a_t \equiv [a_{1t}, \dots, a_{Kt}]'$ is the vector of realizations of sectoral technology shocks.

The monetary policy rule implies $c_t = -p_t^c$, so

$$c_t = (\mathbb{I} - \Lambda') [\mathbb{I} - \delta (\mathbb{I} - \Lambda') \Omega']^{-1} \Omega'_c a_t. \quad (\text{A.24})$$

which may be written in compact form as

$$c_t = \chi' a_t. \quad (\text{A.25})$$

B. Solution of Equation (38)

When inverse-Frisch elasticity $\varphi > 0$, labor supply and demand now jointly determine wages such that

$$w_{kt} = c_t + p_t^c + \varphi l_{kt}^d \quad (\text{A.26})$$

Thus, with monetary policy targeting $c_t + p_t^c = 0$, it no longer holds that sectoral productivity shocks have no effect on wages. Because the labor market is sectorally segmented, wages may differ across sectors. To see the sources of sectoral wage variation, we start from labor demand implied by the sectoral aggregation of the production function and the efficiency condition on the mix between labor and intermediate inputs,

$$l_{kt}^d = y_{kt} - a_{kt} - \delta (w_{kt} - p_t^k). \quad (\text{A.27})$$

Conditioning on sectors' gross output y_{kt} , this equation shows that a positive productivity shock in sector k directly decreases demand for labor in the shocked sector by a_{kt} and indirectly in all sectors by the effect of the productivity shock on sector-specific aggregate prices of intermediate inputs, p_t^k . This latter effect is due to firms substituting labor for cheaper intermediate inputs, the price of which the steady-state I/O linkages of sectors with sector k determine.

To see the way that productivity shocks affect sectors' gross output y_{kt} , we use the log-linear expression for Walras law

$$y_{kt} = \frac{(1 - \psi) \omega_{ck}}{n_k} c_{kt} + \frac{\psi}{n_k} \sum_{k'=1}^K n_{k'} \omega_{k'k} z_{k't}(k), \quad (\text{A.28})$$

such that sectoral gross output depends on households' demand as final goods and all sectors demand as intermediate inputs. The $\{n_k\}_{k=1}^\infty$ are the steady-state shares of sectors in aggregate gross output

$$n_k = (1 - \psi) \omega_{ck} + \psi \sum_{k'=1}^K n_{k'} \omega_{k'k} \text{ for all } k = 1, \dots, K. \quad (\text{A.29})$$

$\psi \equiv Z/Y$ is the fraction of total gross output used as intermediate input in steady state.

Log-linear demands from households and sectors on goods produced in sector k are given

by

$$\begin{aligned} c_{kt} &= c_t - \eta (p_{kt} - p_t^c), \\ z_{k't}(k) &= z_{k't} - \eta (p_{kt} - p_t^{k'}) \text{ for } k' = 1, \dots, K. \end{aligned}$$

Thus, when sector k has a positive productivity shock, its demand from households and firms increases in the extent the price of sector k decreases relative to the price of goods produced in other sectors. This force pushes wages up in the shocked sector and down in all other sectors as households and firms decrease demand for all sectors with no positive shock. The strength of this effect depends on steady-state GDP shares for households' demand and steady-state input-output linkages.

Summing up, these effects create interdependence in the determination of wages. For $\varphi > 0$, wages solve

$$w_t = \Theta^{-1} [\theta_c c_t + \theta_p p_t - \varphi a_t], \quad (\text{A.30})$$

where w_t is the vector of sectoral wages, and the parameters are

$$\begin{aligned} \Theta' &\equiv (1 + \delta\varphi) \mathbb{I} - (1 + \varphi) \psi D^{-1} \Omega' D; \\ \theta_c &\equiv [\mathbb{I} - \psi D^{-1} \Omega' D] \iota + \varphi (1 - \psi) D^{-1} \Omega_c; \\ \theta_p &\equiv [\mathbb{I} - \psi D^{-1} \Omega' D] \iota \Omega'_c - \varphi \eta [\mathbb{I} - (1 - \psi) D^{-1} \Omega_c \Omega'_c] \\ &\quad + \varphi [(\eta - 1) \psi D^{-1} \Omega' D \Omega - \delta \Omega], \end{aligned}$$

where \mathbb{I} is a $K \times K$ identity matrix, D is a $K \times K$ matrix with vector $[n_k]_{k=1}^K$ on its diagonal, Ω_c is a column-vector of GDP shares $\{\omega_{ck}\}_{k=1}^K$, and Ω is the matrix $[\omega_{k'k}]_{k',k=1}^K$ with steady state input-output linkages across sectors.

