ONLINE APPENDIX

" A Theory of Organizational Dynamics: Internal Politics and Efficiency"

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The Online Appendix contains the following five sections. In the first section, we provide a formal proof of the one-shot deviation principle in our context. It is needed because the sincere voting condition (3) in our paper is implicitly built on this principle. In the second section, we develop a method to compute long-term welfare of the equilibrium outcomes. In the third section, we extend our stylized baseline model in various important dimensions to show the main results on long-term welfare are very robust. In the fourth section, we show graphically the testable implications on organization behavior such as expected search length, the fraction of time spent in contentious vs. homogeneous states, and the expected length of time between switches of control. Finally, in the last section, we provide a complete characterization of all equilibria under unanimity voting.

1 A Proof of One-shot Deviation Principle in Our Model

Given any Markovian strategy profile $\sigma = \{\sigma_1, \sigma_2, \sigma_3\}$ and any type profile $b = \{b_1, b_2, b_3\}$, denote $\pi_k(b, \sigma)$ to be member k's searching payoff. We say that under (b, σ), incumbent member k has a profitable deviation if there exists a possibly non-Markovian strategy σ'_k such that $\pi_k(b, \sigma) < \pi_k(b, \sigma'_k, \sigma_{-k})$. We say that k has a profitable one-shot deviation if either of the following two cases occurs: (i) for a candidate with characteristics $(\tilde{v}, \tilde{b})$ belongs to $\mathcal{A}_k$,

$$\pi_k(b, \sigma) - \tau > \frac{2}{3}\left[\tilde{v} + E[\mu|b, \tilde{b}] + E[\pi_k(b', \sigma)|b, \tilde{b}]\right], \tag{1}$$

or (ii) for a candidate with characteristics $(\tilde{v}, \tilde{b})$ does not belong to $\mathcal{A}_k$,

$$\pi_k(b, \sigma) - \tau < \frac{2}{3}\left[\tilde{v} + E[\mu|b, \tilde{b}] + E[\pi_k(b', \sigma)|b, \tilde{b}]\right], \tag{2}$$

where b' denotes the type profile in the next period.

Obviously, a one-shot deviation is a deviation just in a single round of a particular period while a general deviation may involve several deviations in many rounds of many periods. Our next lemma shows that the one-shot deviation principle however still applies in our context.

Lemma 1 *Consider any Markovian strategy profile under which each candidate has a positive probability of getting admitted. Under this strategy profile, member k does not have profitable deviations iff he does not have profitable one-shot deviations.*

Proof. Obviously, if member k has a profitable one-shot deviation, then he must have a profitable deviation. So we only need to show that if k has a profitable deviation, then he has a profitable one-shot deviation as well. Suppose there exists σ'_k such that $\pi_k(b, \sigma'_k, \sigma_{-k}) - \pi_k(b, \sigma) = \epsilon > 0$, and σ'_k may involve several deviations in many rounds of many periods.

First, we will show that the existence of a profitable deviation must imply the existence of a profitable one-period deviation, where member k just deviates in one period (but potentially in many rounds within this period). Set T to be sufficiently large such that $9(\frac{2}{3})^T \bar{v} < \frac{\epsilon}{2}$. Notice that $3\bar{v}$ is the highest possible payoff that k can ever get in each period, and that the random exit plays the role of discounting. So if the above condition is satisfied, we have

$$\sum_{t=T}^{\infty} (\frac{2}{3})^t 3\bar{v} < \frac{\epsilon}{2}.$$

This implies that another $(T-1)$-period deviation $\tilde{\sigma}'_k$ satisfying $\tilde{\sigma}'_k = \sigma'_k$ for $t \leq T-1$ and $\tilde{\sigma}'_k = \sigma_k$ otherwise must be also profitable. Then we can use the standard backward induction argument to show the existence of profitable one-period deviation: if the one-period deviation at $T-1$ is profitable, then we are done; if not, the $(T-2)$-period deviation must be profitable and then we can continue this process until a one-period deviation is found.

Second, if there is a profitable one-period deviation, there must exist a profitable one-shot deviation, where member k just deviates in one round. This is true because under the Markovian strategy profile, a candidate is admitted and hence the period is ended by the same positive probability in each round. This again plays the role of discounting. So similar arguments imply the existence of profitable one-shot deviation. ■

It is not restrictive to consider Markovian strategy profile under which each candidate has a positive probability of getting admitted, because not admitting any candidate yields a searching payoff of $-\infty$ and it is to see that this strategy is dominated.

2 Long-Term Welfare Analysis

Since both the qualities and the types of candidates are uncertain before they arrive, the club's value and type composition are stochastic over time. Our welfare analysis will focus on the long-term (stationary) behavior of these stochastic processes.

With an equilibrium admission policy (v_i^l, v_i^r) in state i, the probability of the newly admitted member being the right type, p_i^r, must satisfy

$$p_i^r = 0.5[1 - F(v_i^r)] + 0.5F(v_i^r)p_i^r + 0.5F(v_i^l)p_i^r.$$

That is, the new member can be of the right type in one of the three events whose probabilities correspond to the three terms above, respectively: (1) the first candidate is of the right type with quality above v_i^r and so is admitted; (2) the first candidate is of the right type with quality below v_i^r and so is rejected but the club admits a right-type candidate eventually; and (3) the first candidate is of the left type with quality below v_i^l and so is rejected, but the club admits a right-type candidate eventually. Solving for p_i^r, we have

$$p_i^r = \frac{1 - F(v_i^r)}{2 - F(v_i^r) - F(v_i^l)} = \frac{\bar{v} - v_i^r}{2\bar{v} - v_i^r - v_i^l}. \tag{3}$$

Similarly, the probability of the newly admitted member in state $i \in \{0, 1, 2, 3\}$ being of the left type is given by

$$p_i^l = \frac{\bar{v} - v_i^l}{2\bar{v} - v_i^r - v_i^l}.$$

The evolution of the state variable, the number of right-type incumbents in the club I, constitutes a Markov chain. Its transition probability matrix can be written as

$$\mathbf{P} = (p_{ij})_{i,j\in\{0,1,2,3\}} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} p_0^l & p_0^r & 0 & 0 \\ \frac{1}{3}p_1^l & \frac{1}{3}p_1^r + \frac{2}{3}p_1^l & \frac{2}{3}p_1^r & 0 \\ 0 & \frac{2}{3}p_2^l & \frac{2}{3}p_2^r + \frac{1}{3}p_2^l & \frac{1}{3}p_2^r \\ 0 & 0 & p_3^l & p_3^r \end{pmatrix} \end{array}.$$

Given $\mathbf{P}$, the stationary probability distribution $\mathbf{Q}$ is given by

$$\mathbf{Q} = \mathbf{P}'\mathbf{Q}, \tag{4}$$

where $\mathbf{Q} = \{q_i\}$ and $q_i \in [0, 1]$ is the stationary probability of the state i for $i \in \{0, 1, 2, 3\}$ such that $\sum_i q_i = 1$, and $\mathbf{P}'$ is the transpose of $\mathbf{P}$.

