Online Appendix for "Laboratories of Democracy: Policy Experimentation under Decentralization" by Chen Cheng and Christopher Li

Two-Period Extension of the Model

In the benchmark setting, we assume that the voter's objective for the election outcome is to vote into office the policymaker with the maximum expected competence level. Moreover, we define voter welfare as the expected output following the election. In this section, we describe in more detail a two-period version of our model, which helps justify our modeling choices.

Let the players live for one additional period after the election. Define the output in Period 2 as ϕ_2^i similarly to ϕ^i—i.e., the output is the sum of the (elected) policymaker's competence and the policy payoff. We assume that the policymaker in Period 2 chooses the optimal policy, given the observed outputs in Period 1. This assumption is innocuous, since the game ends after Period 2. Finally, the utility of the voter in district i is equal to the output in district i. Thus, the voter's objective is to maximize the expected output in Period 2. This means that the optimal election rule is the same as in (2). Consequently, the policymaker's equilibrium behavior in this two-period model is the same as in the benchmark setting. Moreover, in the two-period setting, the voter's welfare is naturally defined as the expected output in Period 2, which is in accordance with the definition presented in Section 3.5 of the paper.

Endogenizing the Incumbents' Payoff

As noted in the main paper, a key determinant of the policymakers' incentives is the nature of their payoffs. More specifically, the incumbent's payoff is zero if they are voted out of office, and increasing in their reputation if they are re-elected. In this section, we show that this feature of the politician's payoff naturally arises when the incumbent takes into account the possibility of remaining in office for multiple terms. More specifically, we examine a dynamic model with multiple elections and show that the incumbent's continuation payoff has the bonus and limited liability feature even when the static payoff does not.

Consider a model of elections with $T > 2$ periods. As before, we assume that there is prior uncertainty about the policymakers' competence. In each period t, the voter observes an *i.i.d.* normal signal s_t in the form of the sum of the incumbent's competence and some normal noise.[24] Let h^t denote the set of signals observed up to time t. There is an election at the end of every period. We denote the voter's posterior belief about the incumbent at the time of the election as follows: $\tilde{\theta}_t \sim N\left(\mathbb{E}\left[\theta_t|h^t\right], \sigma\left[\theta_t|h^t\right]\right)$. We assume the voter's objective is to maximize the expected sum of the incumbent's competence: $\sum_t \mathbb{E}\left[\theta_t\right]$.

We define the static payoff for the incumbent as follows:

$$u(h^t) = \begin{cases} 0 & \lambda_t(h^t) = 0 \\ 1 & \lambda_t(h^t) = 1, \end{cases} \tag{5}$$

where $\lambda_t(\cdot)$ is the electoral rule. Note that the static utility $u(h^t)$ is different in nature from the payoff in the benchmark model. In particular, the in-office payoff is constant, and, therefore, the incumbent does not obtain "bonuses" for obtaining high realizations of the signals. One can show that if the payoff in the benchmark model were of the same form as (5), then the policymakers in our benchmark model would not exhibit risk-seeking behaviors. However, in a dynamic setting as that described above, the continuation payoff for the incumbent does have the bonuses and limited liability nature of the payoff in the benchmark model (see Lemma OA.1 below). Moreover, we can show that for $T = 3$, the continuation payoff satisfies the conditions in Proposition A.1 and, therefore, the incumbent would seek risk, as in the benchmark model.

The following lemma shows that the incumbents' continuation payoff exhibits the bonuses and limited liability feature when voters adopt the optimal electoral rule. For simplicity, we focus on the continuation payoff for the initial (date 1) incumbent, denoted as $v(s_1)$.[25]

Lemma OA.1. *The optimal electoral rule is a threshold rule with respect to s_1. In particular, there exists some constant c such that $\lambda(\tilde{\theta}_1) = 1$ if and only if $s_1 \geq c$. And the resulting continuation payoff for the initial incumbent is as follows:*

$$v(s_1) = \begin{cases} 0 & s_1 < c \\ w(s_1) & s_1 \geq c, \end{cases} \tag{6}$$

[24] The equivalent of s_t in our benchmark model would be the sufficient statistic s_c or s_d.

[25] At date 1, $h^1 = \{s_1\}$.

where $w(s_1)$ is an increasing function of s_1.