This expression collapses to $w_{kt} = c_t + p_t^c$ when $\varphi = 0$. In the special case when $\delta = 0$ (i.e., no intermediate inputs), sectoral wages solve

$$w_t = (1 + \varphi) \iota c_t + [(1 + \varphi \eta) \iota \Omega'_c - \varphi \eta \mathbb{I}] p_t - \varphi a_t. \quad (\text{A.31})$$

Although the interaction between price rigidity and sector size and I/O linkages is more involved, as these effects jointly create an interdependence of labor demand across sectors, our key insight from Section III remains: heterogeneity in price rigidity affects the propagation of

idiosyncratic sectoral productivity shocks by affecting the responsiveness of sectoral prices to these shocks together with GDP shares and input output linkages.

The general solution for χ' when $\delta > 0$ now becomes

$$\chi' = \Omega'_c [\mathbb{I} - \delta (\mathbb{I} - \Lambda) \Omega - (1 - \delta) (\mathbb{I} - \Lambda) \Theta^{-1} (\theta_p - \theta_c \Omega'_c)]^{-1} (\mathbb{I} - \Lambda) [\mathbb{I} + (1 - \delta) \varphi \Theta^{-1}]. \quad (\text{A.32})$$

Although now functional forms are more involved, sectoral price rigidity affects the aggregate propagation of sectoral productivity shocks through distorting the effect of the distribution of GDP shares and input-output linkages.

From a different angle, to further explore the effect of positive inverse-Frisch elasticity, consider the special case of no input-output linkages ($\delta = 0$), so

$$\chi' = \Omega'_c (\mathbb{I} - \Phi), \quad (\text{A.33})$$

where Φ is a diagonal matrix with entries

$$\frac{1 - \lambda_k}{1 + \varphi \eta (1 - \lambda_k)} \left[1 - \varphi (\eta - 1) \sum_{k'=1}^K \frac{\omega_{ck'} (1 - \lambda_{k'})}{1 + \varphi \eta (1 - \lambda_{k'})} \right]^{-1}, \quad (\text{A.34})$$

for $k = 1, \dots, K$ on its diagonal. Note $\Phi = \Lambda$ when $\varphi = 0$. According to equation (28), the inverse-Frisch elasticity φ has two opposite effects on the capacity of price rigidity to generate aggregate volatility from sectoral productivity shocks. On the one hand, if sector k has more flexible prices, its demand responds by more to its own productivity shocks, so wages in the shocked sector respond by more. This effect is captured by the denominator of the term outside the brackets. On the other hand, the response of prices in the shocked sector has an effect on the demand of other sectors. This effect is captured by the term in brackets which is common to all sectors. Thus, in the absence of input-output linkages ($\delta = 0$), more elastic labor supply reduces the quantitative importance of price rigidity to generate fluctuations. However, because both effects operate through sectoral demand, the effect of φ depends on the elasticity of substitution across sectors, η . Quantitatively, empirical estimates suggest η is small (see Atalay (2017) and Feenstra, Luck, Obstfeld, and Russ (2018)).

A.3 Proofs

Most proofs below are modifications of the arguments in Gabaix (2011), Proposition 2, which rely heavily on the Levy's Theorem (as in Theorem 3.7.2 in Durrett (2013) on p. 138).

Theorem 5 (Levy's Theorem) *Suppose X_1, \dots, X_K are i.i.d. with a distribution that satisfies*

- (i) $\lim_{x \rightarrow \infty} \Pr [X_1 > x] / \Pr [|X_1| > x] = \theta \in (0, 1)$
- (ii) $\Pr [|X_1| > x] = x^{-\zeta} L(x)$ with $\zeta < 2$ and $L(x)$ satisfies $\lim_{x \rightarrow \infty} L(tx) / L(x) = 1$.

Let $S_K = \sum_{k=1}^K X_k$,

$$a_K = \inf \{x : \Pr [|X_1| > x] \leq 1/K\} \text{ and } b_K = K \mathbb{E} [X_1 \mathbf{1}_{|X_1| \leq a_K}], \quad (\text{A.35})$$

As $K \rightarrow \infty$, $(S_K - b_K) / a_K \xrightarrow{d} u$, where u has a nondegenerated distribution.