Given $\mathbf{Q}$ and the club's equilibrium admission policy in each state, we can evaluate the club's long-term welfare. First, the long-term *expected quality* of a representative club member in the club, denoted by s, can be calculated as follows:

$$s = \sum_{i=0}^{i=3} q_i \left(p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l] \right); \tag{5}$$

where for each state i, q_i is the probability that state i happens, p_i^r (resp., p_i^l) is the probability that a newly admitted member will be of the right type (resp. left type), and $E[v|v \geq v_i^r]$ (resp., $E[v|v \geq v_i^l]$) is the expected quality of a newly admitted right-type (resp. left-type) member. Notice that the expression $p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l]$ is the expected quality of a new member in a given state i under the equilibrium admission policy (v_i^l, v_i^r). Taking expectation over the states using the stationary probability distribution thus gives the expected quality of a representative member in the club.

To calculate the expected search cost in the long-term stationary world, note that for any given state i and equilibrium admission policy (v_i^l, v_i^r), a candidate is admitted with probability of

$$x_i = 0.5(1 - F(v_i^r)) + 0.5(1 - F(v_i^l)) = 1 - 0.5F(v_i^r) - 0.5F(v_i^l).$$

The expected delay in state i is then given by

$$E[d_i] = \sum_{d=1}^{\infty} x_i(1 - x_i)^d d = (1 - x_i)/x_i.$$

Thus, the expected delay in the long-term stationary world is

$$D = \sum_{i=0}^{i=3} q_i E[d_i]. \tag{6}$$

The long-term welfare of the club can be measured as follows:

$$U = 3s - \tau D = 3 \sum_{i=0}^{i=3} q_i \left(p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l] \right) - \tau \sum_{i=0}^{i=3} q_i E[d_i]. \tag{7}$$

Once we have solved the equilibrium admission policy, the above formula can be used to compare long-term welfare under different voting rules.

3 Robustness Check

3.1 Exponential Distribution of Quality

The existence of the glass-ceiling equilibrium is a direct result of the assumption that v has bounded support. Apparently, if the support is unbounded, Condition (A.18) can never be satisfied, and hence all equilibria would be "power-switching." In this section, we replace the uniform distribution assumption by assuming that a player's quality, v, is distributed according to an exponential distribution with parameter λ.

We can similarly derive the equilibrium conditions under both majority voting and unanimity voting. The detailed derivations of the following and all subsequent equilibrium conditions are omitted, and can be obtained upon request. Under majority voting, the equilibrium cutoffs $(v_3^r, v_3^l, v_2^r, v_2^l)$ should satisfy the following system of four equations:

$$\begin{aligned}
\frac{e^{-\lambda v_3^r}}{2\lambda} + \frac{e^{-\lambda v_3^l}}{2\lambda} &= \tau \\
\frac{e^{-\lambda v_2^r}}{2\lambda} + \frac{e^{-\lambda v_2^l}}{2\lambda} &= \tau \\
2v_3^r - v_2^r - v_3^l &= \frac{B}{6} \\
\frac{1}{2}e^{-\lambda v_2^r}(v_2^l - v_3^r) + e^{-\lambda v_2^l}\left(2v_2^l - \frac{3}{2}v_2^r - \frac{1}{2}v_3^l\right) &= \frac{B}{6}\left(e^{-\lambda v_2^r} + e^{-\lambda v_2^l}\right).
\end{aligned} \tag{8}$$

Under unanimity voting, the equilibrium cutoffs $(v_3^r, v_3^l, v_2^r, v_2^l, v_1^r, v_1^l)$ should satisfy the following system of six equations:

$$\begin{aligned}
\frac{e^{-\lambda v_3^r}}{2\lambda} + \frac{e^{-\lambda v_3^l}}{2\lambda} &= \tau \\
\frac{e^{-\lambda v_2^l}}{2}\left(v_2^l - v_1^r + \frac{1}{\lambda}\right) + \frac{e^{-\lambda v_1^l}}{2\lambda} &= \tau \\
\frac{e^{-\lambda v_1^l}}{2}\left(v_1^l - v_2^r + \frac{1}{\lambda}\right) + \frac{e^{-\lambda v_2^l}}{2\lambda} &= \tau \\
2v_3^r - v_2^r - v_3^l &= \frac{B}{6} \\
v_1^1 + v_2^l - v_2^r - v_3^r &= \frac{B}{3} \\
2v_1^l - v_1^r - v_2^l &= \frac{B}{2}.
\end{aligned} \tag{9}$$

In the baseline model, the club's degree of incongruity can be measured by a single variable $c = B/(12\sqrt{a\tau})$. It is impossible to define a similar variable in this exponential distribution case. Therefore, in the following numerical example, we fix $\tau = 0.1$, $\lambda = 1$ and explore the impact of B on the long-term welfare of the organization.

Figure A.1 illustrates the comparison of long-term welfare under different cases. It is not surprising to see that the harmonious equilibrium still dominates majority voting in terms of long-term welfare. Under majority voting, as evident from Equation (8), admission standards for candidates of both types are distorted: one is favored, the other is discriminated against. This intuition does not depend on the existence of the glass-ceiling equilibrium, hence is independent of the assumption of quality having bounded support. This divergence of admission standards leads to lower long-term welfare under majority voting than in the harmonious equilibrium. Since the glass-ceiling equilibrium does not exist, long-term welfare changes continuously in B: there is no drop in welfare as shown in Figure 2.

Under unanimity voting, the presence of internal politics can enhance welfare relative to the harmonious equilibrium as in the baseline model. This is not surprising either. Under unanimity voting, incumbent members of both types have relatively balanced power and try to set relatively high standards for candidates of both types, mitigating the intertemporal free-riding problem. Moreover, under unanimity voting, it is the power-switching equilibrium that yields greater long-term welfare than the harmonious equilibrium, and hence the assumption that quality has unbounded support should not change this result.

3.2 Different Ways of Allocating Rent

In this section, we consider three different ways of allocating rent. Our main results are still robust in each of these extensions.

3.2.1 Each Majority Member Receives a Fixed Amount of Rent

To make it easy to compare welfare, we assume in the baseline model that there is a fixed amount of total rent B, no matter how large the majority is. This makes the rent B irrelevant in the welfare comparison. This section considers another case where each majority member receives a fixed amount of rent $\frac{B}{3}$. Then the total rent is $2B/3$ in contentious states and B in homogeneous states. To compare welfare with the baseline model, in the following we do not include the amount of total rent in the welfare calculation.

For $i = 1, 2, 3$ and $j = l, r$, let $x_i^j = \frac{\bar{v} - v_i^j}{a}$, $y_i^j = \sqrt{\frac{a}{4\tau}} x_i^j$ and $c = \frac{B}{12\sqrt{a\tau}}$, just as before. Then the power-switching equilibrium conditions under majority voting are:

$$
\begin{aligned}
(y_3^r)^2 + (y_3^l)^2 &= 1 \\
(y_2^r)^2 + (y_2^l)^2 &= 1 \\
y_2^r + y_3^l - 2y_3^r &= 0 \\
\frac{1}{2} y_3^r (y_2^r + y_2^l) - (y_2^l)^2 &= c(3y_2^r + y_2^l)/3.
\end{aligned}
\tag{10}
$$

Comparing Equation (10) and Equation (A.17) in the paper, we can see that the first two equations are the same. The last two equations become different due to the change of rent specification. In particular, the third equation becomes $y_2^r + y_3^l - 2y_3^r = 0$ because there is no dilution of rent when the majority members move from the contentious state to the homogeneous state.