Proof. It is straightforward to see that the optimal electoral rule at any given date would necessarily have the following property: given the variance of $\sigma\left[\theta_t|h^t\right]$, the electoral rule is a threshold rule with regard to $\mathbb{E}\left[\theta_t|h^t\right]$. Since $\mathbb{E}\left[\theta_1|h^1\right]=\mathbb{E}\left[\theta_t|s_1\right]$ and $\mathbb{E}\left[\theta_1|s_1\right]$ is monotonic in s_1, the voter would adopt a threshold rule s_1 at date 1, similar to that of the threshold model. Clearly, below the threshold—call it c—the continuation payoff for the incumbent is 0. However, if s_1 is above the threshold, then the continuation value would be positive; we denote it as $w(s_1)$.

We now argue that $w\left(s_1\right)$ is a strictly increasing function. This is equivalent to showing that the expected length of tenure is increasing in the realization of s_1. Formally, the expected length of tenure is $\mathbb{E}\left[\sum_t \mathbf{1}\left[\theta_t=\theta_1\right]\right]$,where $\mathbf{1}\left[\theta_t=\theta_1\right]$ is an indicator function that takes the value of 1, if the initial incumbent is in office at time t. Clearly, for the initial incumbent, the expected length of tenure is an increasing function of $\mathbb{E}\left[\theta_1|h^t\right]$ for all t, since the optimal electoral rule in every period is a cut-off rule in $\mathbb{E}\left[\theta_t|h^t\right]$. Now, the distribution of signals $(s_2,\ldots,s_T)$ is independent of s_1; the properties of Bayesian updating imply that for all t, $\mathbb{E}\left[\theta_1|h^t\right]$ is strictly increasing in the realization of s_1. This would imply that the length of tenure would be strictly increasing in s_1. □

Although we have shown that the continuation payoff is the same in nature as the payoff in the benchmark model, we cannot yet conclude that the incumbent prefers to take risks. In particular, it is not easy to verify whether (6), in general, satisfies the conditions in Proposition A.1. However, for the case of $T=3$, we can show that the continuation payoff for the initial incumbent, $v(s_1)$, satisfies the conditions in Proposition A.1.

Proposition OA.1. *For $T=3$, the incumbent's expected continuation value $\mathbb{E}\left[v(s_1)\right]$ is increasing in the variance of s_1.*

Proof. First, we show that the continuation value for the voter, $\pi(\tilde{\theta}_1)$, is increasing in the mean and variance of $\tilde{\theta}_1$—the competence of the second-period incumbent prior to observing s_2. This would, then, imply that for the election at the end of $t=1$, the voter would choose a replacement if the posterior competence in the incumbent is non-positive, because the variance of $\tilde{\theta}_1$ is greater with a replacement than with the incumbent. Now, the continuation value $\pi\left(s_1\right)$ can be written as follows:

$$\pi\left(\tilde{\theta}_1\right)=\int_{s_2}\max\left\{\mathbb{E}\left[\tilde{\theta}_2\right],0\right\}dF\left(s_2|\tilde{\theta}_1\right),$$

where $\mathbb{E}\left[\tilde{\theta}_2\right] = \mathbb{E}\left[\tilde{\theta}_1 | s_2\right]$ is the posterior expectation of the second-period incumbent, and $s_2|\tilde{\theta}_1$ denotes the distribution of s_2 provided the competence component is $\tilde{\theta}_1$ (it is not a conditional distribution). This expression relies on the fact that at the optimum, the voter re-elects the Period-2 incumbent, if and only if their expected competence is above 0. Now, observe that prior to the realization of s_2, the quantity $\mathbb{E}\left[\tilde{\theta}_2\right]$ is a normal random variable with mean $\mathbb{E}\left[\tilde{\theta}_1\right]$. In fact, $\mathbb{E}\left[\tilde{\theta}_2\right]$ would be a weighted sum of s_2 and $\mathbb{E}\left[\tilde{\theta}_1\right]$. Thus, the optimal re-election rule in Period 2 implies that there is some constant c' such that:

$$\max\left\{\mathbb{E}\left[\tilde{\theta}_2\right], 0\right\} = \begin{cases} 0 & s_2 < c' \\ \mathbb{E}\left[\tilde{\theta}_2\right] & s_2 \geq c'. \end{cases}$$

Treating $\max\left\{\mathbb{E}\left[\tilde{\theta}_2\right], 0\right\}$ as $u(s)$, we can apply the second condition of Proposition A.1, and, consequently, $\pi\left(\tilde{\theta}_1\right)$ is increasing in the variance of $s_2|\tilde{\theta}_1$. Now, the variance of $s_2|\tilde{\theta}_1$ is increasing in the variance of $\tilde{\theta}_1$, since s_2 is the sum of $\tilde{\theta}_1$ and some noise. Therefore, $\pi\left(\tilde{\theta}_1\right)$ is increasing in the variance of $\tilde{\theta}_1$. Finally, we want to show that $\pi\left(\tilde{\theta}_1\right)$ is increasing in the mean of $\tilde{\theta}_1$. It follows from the observation that $\mathbb{E}\left[\tilde{\theta}_2\right]$, as a normal random variable, has mean $\mathbb{E}\left[\tilde{\theta}_1\right]$; therefore, can be ranked according to $\mathbb{E}\left[\tilde{\theta}_1\right]$ in terms of first-order stochastic dominance. Thus, $\pi\left(\tilde{\theta}_1\right)$ would be increasing in $\mathbb{E}\left[\tilde{\theta}_1\right]$.