A. Proof of Proposition 1

In the following proofs, we go through three cases: first, when both first and second moments exist, second, when only the first moment exists, and third, when neither first nor second moments exist.

Generally, when there are no intermediate inputs ($\delta = 0$) and price rigidity is homogeneous across sectors ($\lambda_k = \lambda$ for all k),

$$\|\chi\|_2 = \frac{1 - \lambda}{K^{1/2} \bar{C}_k} \sqrt{\frac{1}{K} \sum_{k=1}^K C_k^2}. \quad (\text{A.36})$$

Given the power-law distribution of C_k , the first and second moments of C_k exist when $\beta_c > 2$, so

$$K^{1/2} \|\chi\|_2 \longrightarrow \frac{\sqrt{\mathbb{E} [C_k^2]}}{\mathbb{E} [C_k]}. \quad (\text{A.37})$$

In contrast, when $\beta_c \in (1, 2)$, only the first moment exists. In such cases, by the Levy's theorem,

$$K^{-2/\beta_c} \sum_{k=1}^K C_k^2 \xrightarrow{d} u_0^2, \quad (\text{A.38})$$

where u_0^2 is a random variable following a Levy's distribution with exponent $\beta_c/2$ since $\Pr [C_k^2 > x] = x_0^\beta x^{-\beta_c/2}$.

Thus,

$$K^{1-1/\beta_c} \|\chi\|_2 \xrightarrow{d} \frac{u_0}{\mathbb{E}[C_k]}. \quad (\text{A.39})$$

When $\beta_c = 1$, the first and second moments of C_k do not exist. For the first moment, by Levy's theorem,

$$(\bar{C}_k - \log K) \xrightarrow{d} g, \quad (\text{A.40})$$

where g is a random variable following a Levy distribution.

The second moment is equivalent to the result above and hence

$$(\log K) \|\chi\|_2 \xrightarrow{d} u'. \quad (\text{A.41})$$

B. Proof of Proposition 4

When $\delta \in (0, 1)$, $\lambda_k = \lambda$ for all k , and $\Omega_c = \frac{1}{K}\iota$, we know

$$\begin{aligned} \|\chi\|_2 &\geq \frac{1-\lambda}{K} \sqrt{\sum_{k=1}^K [1 + \delta' d_k + \delta'^2 q_k]^2} \\ &\geq (1-\lambda) \sqrt{\frac{1 + 2\delta' + 2\delta'^2}{K} + \frac{\delta'^2}{K^2} \sum_{k=1}^K [d_k^2 + 2\delta' d_k q_k + \delta'^2 q_k^2]}. \end{aligned}$$

Following the same argument as in Proposition 2,

$$\begin{aligned} K^{-2/\beta_d} \sum_{k=1}^K d_k^2 &\longrightarrow u_d^2, \\ K^{-2/\beta_q} \sum_{k=1}^K q_k^2 &\longrightarrow u_q^2, \\ K^{-1/\beta_z} \sum_{k=1}^K d_k q_k &\longrightarrow u_z^2, \end{aligned}$$

where u_d^2 , u_q^2 and u_z^2 are random variables. Thus, if $\beta_z \geq 2 \min\{\beta_d, \beta_q\}$,

$$v_c \geq \frac{u_3}{K^{1-1/\min\{\beta_d, \beta_q\}}} v \quad (\text{A.42})$$

where u_3^2 is a random variable.

A.4 Input-Output Linkages

We combine the make and use tables to construct an industry-by-industry matrix that details how much of an industry's inputs other industries produce. We use the make table (*MAKE*) to determine the share of each commodity each industry k produces. We define the market share (*SHARE*) of industry k 's production of commodities as

$$SHARE = MAKE \oslash (\mathbb{I} \times MAKE),$$

where \mathbb{I} is a matrix of ones with suitable dimensions and \oslash represents the Hadamard division (element by element).

We multiply the share and use tables (*USE*) to calculate the dollar amount industry k' sells to industry k . We label this matrix revenue share (*REVSHARE*), which is a supplier industry-by-consumer industry matrix,

$$REVSHARE = SHARE \times USE.$$

We then use the revenue-share matrix to calculate the percentage of industry k inputs purchased from industry k' , and label the resulting matrix *SUPPSHARE*

$$SUPPSHARE = [REVSHARE \oslash (\mathbb{I} \times USE)]'. \tag{A.43}$$

The input-share matrix in this equation is an industry-by-industry matrix and therefore consistently maps into our model.¹

¹Ozdogli and Weber (2016) follow a similar approach.