Similar to the baseline model, there also exists a glass-ceiling equilibrium under majority voting. The conditions for such an equilibrium become

$$\begin{aligned} (y_3^r)^2 + (y_3^l)^2 &= 1 \\ (y_2^r)^2 + (y_2^l)^2 &= 1 \\ y_2^r + y_3^l - 2y_3^r &= 0 \\ y_2^l = 0. \end{aligned} \tag{11}$$

It is straightforward to solve $(y_3^r, y_3^l) = (\frac{4}{5}, \frac{3}{5})$, and $(y_2^r, y_2^l) = (1, 0)$ in this glass-ceiling equilibrium.

The equilibrium conditions under unanimity voting are

$$\begin{aligned} (y_3^r)^2 + (y_3^l)^2 &= 1 \\ y_2^r + y_3^l - 2y_3^r &= 0 \\ y_3^r - y_2^l + y_2^r - y_1^l &= 2c \\ y_2^l + y_1^r - 2y_1^l &= 2c \\ (y_2^r)^2 + (y_2^l)^2 &= 1 + (y_2^r - y_1^l)^2 \\ (y_1^r)^2 + (y_1^l)^2 &= 1 + (y_1^r - y_2^l)^2. \end{aligned} \tag{12}$$

We solve the most efficient equilibrium under each voting rule. Figure A.2 depicts the relative frequency of homogeneous states over contentious states, q_3/q_2, under both majority voting and unanimity voting. Comparing Figure A.2 and Figure 4, we can see that when each majority member receives a fixed amount of rent, the fraction of time spent in the homogeneous states will be greater. Since there is no "dilution" of rent among majority-type members, the incumbent members are more likely to favor candidates of the same type than in the baseline model.

In terms of long-term welfare, Figure A.3 exhibits a pattern similar to that shown in Figure 2 in the paper. Under majority rule, although Equation (10) is different from Equation (A.17), the solutions to these two systems of equations share a similar pattern: either y_i^r or y_i^l is larger than $\frac{\sqrt{2}}{2}$ while the other is smaller than $\frac{\sqrt{2}}{2}$. In other words, the equilibrium admission standards under majority voting are biased in opposite directions relative to those in the harmonious equilibrium. As a result, the long-term welfare under majority voting is always lower than that in the harmonious equilibrium.

As in the baseline model, unanimity voting can dominate the harmonious equilibrium in terms of long-term welfare when the degree of incongruity c is relatively low, because internal politics under unanimity voting leads to higher admission standards for both types of candidates and thus offsets the intertemporal free riding in the harmonious equilibrium. This implies that our main results are not driven by the dilution of rent in the homogeneous states.

Under unanimity voting, it is also interesting to notice that when the degree of incongruity is small, the long-term welfare becomes lower if each majority member receives a fixed amount of rent. When the degree of incongruity is small, the long-term welfare is increasing in the degree of incongruity. Hence, a more unequal distribution of rent increases the long-term welfare by intensifying internal politics. Compared with the case when each majority member receives a fixed amount of rent, our baseline model leads to a more unequal distribution of rent $((\frac{B}{2}, \frac{B}{2}, 0)$ instead of $(\frac{B}{3}, \frac{B}{3}, 0))$ and thus higher long-term welfare. When the degree of incongruity is large, the long-term welfare is higher if each majority member receives a fixed amount of rent. In particular, when the degree of incongruity is in the range from 0.43 to 0.6, always giving each majority member the same amount of rent yields a long-term welfare even higher than the one in the harmonious equilibrium, which in turn is higher than the long-term welfare in our baseline model. When the degree of incongruity is large, internal politics decreases welfare, because both types of incumbent members will set standards that are too stringent to admit candidates of the opposite type. Thus, a more unequal distribution of rent decreases the long-term welfare. Consequently, always giving each majority member the same amount of rent yields a higher long-term welfare when the club is sufficiently incongruous. This finding suggests that if the organization can commit ex ante to the form of rent distribution, the form of the most desirable rent distribution will depend on the degree of incongruity.

3.2.2 Rent is Allocated According to Seniority in Homogeneous States

Another possible concern is that in the homogeneous states, the majority members may not want to share the rent equally. In reality, rent can be allocated according to seniority in homogeneous states: the senior members can form a coalition, and exclude the new member from diluting their rent.

In this section, we consider a case where the two senior members each receive rent $B/2$ in homogeneous states, while the new member receives no rent. In contentious states, it is still assumed that the majority members each receive rent $B/2$ as we did in the benchmark model. Notice that homogeneous states occur only when the majority members admit the same type candidate in homogeneous states or the majority members admit the same type candidate in contentious states and the minority member retires. Therefore, when the incumbent members make the admission decisions, they know that they will be the senior members in homogeneous states, and receive rent $B/2$.

Therefore, when rent is allocated according to seniority in homogeneous states, the analysis is similar to the one when each majority member receives a fixed amount of rent. The only difference is that now each majority incumbent member receives rent $B/2$ instead of $B/3$. Since we can redefine $c = \frac{3B/2}{12\sqrt{a\tau}}$ to obtain the same system of equations (10) to (12), it is immediate to see that our main results still hold: the long-term welfare under majority voting is always lower than that in the harmonious equilibrium, and unanimity voting can dominate the harmonious equilibrium in terms of long-term welfare when the degree of incongruity c is relatively low.

3.2.3 General Rent Allocations in Contentious States via Nash Bargaining

Finally, we consider fairly general rent allocations via Nash bargaining. We allow the bargaining power to be type-dependent, and assume that: 1) members of the same type have the same bargaining power, and 2) majority members have greater bargaining power than minority member. Regarding to the first assumption, it is natural to assume that members with the same type are treated equally, and as seen from the previous section, adding seniority does not change our main results. The second assumption captures the essence of internal politics in our model: the majority members have an advantage over the minority member in rent division.

We allow an arbitrary allocation of rent satisfying the above two assumptions. Hence, in homogeneous states, the members each receive rent $B/3$; and in contentious states, the majority members each receive rent νB where $\nu > \frac{1}{3}$ while the minority member receives $(1-2\nu)B$.

Under majority voting, if the equilibrium is power-switching, we can write down equations similar to Equations (A.6)-(A.12). Define $x_i^{b'} \equiv (\overline{v} - v_i^{b'})/a$, and then we obtain:

$$\begin{aligned} (x_3^r)^2 + (x_3^l)^2 &= 4\tau/a; \\ (x_2^r)^2 + (x_2^l)^2 &= 4\tau/a; \\ x_2^r + x_3^l - 2x_3^r &= \frac{(\nu - \frac{1}{3})B}{a}; \\ 6(x_2^r + x_2^l)\frac{(\nu - \frac{1}{3})B}{a} &= 3x_2^r x_3^r + 3x_2^l x_3^l + 6x_2^l x_2^r - 12(x_2^l)^2. \end{aligned} \tag{13}$$

The above equations coincide with the equilibrium conditions in the benchmark model when $\nu = \frac{1}{2}$. When $\nu = \frac{1}{3}$, we have $x_3^r = x_3^l = x_2^r = x_2^l$, and the resulting equilibrium is the same as the harmonious equilibrium. If each member always receives the same rent no matter what the state is, the equilibrium play is the same as if there was no rent.