In sum, we have shown that the voter's continuation payoff, $\pi(\tilde{\theta}_1)$, is increasing in the mean and variance of $\tilde{\theta}_1$. It follows that if the initial incumbent has posterior expectations that are less than 0 at the time of Period-1 election, then the voter would have a strict preference for a replacement whose competence has a mean of 0 and a greater variance. In other words, the threshold for re-election at $t = 1$ must be strictly positive, and we can apply the first condition of Proposition A.1 to get our results. □

Higher-Office Motivation[26]

In the main paper, we assume that, under decentralization, district incumbents are motivated to retain district office. However, in reality, local politicians often aspire to run for

[26] We thank Roger Myerson for suggesting this extension.

higher office.[27] In this section, we examine the implications of this type of higher-office motivation. We show that the characterization of the equilibrium policy profile remains as in the benchmark model. However, unlike in the benchmark model, decentralization induces voters' greater learning about politicians than centralization does.

To incorporate the higher-office motivation, we make the following modification to our benchmark model. We assume that under decentralization, there is an election for higher office instead of local elections. The electoral candidates are the two district incumbents. Note that since the modification affects only decentralization, it immediately follows that the equilibrium under centralization is unaffected. Proposition OA.2, below, shows that the equilibrium under decentralization also remains the same.

Proposition OA.2. *Diverse policies remain as the equilibrium policy profile under decentralization in the presence of higher-office motivation.*

Proof. We show that district A's incumbent (and analogously for district B's incumbent) has the incentive to decrease the correlation between the outputs of the two districts. Equilibrium behavior then follows from our discussion in Section 3.2 of the paper. Note, first, that because of symmetry, the voter will elect district A's incumbent, if and only if $\phi^A > \phi^B$. Let $s \triangleq \phi^A - \phi^B$, and let voters conjecture that the politicians' policies are diverse; thus, the expected utility for the incumbent can be written as follows:

$$\begin{aligned}\mathbb{E}\left(u|\mathsf{p}\right) &= \int_{s>0} \mathbb{E}_{\phi^A}\left[\mathbb{E}_{voter}\left(\theta|\phi^A\right)|s\right] dF(s|\mathsf{p}) \\ &= \frac{\text{cov}\left(\phi^A,\theta\right)}{\text{var}\left(\phi^A\right)} \int_{s>0} \mathbb{E}\left[\phi^A|s\right] dF(s|\mathsf{p}),\end{aligned}$$

where the first inequality follows from the law of iterated expectations, and the second equality follows from the fact that $\mathbb{E}_{voter}\left(\theta|\phi^A\right) = \frac{\text{cov}(\phi^A,\theta)}{\text{var}(\phi^A)} \cdot \phi^A$. Now, since s and ϕ^A are correlated normal random variables, we have that $\mathbb{E}\left[\phi^A|s\right] = \frac{v_\theta+1-\rho}{2(v_\theta+1)-2\rho}s$. Noting that $s \sim N\left(0, 2\left(v_\theta+1\right)-2\rho\right)$, we have that

$$\begin{aligned}\int_{s>0} \mathbb{E}\left[\phi^A|s\right] dF(s) &= \frac{v_\theta+1-\rho}{2\left(v_\theta+1\right)-2\rho} \int_{s>0} sF(s) \\ &= \frac{\sqrt{v_\theta+1-\rho}}{2\sqrt{\pi}}.\end{aligned}$$

[27] This is a setting found in many papers on political economy; see Myerson (2006) and those cited within.