In the glass-ceiling equilibrium, we similarly obtain the equilibrium conditions as:

$$\begin{aligned} (x_3^r)^2 + (x_3^l)^2 &= 4\tau/a; \\ (x_2^r)^2 + (x_2^l)^2 &= 4\tau/a \quad \text{and} \quad x_2^l = 0; \\ x_2^r + x_3^l - 2x_3^r &= \frac{(\nu - \frac{1}{3})B}{a}. \end{aligned} \tag{14}$$

And under unanimity voting, if the equilibrium is power-switching, the equilibrium conditions are:

$$
\begin{aligned}
(x_3^r)^2 + (x_3^l)^2 &= 4\tau/a; && (15)\\
x_2^r + x_3^l - 2x_3^r &= \frac{(\nu - \frac{1}{3})B}{a}; && (16)\\
x_3^r - x_2^l + x_2^r - x_1^l &= \frac{2(\nu - \frac{1}{3})B}{a}; && (17)\\
x_2^l + x_1^r - 2x_1^l &= \frac{3(\nu - \frac{1}{3})B}{a}; && (18)\\
(x_2^r)^2 + (x_2^l)^2 &= \frac{4\tau}{a} + (x_2^r - x_1^l)^2; && (19)\\
(x_1^r)^2 + (x_1^l)^2 &= \frac{4\tau}{a} + (x_1^r - x_2^l)^2. && (20)
\end{aligned}
$$

Define $y_i^j = \sqrt{\frac{a}{4\tau}} x_i^j$ and $c = \frac{(\nu - \frac{1}{3})B}{2\sqrt{a\tau}}$. Then the resulting systems of equations are exactly the same as equilibrium conditions in the benchmark model. Therefore, we conclude that our main results still hold as long as $\nu > \frac{1}{3}$. This implies that the main driving force in our model is the unequal allocation of rent between majority and minority members in the contentious states, and has nothing to do with rent dilution in the homogeneous states.

This extension has another interesting implication on organizational design: even when the parameters a, τ and B are all exogenous, the organization can still achieve the right degree of incongruity by adjusting the value of ν. Notice that for $\nu < \frac{1}{2}$, $c = \frac{(\nu - \frac{1}{3})B}{2\sqrt{a\tau}}$ is always less than $\frac{B}{12\sqrt{a\tau}}$, the degree of incongruity defined in our benchmark model. Similar to the finding in Section 2.2.1, this implies that a more equal distribution of rent $(\nu, \nu, 1-2\nu)$ will decrease the long-term welfare when the degree of incongruity $\frac{B}{12\sqrt{a\tau}}$ is low, but increase the long-term welfare when the degree of incongruity is high. In particular, as the long-term welfare is maximize at $c = 0.24$, the ν which maximizes the long-term welfare should be $\frac{1}{2}$ when $\frac{B}{12\sqrt{a\tau}} < 0.24$, and should satisfy $\frac{(\nu - \frac{1}{3})B}{2\sqrt{a\tau}} = 0.24$ when $\frac{B}{12\sqrt{a\tau}} \geq 0.24$.

3.3 Different Discount Factor

For simplicity, we assume in the paper that there is no discounting and the effective discount factor is the probability of staying in the club, $\frac{2}{3}$. To relax this assumption, we now assume that, in any period, exiting by incumbent members does not always occur. Specifically, at the beginning of each period, with probability p, the three incumbents know that there is an exit in this period, and they have to search for a candidate within this period. With the complementary probability, there is no exit and the game moves to the next period. Probabilistic exit occurs when the incumbent member receives a random chance of promotion or an outside option that dominates staying in the club. Regardless of whether there is exit or not, there is rent B to be distributed in every period.

Obviously, in this extension, the effective discount factor becomes $1 - \frac{1}{3}p$, which can be arbitrarily close to 1 as p approaches 0. The first best solution thus maximizes $3E[v|v \geq v^*] - p\tau F(v^*)/(1 - F(v^*))$, and it is straightforward to obtain the interior solution $v^* = \bar{v} - \sqrt{2pa\tau/3}$. In the harmonious equilibrium, the expected value to an incumbent member if a candidate with quality $\hat{v}$ is admitted is:

$$\frac{2}{3}\left(1 + \left(1 - \frac{2}{3}p\right) + \left(1 - \frac{2}{3}p\right)^2 + ...\right)\hat{v} = \frac{\hat{v}}{p}.$$

This implies that in an interior solution, $\hat{v} = \bar{v} - \sqrt{2pa\tau}$.

Now we analyze the equilibrium with internal politics. Denote π_i^R to be the right-type incumbent's searching payoff at state i when there is a random exit, and $\tilde{\pi}_i^R$ to be the right-type incumbent's expected searching payoff at state i. Obviously,

$$\tilde{\pi}_i^R = p\pi_i^R + (1 - p)(\tilde{\pi}_i^R + \mu_i),$$

where μ_i is the rent received by the right-type incumbent. With probability p, there is an exit and the searching payoff is π_i^R; with the complementary probability, there is no exit and the searching payoff is $\tilde{\pi}_i^R + \mu_i$.

Under majority voting, Equation (A.6) becomes

$$\frac{v_3^r}{p} + \frac{2}{3}\left[\frac{B}{3} + \tilde{\pi}_3^R\right] = \pi_3^R - \tau,$$

which implies

$$\frac{v_3^r}{p} + \frac{2}{3}\left[\frac{B}{3p} + \pi_3^R\right] = \pi_3^R - \tau. \tag{21}$$

Similarly, Equations (A.7)-(A.9) should be rewritten as:

$$\frac{v_3^l}{p} + \frac{2}{3}\left[\frac{B}{2p} + \pi_2^R\right] = \pi_3^R - \tau; \tag{22}$$

$$\frac{v_2^r}{p} + \frac{2}{3}\left[\frac{5B}{12p} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] = \pi_2^R - \tau; \tag{23}$$

$$\frac{v_2^l}{p} + \frac{2}{3}\left[\frac{B}{4p} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] = \pi_2^R - \tau; \tag{24}$$

and Equations (A.10)-(A.12) should be rewritten as:

$$
\begin{aligned}
\pi_3^R &= \frac{\overline{v}-v_3^r}{3a}\left[\frac{B}{3p}+\pi_3^R\right]+\frac{\left[\overline{v}^2-(v_3^r)^2\right]}{4ap}+\frac{v_3^r-\underline{v}}{2a}[\pi_3^R-\tau] \\
&\quad +\frac{\overline{v}-v_3^l}{3a}[\frac{B}{2p}+\pi_2^R]+\frac{\left[\overline{v}^2-\left(v_3^l\right)^2\right]}{4ap}+\frac{v_3^l-\underline{v}}{2a}[\pi_3^R-\tau]; \qquad (25)\\
\pi_2^R &= \frac{\overline{v}-v_2^r}{3a}\left[\frac{5B}{12p}+\frac{1}{2}\pi_2^R+\frac{1}{2}\pi_3^R\right]+\frac{\left[\overline{v}^2-(v_2^r)^2\right]}{4ap}+\frac{v_2^r-\underline{v}}{2a}[\pi_2^R-\tau] \\
&\quad +\frac{\overline{v}-v_2^l}{3a}\left[\frac{B}{4p}+\frac{1}{2}\pi_1^R+\frac{1}{2}\pi_2^R\right]+\frac{\left[\overline{v}^2-\left(v_2^l\right)^2\right]}{4ap}+\frac{v_2^l-\underline{v}}{2a}[\pi_2^R-\tau]; \qquad (26)\\
\pi_1^R &= \frac{\overline{v}-v_2^r}{3a}\pi_1^R+\frac{\left[\overline{v}^2-(v_2^r)^2\right]}{4ap}+\frac{v_2^r-\underline{v}}{2a}[\pi_1^R-\tau] \\
&\quad +\frac{\overline{v}-v_2^l}{3a}[\frac{B}{2p}+\pi_1^R]+\frac{\left[\overline{v}^2-\left(v_2^l\right)^2\right]}{4ap}+\frac{v_2^l-\underline{v}}{2a}[\pi_1^R-\tau]. \qquad (27)
\end{aligned}
$$

Rearranging the above seven equations yields (we similarly define $x_i^{b'} \equiv (\overline{v}-v_i^{b'})/a$)

$$
\begin{aligned}
(x_3^r)^2+(x_3^l)^2 &= 4p\tau/a; \\
(x_2^r)^2+(x_2^l)^2 &= 4p\tau/a; \qquad (28)\\
x_2^r+x_3^l-2x_3^r &= \frac{B}{6a}; \\
(x_2^r+x_2^l)\frac{B}{a} &= 3x_2^r x_3^r+3x_2^l x_3^l+6x_2^l x_2^r-12(x_2^l)^2.
\end{aligned}
$$

Notice that this model is equivalent to our baseline model with effective cost of delay $\tau'=p\tau$. Let $y_i^{b'}=x_i^{b'}\sqrt{a/(p\tau)}/2$, for $i=1,2,3,4$ and $b'=l,r$, and $c'=B/(12\sqrt{ap\tau})$. Then we derive the following four equations which are exactly the same as Equation (A.17) in the paper (after replacing c with c'):

$$
\begin{aligned}
(y_3^r)^2+(y_3^l)^2 &= 1 \\
(y_2^r)^2+(y_2^l)^2 &= 1 \qquad (29)\\
y_2^r+y_3^l-2y_3^r &= c' \\
y_2^r y_3^r+y_2^l y_3^l+2y_2^l y_2^r-4(y_2^l)^2 &= 2c'(y_2^r+y_2^l).
\end{aligned}
$$

Under unanimity voting, we can derive the same equilibrium conditions about $y_3^r, y_3^l, y_2^r, y_2^l, y_1^r, y_1^l$ using the above definitions of $y_i^{b'}$ and c'.

Finally, using some algebra, we recalculate the long-term welfare in each case. Similar to the baseline model,

$$U = 3Ev + \frac{3}{2}a + p\tau - 2\sqrt{pa\tau}\gamma, \tag{30}$$

where γ summarizes the total long-term expected welfare loss for the club in each case. Furthermore, the expressions of γ are exactly the same as the ones in the baseline model. Equation (30) implies that as p approaches zero, $\sqrt{pa\tau}$ goes to zero, and the long-term welfare in each case converges to the same value. However, as p enters into Equation (30) in the same way, the comparisons of the long-term welfare do not directly depend on p, and we still only need to compare γ.

Therefore, Proposition 5 in our main context still holds if we replace c with the newly defined variable c'. Since $c' = c/\sqrt{p} > c$ for all $p < 1$, the range of c for which unanimity voting can yield greater long-term welfare than the harmonious equilibrium (that is, the range in which internal politics can improve welfare) shrinks compared to the baseline model. Moreover, this range is smaller as p becomes lower. The reason for this result is the following. As the discounting factor increases, the intertemporal free-riding problem becomes less severe since the effective cost of delay $p\tau$ becomes smaller. As a result, internal politics becomes less beneficial.

3.4 Larger Committee Size: $n = 5$

In the baseline model we consider a club with three members. In this section, we extend our baseline model to a club with five members ($n = 5$), in order to show that the main results of the baseline model can be generalized to larger committees.[1] Increasing the club size from three to five effectively increases the discount factor from 2/3 to 4/5, thus the analysis below also provides a different way of increasing the effective discount factor than from the way of introducing probabilistic exit considered in the preceding section.

It is straightforward to verify that, just as before, majority voting is dominated by the harmonious equilibrium when $n = 5$, because the equilibrium admission standards under majority voting are biased in opposite directions relative to those in the harmonious equilibrium. To save space, we omit the details of the derivation of this result.

Let $y_i^{b'} = x_i^{b'}\sqrt{a/\tau}/2$, for $i = 1, 2, 3, 4$ and $b' = l, r$, and $c' = B/(12\sqrt{a\tau})$. Under unanimity voting, we can derive ten equilibrium conditions about $y_5^r, y_5^l, y_4^r, y_4^l, y_3^r, y_3^l, y_2^r, y_2^l$, y_1^r, y_1^l

[1]From the analysis of the model with $n = 5$, it is clear that solving the model with an arbitrary n is too tedious. However, it is not hard to see that the insights of our baseline model should still hold.

$$
\begin{aligned}
(y_5^r)^2 + (y_5^l)^2 &= 1 \\
(y_4^r)^2 + (y_4^l)^2 &= 1 + (y_4^r - y_1^l)^2 \\
(y_3^r)^2 + (y_3^l)^2 &= 1 + (y_3^r - y_2^l)^2 \\
(y_2^r)^2 + (y_2^l)^2 &= 1 + (y_2^r - y_3^l)^2 \\
(y_1^r)^2 + (y_1^l)^2 &= 1 + (y_1^r - y_4^l)^2 \\
2y_4^r + y_5^l - 3y_5^r &= \frac{3}{5}c \\
y_4^l + y_3^r - y_4^r - y_5^r &= \frac{4}{5}c \\
2y_2^l + y_1^r - 3y_1^l &= 0 \\
y_2^r + y_3^l - y_1^l - y_2^l &= 2c \\
\frac{25}{3}(x_5^r - x_1^l) - \frac{5}{3}(x_5^l + x_4^l - x_1^r - x_2^r) &= \frac{11}{2}c.
\end{aligned} \tag{31}
$$

We can similarly use γ to summarize the total long-term expected welfare loss for the club. Here, γ^u changes to

$$\gamma^u \equiv 6q_5/(y_5^r + y_5^l) + q_4(1 + 5(y_1^l)^2 + 5(y_4^l)^2)/(y_1^l + y_4^l) + q_3(1 + 5(y_2^l)^2 + 5(y_3^l)^2)/(y_2^l + y_3^l).$$

And in the harmonious equilibrium $\hat{\gamma} = \frac{3}{2}\sqrt{2}$.

Figure A.4 illustrates the the comparison of welfare losses γ between unanimity voting and the harmonious equilibrium. Similar to the baseline model, unanimity voting yields greater long-term welfare than the harmonious equilibrium when c is relatively low. Moreover, the range of c for which this property holds obviously becomes larger (we require $c \leq 0.42$ in the $n = 3$ case). The reason for this result is the following. On the one hand, larger club size increases the discounting factor, and hence the intertemporal free-riding problem becomes less severe as in the previous section. On the other hand, the incentives of politicking become less severe with larger club size as the total amount of rent is fixed. As in the fixed per-capita rent case, less politicking due to larger club sizes increases the long-term welfare when the degree of incongruity is large.