Clearly, the expression is decreasing in the covariance between ϵ^A and ϵ^B. The incumbents have an incentive to adopt diverse policies. This justifies diverse policies as an equilibrium policy profile. Now, we need to show that uniform policies cannot be sustained in equilibrium. Under uniform policies, the voter's posterior expectation is $\mathbb{E}_{voter}(\theta|\varphi) = \frac{v_\theta}{v_\theta + \frac{1+v_\theta-\rho^2}{1+v_\theta}} \cdot s_d$ where $s_d = \phi^A - \left(\frac{\rho}{v_\theta+1}\right) \cdot \phi^B$. Denote $M \equiv \frac{v_\theta}{v_\theta + \frac{1+v_\theta-\rho^2}{1+v_\theta}}$; we can write the incumbents' expected payoff as follows:

$$\mathbb{E}(u|\mathsf{p}) = M \int_{s>0} \mathbb{E}[s_d|s]\, dF(s|\mathsf{p}).$$

Now, the standard results imply that $\mathbb{E}[s_d|s] = \frac{\text{cov}(s_d,s)}{\text{var}(s)} s$. Thus, $\mathbb{E}(u|\mathsf{p}) = M \cdot \frac{\text{cov}(s_d,s)}{\text{var}(s)} \cdot \int_{s>0} s dF(s|\mathsf{p})$. Since s is a normal random variable with a mean of 0, we have that the quantity $\int_{s>0} s dF(s|\mathsf{p})$ is increasing in the variance in s. It follows that the expected utility is greater under diverse policies than under uniform policies, and the incumbents would have an incentive to deviate from uniform policies. □

It is straightforward to see that, given that the equilibrium policy profiles are unchanged, our observations on policy learning are robust to the motivation to attain higher-office.[28] The same goes for voters' learning about politicians under centralization. However, learning about politicians under decentralization (i.e., U_d as defined in Section 3.4 of the paper) needs to be recomputed to account for the change in the nature of the election. In particular, voters are selecting between two district incumbents instead of between the incumbent and an unknown challenger.

Formally, $U_d \triangleq \mathbb{E}\left[\theta_2^d\right]$ is now defined as $\int_\varphi \max\left\{\mathbb{E}\left[\theta_1^B|\varphi\right], \mathbb{E}\left[\theta_1^B|\varphi\right]\right\} dF(\varphi)$. And recall that in the benchmark model, $U_d = \int_\varphi \max\left\{\mathbb{E}\left[\theta_1^d|\varphi\right], 0\right\} dF(\varphi)$. Consequently, under decentralization, voters have information on both candidates. Proposition OA.3 shows that this results in voters' greater learning about politicians under decentralization than they do under centralization. It is worth noting that our result concurs with an observation by Myerson (2006). He argues that the competition for higher office between local politicians improves the selection of national leaders.

Proposition OA.3. *Learning about politicians is greater under decentralization than under centralization.*

[28] The definition of policy learning as described in Section 3.3 of the paper depends only on the policy choices.

Proof. Following the computation in Section 3.4 of the paper, voters' learning about politicians under centralization is $\frac{v_\theta}{\sqrt{2\left(\frac{1+\rho}{2}+v_\theta\right)\pi}}$. For decentralization, voters' learning about politicians is as follows:

$$\mathbb{E}^d\left[\theta_2\right] = \int_{\phi^A,\phi^B} \max\left\{\mathbb{E}\left[\theta^A|\phi^A\right], \mathbb{E}\left[\theta^B|\phi^B\right]\right\} dF(\phi^A,\phi^B),$$

where the equality follows from the law of iterated expectations. The posterior expectations are a linear function of the signal: $\mathbb{E}\left[\theta^i|\phi^i\right] = \frac{v_\theta}{v_\theta+1}\phi^i$, $i \in \{A, B\}$. Thus, one can rewrite the expression above as follows:

$$\mathbb{E}^d\left[\theta_2\right] = \frac{v_\theta}{v_\theta+1}\int_{\phi^A,\phi^B} \max\left\{\phi^A,\phi^B\right\} dF(\phi^A,\phi^B).$$

According to Nadarajah and Kotz (2008), $\int_{\phi^A,\phi^B} \max\left\{\phi^A,\phi^B\right\} dF(\phi^A,\phi^B) = \sqrt{\frac{v_\theta+1}{\pi}}$. Thus, it follows that $\mathbb{E}^d\left[\theta_2\right] = \frac{v_\theta}{\sqrt{(v_\theta+1)\pi}}$. This quantity is clearly greater than the electoral accountability under centralization, which is as follows: $\frac{v_\theta}{\sqrt{2\left(\frac{1+\rho}{2}+v_\theta\right)\pi}}$. □

Asymmetric Information with Different Payoff Structures

Asymmetric Information with Constant In-Offices Payoffs

In Proposition 5 of the paper, we show that our results are robust to whether uncertainty about the politician's type is symmetric or asymmetric. This is driven by the payoff structure and policy non-observability. To aid our understanding, we show in this section and the next that our results were to change if the politician's payoff structure changes. Specifically, instead of letting the in-office payoff be increasing in the politician's reputation—e.g., $w(s) = \mathbb{E}[\theta|s]$—we consider two cases: one in which the in-office payoff is a constant, and the other in which the in-office payoff is type-dependent. Recall that s is the sufficient statistic that voters use to update their beliefs about θ.