4 Testable Implications on Organization Behavior

Our model also allows us to conduct positive analysis of the equilibrium behavior of the organization, and derive implications that are potentially testable in empirical contexts. There are at least three aspects of organizational behavior to investigate. First of all, Figure A.5 depicts the expected search length, measured by the weighted average of the expected length in each state, where the weight is the long-term probability of each state. Figure A.5 shows that under both majority and unanimity voting, the expected search length is increasing in c. That is, more incongruous organizations have longer decision time. Moreover, unanimity voting always yields a higher expected search length than majority voting. This is because majority candidates in contentious states face more stringent admission standards under unanimity voting. Under unanimity voting, the equilibrium admission standards for each type of candidates in contentious states are determined by the incumbent members of the opposite type (as shown by Lemma 1 in the Appendix), and both types of incumbent members set stringent standards for the admission of candidates of the opposite type. However, under majority voting, the majority incumbent members favor their own type.

Secondly, Figure A.6 depicts the fraction of time spent in contentious vs. homogeneous states, measured by the relative frequency in the long run: q_3/q_2. It can be seen that this relative frequency is discontinuous in c under both majority and unanimity voting. For majority voting, the discontinuity reflects the switch from the power-switching equilibrium to the glass-ceiling equilibrium; for unanimity voting, the discontinuity reflects the switch from the pro-minority equilibrium to the pro-majority equilibrium. Moreover, it is interesting to notice that this relative frequency is non-monotonic in c under both voting rules. This reflects two opposite effects caused by an increase of c. For example, when c becomes larger, the majority incumbent members on the one hand have higher incentives to stay in the homogeneous states to avoid the risk of losing control. But on the other hand they also have higher incentives to switch to the contentious states to avoid "dilution" of rent.

Finally, Figure A.7 depicts the expected length of time between switches of control. Under majority voting, this expected length is increasing in c when $c < 0.43$: as c becomes larger, the majority incumbent members are more reluctant to lose control. For $c > 0.43$, the expected length is infinity as there is no switch in the glass-ceiling equilibrium. Under unanimity voting, the expected length is decreasing in c in the pro-minority power-switching equilibrium. However, in the pro-majority power-switching equilibrium, the expected length is again non-monotonic in c as shown in Figure A.6.

5 Equilibrium Characterization under Unanimity Voting

Based on the derivation procedure of Proposition 4 in the Appendix of the main text, we can solve for all possible equilibria under unanimity voting. The proposition below provides a complete characterization of all equilibria under unanimity voting. (Details of the derivation are available upon request.)

Proposition A.1 *Under unanimity voting rule, there are five different kinds of equilibria.*

1. *When* $0 < c \equiv B/12\sqrt{a\tau} < 2/3$*, a "pro-minority power-switching" equilibrium exists in*

which candidates of both types are admitted in each state with positive probabilities. However, candidates of the majority type in the contentious state have a lower probability of admission (higher admission standard) than those of the minority type, and are less likely to be admitted as c increases. They are never admitted when c goes to $2/3$*.*

2. *When* $10/29 < c < 2.839$*, a "pro-majority power-switching" equilibrium exists in which, as in the pro-minority power-switching equilibrium, candidates of both types are admitted in each state with positive probabilities and candidates of the majority type in the contentious state have a higher probability of admission (lower admission standard) than those of the minority type.*

3. *When* $c > 10/29$*, the glass-ceiling equilibrium exists and it is the same as in the majority case. In the long run, the system switches between state 3 and state 2 (resp. state 1 and state 0) if the initial state is 3 or 2 (resp. 1 or 0).*

4. *When* $c > 2/3$*, a "minority tyranny" equilibrium exists in which, in the contentious state, only candidates of the minority type are admitted. In the long run, the club only switches between state 2 and state 1.*

5. *When* $c > 2.839$*, an "exclusive" equilibrium exists in which the incumbent members in homogeneous states 3 and 0 admit only candidates of their own type. In the long run, the club stays at either state 3 or state 1.*

Proposition A.1 characterizes the range of c each of the five equilibria exists. The following table then characterizes the set of equilibria for each value of c.

c	$\left(0, \frac{10}{29}\right)$	$\left(\frac{10}{29}, \frac{2}{3}\right)$	$\left(\frac{2}{3}, 2.839\right)$	(> 2.839)
Pro-minority Power-switching	✓	✓		
Pro-majority Power-switching		✓	✓	
Glass Ceiling		✓	✓	✓
Minority Tyranny			✓	✓
Exclusive				✓

Figure A.8 depicts the long-term welfare comparison of different equilibria under unanimity voting. The welfare in the most efficient equilibrium depicted in Figure 2 of the main text is obtained from taking the lower envelope of Figure A.8.

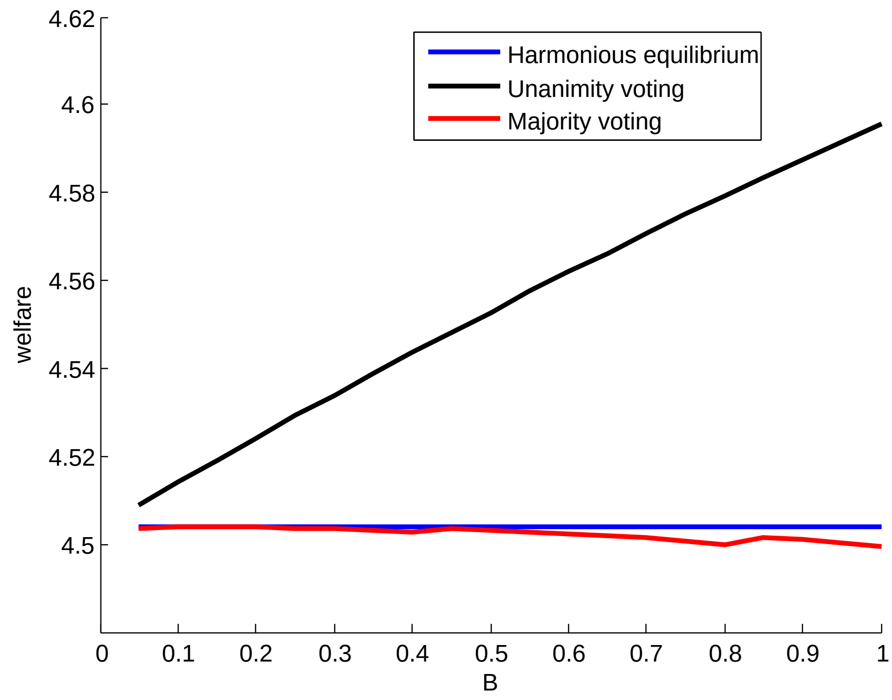

Figure A.1: Long-Term Welfare Comparison Under Exponential Distribution

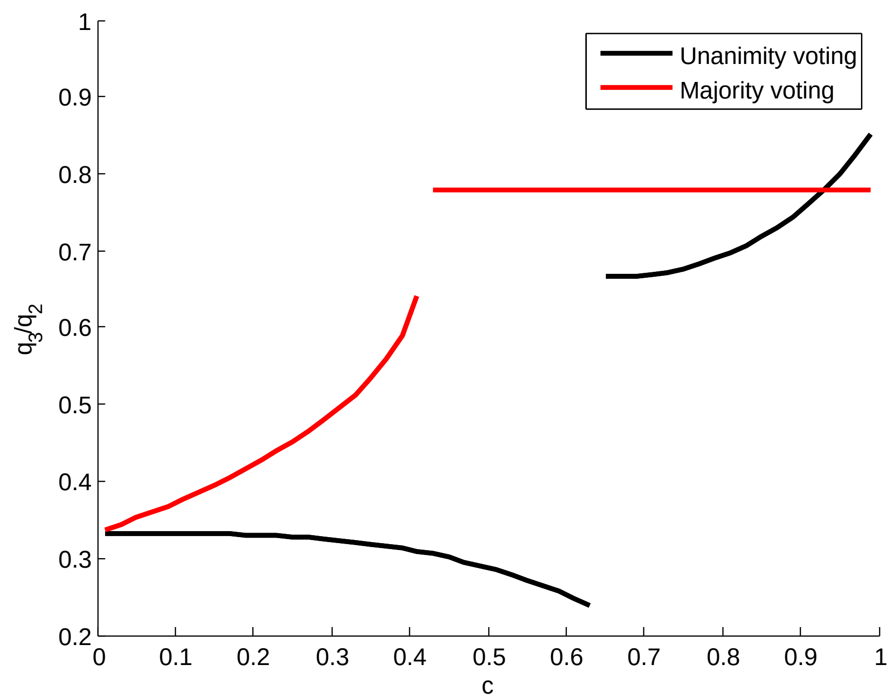

Figure A.2: Relative State Frequency Under Fixed Per-capita Rent

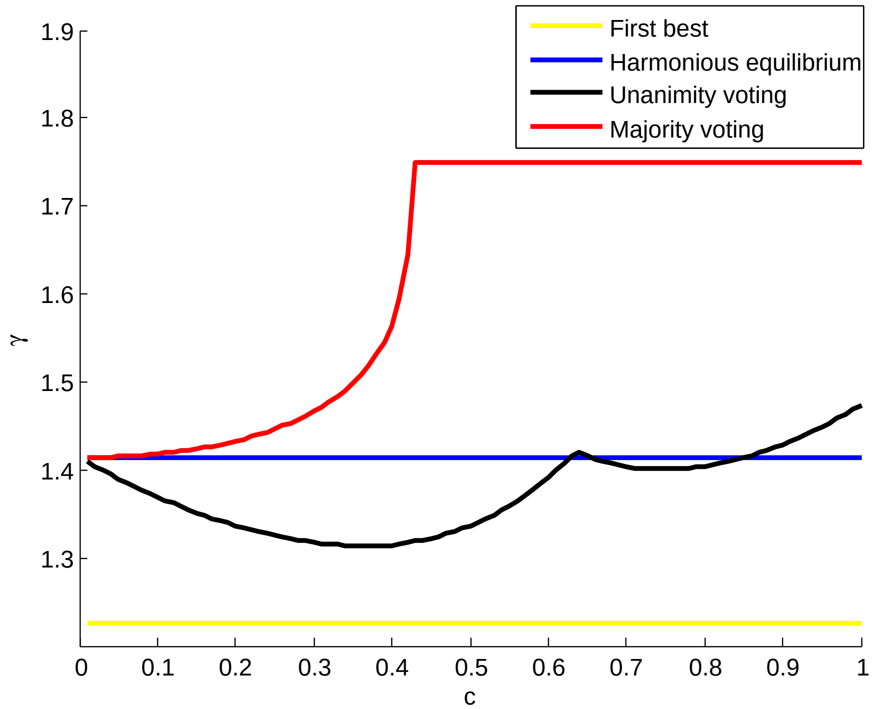

Figure A.3: Long-Term Welfare Loss Comparison Under Fixed Per-capita Rent

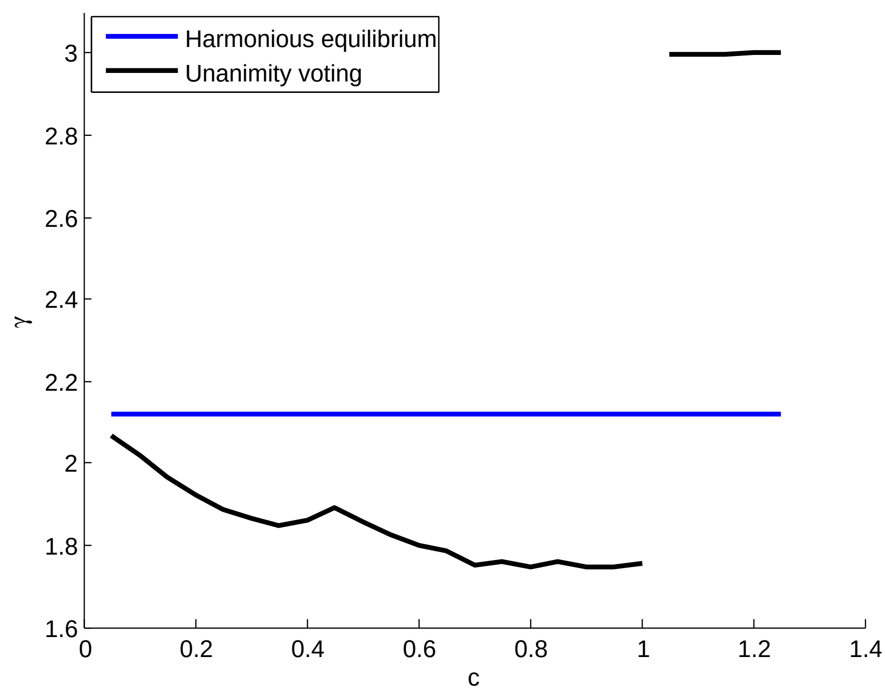

Figure A.4: Long-Term Welfare Loss When $n = 5$

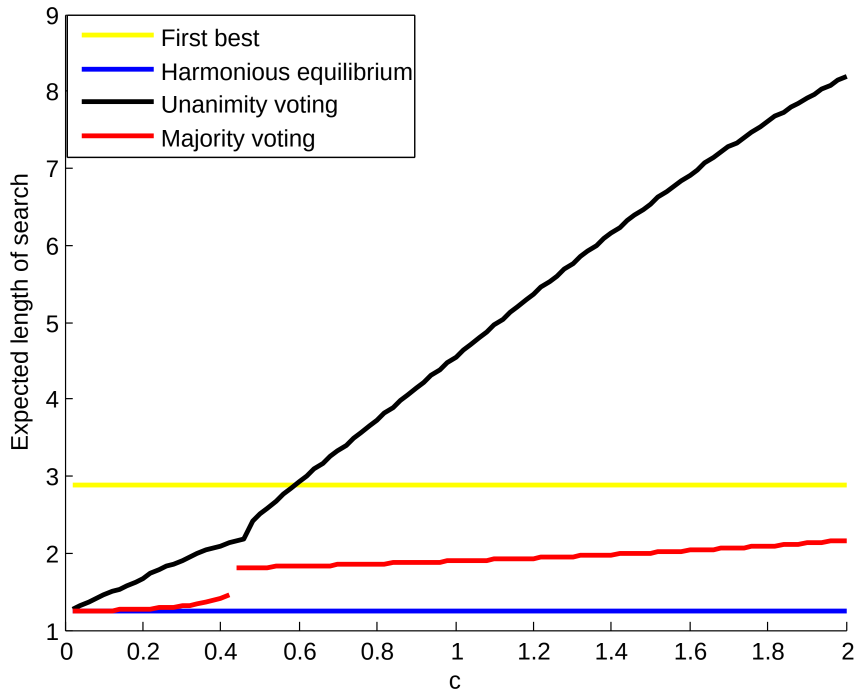

Figure A.5: Long-Term Expected Delay in the Most Efficient Equilibrium, Unanimity Voting Always Causes a Longer Expected Delay than Majority Voting and Can Even Cause a Longer Expected Delay than First Best when c is Sufficiently Large