Let's consider the first case here. Suppose that politicians have the following Bernoulli utility function:

$$u(s) = \begin{cases} w(s) = 1 & \text{if re-elected} \\ 0 & \text{if replaced.} \end{cases} \tag{7}$$

We call politicians with type $\theta > 0$ competent and those with $\theta < 0$ incompetent. Thus, we arrive at the following proposition.

Proposition OA.4. *Suppose that politicians have the constant in-office payoffs shown in formula (7). When politicians know their own type θ, competent politicians will behave as if they are risk averse—i.e., they will choose policy profiles that minimize the variance of s—and incompetent politicians will behave as if they are risk-loving—i.e., they will maximize the variance of s. In other words, our main results hold for incompetent politicians but they are the reverse for competent politicians.*

Proof. Politicians still get re-elected if and only if $s > 0$. Given this electoral rule, politicians maximize their following payoff:

$$\int_{s>0} 1 dF(s). \tag{8}$$

Take the case of centralization for example. We know from Lemma 1 that the sufficient statistic s in this case is the average of the district outputs $\frac{\phi^A+\phi^B}{2}$, which is $\theta + \frac{\epsilon^A+\epsilon^B}{2}$. Let $\frac{\epsilon^A+\epsilon^B}{2}$ be R. When the politician knows his type, θ, equation (8) becomes $\int_{R>-\theta} dF(\theta + R)$. We know $R \sim N(0, \sigma^2)$, hence $\theta + R \sim N(\theta, \sigma^2)$. Then, equation (8) further becomes $1 - F'(-\theta)$, where F' is the cdf for $\theta + R$. We know $F'(z) = \frac{1}{2}[1 + \text{erf}(\frac{z-\theta}{\sigma\sqrt{2}})]$ and $\text{erf}(w) = \frac{2}{\sqrt{\pi}} \int_0^w e^{-t^2} dt$. It is not hard to show that when $\theta > 0$, $F'(-\theta)$ is increasing in σ; hence, $1 - F'(-\theta)$ is decreasing in σ, so politicians with a type that is larger than 0 would like to minimize σ. Similarly, when $\theta < 0$, $F'(-\theta)$ is decreasing in σ, so $1 - F'(-\theta)$ is increasing in σ. Hence, politicians with a type that is smaller than 0 would like to maximize σ. The case for decentralization follows. □

The intuition for the above observation is as follows: if the in-office payoff is constant, then the incumbents' objective is solely to maximize the probability of re-election. When the incumbent knows that his competence is high, then a larger variance of the signals makes bad signals more likely to occur, thereby reducing the probability of re-election, which is higher when the variance is small. The opposite is true for incumbents whose competence is low.

Asymmetric Information with Type-Dependent In-Office Payoffs

Now, suppose that politicians have the following type-dependent payoffs:

$$u(s) = \begin{cases} \theta & \text{if re-elected} \\ 0 & \text{if replaced.} \end{cases} \tag{9}$$

s still stands for the sufficient statistic that voters use to make their inferences about the politician's type.

Proposition OA.5. *Suppose that politicians have the type-dependent in-office payoffs shown in formula (9). When the politicians know their own type, θ, then both the competent and incompetent politicians will be risk-averse—i.e., they will choose policy profiles that minimize risk.*

Proof. The voters' re-election rule is as before: politicians get re-elected if and only if $s > 0$. We take centralization as an example. Now, $s = \theta + \frac{\epsilon^A + \epsilon^B}{2}$ is the average of the district outputs. Let $R = \frac{\epsilon^A + \epsilon^B}{2}$. Given the electoral rule and the politicians' payoffs, these politicians now maximize $\int_{R>-\theta} \theta dF(\theta + R) = \theta(1 - F'(-\theta))$, where F' is as defined in the previous proof for Proposition OA.4. From the previous proof we know that $1 - F'(-\theta)$ is decreasing in the variance of R if $\theta > 0$ and increasing in the variance of R if $\theta < 0$. One can immediately see that $\theta(1 - F'(-\theta))$ is decreasing in the variance of R. Hence, all politicians will choose policy profiles that minimize risk. □

When the politician's in-office payoff is solely a function of their true type, their incentive is perfectly aligned with that of the voters: they want to maximize their probability of re-election when they are competent and they do not want to be in office when they are not competent. This will be the case when the signal s is very informative of θ, which is equivalent to minimizing the variance of s.