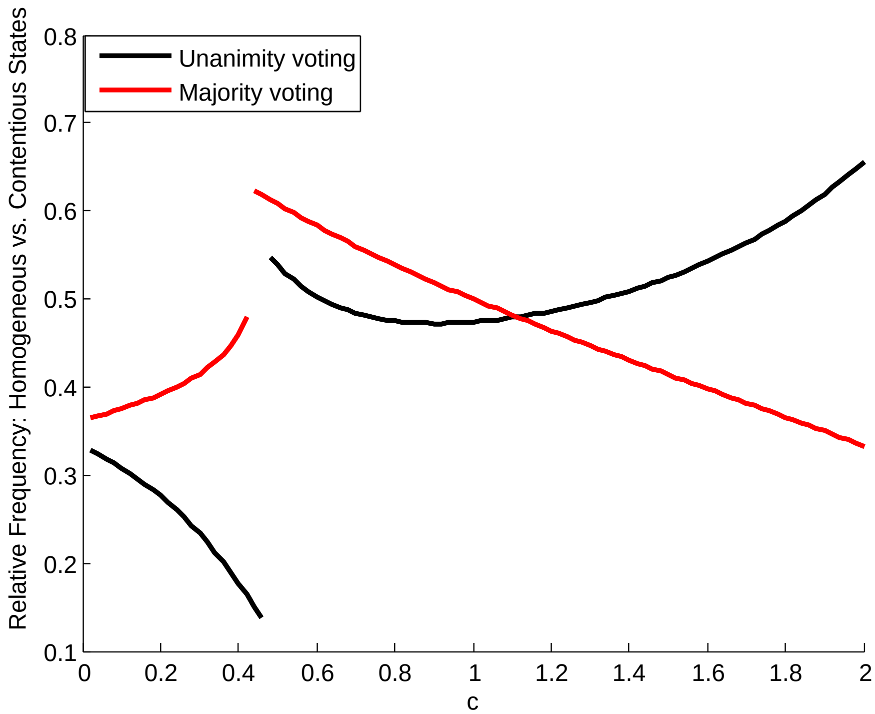

Figure A.6: Relative State Frequency in the Most Efficient Equilibrium, Non-Monotonic Relationship in c under Both Majority Voting and Unanimity Voting

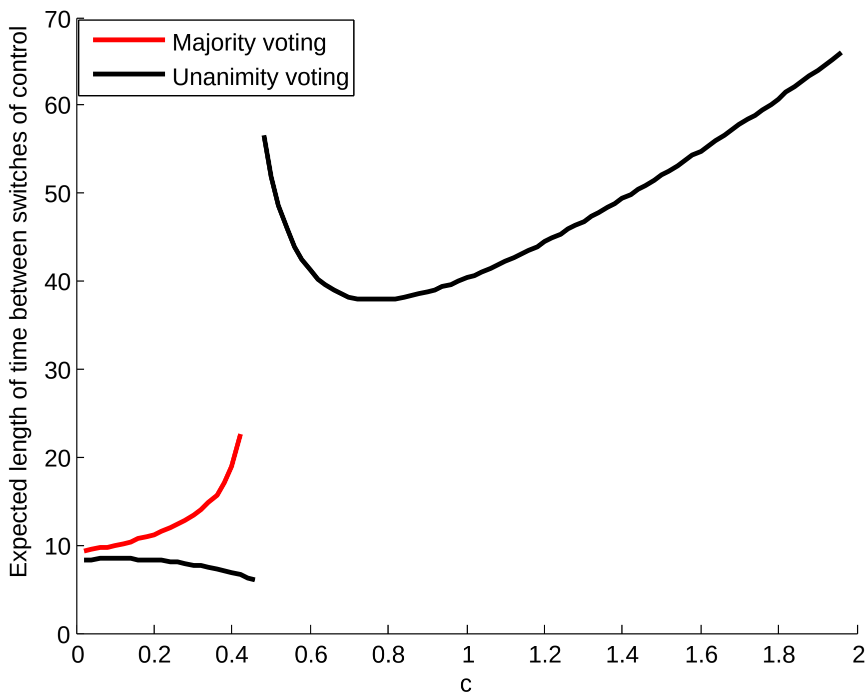

Figure A.7: Expected Length of Time between Switches of Control in the Most Efficient Equilibrium, Non-Monotonic Relationship in c under Unanimity Voting

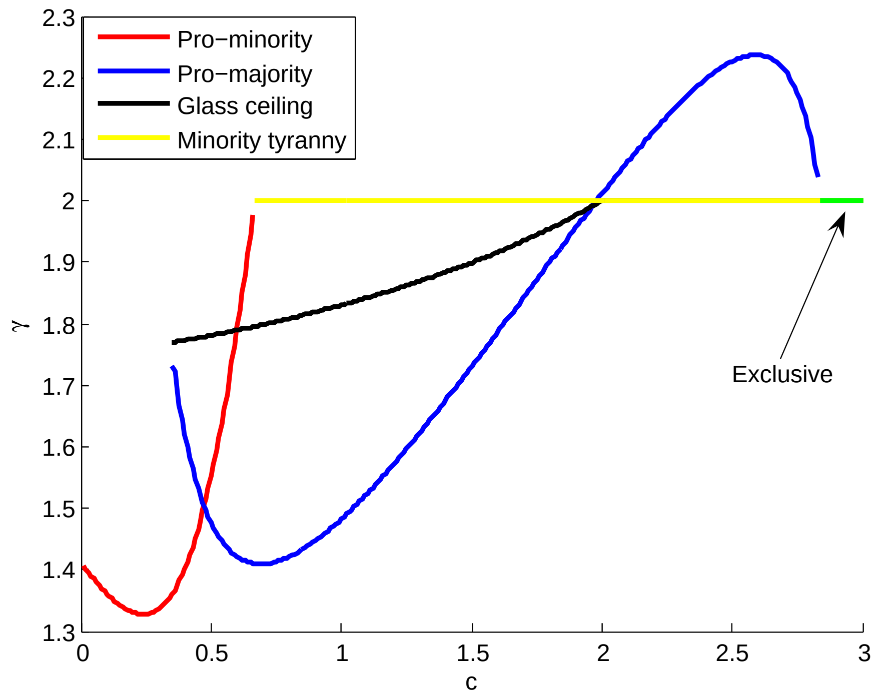

Figure A.8: Long-Term Welfare Comparison of Different Equilibria under Unanimity Voting