Online Appendix

"Market-Share Contracts, Exclusive Dealing, and the Integer Problem"

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A. Optimal Pricing under MS

We show that the optimal per-unit price under MS is equal to v. Suppose the incumbent offers $C = \{s, x, p\}$ to K buyers with $s < 1$ and all K buyers accept the contract in the PCPNE of the continuation game. We restrict to the scenario with $p \geq c$, as any price below its unit cost is allegable for predatory pricing and could be prohibited by antitrust laws. Consider two cases:

Case (1): The incumbent charges a price $p \leq v$. In this case, the signed buyers will purchase all of their demands from the incumbent when the entrant does not enter. Note that the profit from unsigned buyers is independent of the committed price p, whereas the profit from signed buyers depends on p. To see further the impact of price change on the incumbent's profit, let $x^*(s, p) = x_m(s, p) = \alpha_{m-1}(s)(v - c) - (1 - \alpha_m(s))(v - p) - \alpha_m(s)(v - p_a)$ and substitute into the profit from each signed buyer, we have

$$
\begin{aligned}
\Pi^S(K, s, p) &= s(p - c) + (1 - \alpha_K(s))(1 - s)(p - c) - x^*(s, p) \\
&= (1 - \alpha_{m-1}(s))(v - c) + (1 - s)(\alpha_m(s) - \alpha_K(s))(p - c)
\end{aligned}
$$

Differentiating $\Pi^S(K, s, p)$ with respect to p, we obtain

$$
\frac{\partial \Pi^S(K, s, p)}{\partial p} = (1 - s)(\alpha_m(s) - \alpha_K(s)).
$$

The above derivative is strictly positive for any $s < 1$ and $\alpha_m > \alpha_K$, thus, the profit from signed buyers is always increasing in p, and the optimal price under MS is equal to v. If $\alpha_m = \alpha_K$, then $\Pi^S(K, s, p) = (1 - \alpha_{m-1}(s))(v - c)$ and the incumbent's profit is independent of p.

Case (2): Suppose the incumbent charges a price $p > v$. Then each signed buyer will only purchase s units regardless whether the entrant enters or not, as buying more units incurs more loss to the signed buyer. When the entrant enters, each signed buyer will purchase the residual demand of $1 - s$ units from the entrant at a per unit price c, and obtains a surplus $v - p_a$. When the entrant stays out, each signed buyer will only purchase s units from the incumbent and obtains a (negative) surplus $s(v - p)$. Thus, the expected surplus of a buyer who agrees to

the incumbent's offer when $n - 1$ other buyers are also agreeing to the incumbent's offer is

$$
\begin{aligned}
U_A(n) &= (1 - \alpha_n) s (v - p) + \alpha_n (v - p_a) + x \\
&= s(v - p) + \alpha_n (1 - s)(v - c) + x.
\end{aligned}
$$

The expected surplus of this same buyer if it rejects the incumbent's offer is

$$
U_R(n - 1) = \alpha_{n-1}(v - c).
$$

Thus, the compensation that just makes this buyer indifferent between accepting or rejecting is

$$
\tilde{x}_n(s, p) = \alpha_{n-1}(v - c) - s(v - p) - \alpha_n(1 - s)(v - c).
$$

Let $\tilde{x}^*(s, p) = \tilde{x}_m(s, p)$ be the maximum payment. Then, the incumbent's profit from each signed buyer is

$$
\begin{aligned}
\tilde{\Pi}^S(K, s, p) &= s(p - c) - \tilde{x}^*(s, p) \\
&= s(p - c) - \alpha_{m-1}(v - c) + s(v - p) + \alpha_m (1 - s)(v - c) \\
&= (s + \alpha_m (1 - s) - \alpha_{m-1})(v - c).
\end{aligned}
$$

It follows that the incumbent's profit is independent of p for any $p > v$. Since $s + \alpha_m (1 - s) - \alpha_{m-1} < 1 - \alpha_{m-1}$, we have

$$
\tilde{\Pi}^S(K, s, p) < (1 - \alpha_{m-1})(v - c) + (1 - s)(\alpha_m - \alpha_K)(v - c) = \Pi^S(K, s, v).
$$

Therefore, charging $p > v$ is strictly dominated by setting $p = v$. **Q.E.D.**

B. Elastic demand

Consider now that each buyer faces a downward sloping demand $q(p)$. Let $\pi(p) = (p - c)q(p)$ denote the incumbent's profit gross of any fixed payments, and let p_m denote its argmax.

Proposition A.1 *When the incumbent offers contract $C = \{1, x, p\}$ to a subset of buyers and each buyer faces a downward-sloping demand curve, it is optimal to charge $p = c$. Moreover, the optimal number of signed buyers, K^{ED}, is independent of p, and is not an integer in generic.*

Proof: Suppose the incumbent offers ED to K buyers. If a buyer accepts the contract, it will have to purchase all of its demand from the incumbent at the price p, regardless whether the entrant

enters or not. The signed buyer then gains a surplus of $S(p) + x$, where $S(p) \equiv \int_p^{+\infty} q(v)dv$ denotes the buyer's surplus at price p. If, instead, this buyer rejects the offer jointly with other $K - 1$ buyers, the entrant will enter for sure and it gains a surplus of $S(c)$. Thus, all K buyers will accept the ED contract in PCPNE if and only if the incumbent's inducement x satisfies

$$x \geq \hat{x}^*(p) = S(c) - S(p).$$

When all K buyers accept the incumbent's contract, the entrant's profit is reduced to $\Pi_E(K) = (N - K)\delta q(c)$. Thus, the probability of entry is reduced to $G((N - K)\delta q(c))$. Note that the entrant's profit is not affected by p, because the entrant can only contest for the unsigned buyers, and competition for the unsigned buyers drives the spot-market price down to c.

The incumbent earns $\pi(p)$ from each signed buyer, but it has to offer each a payment of $\hat{x}^*(p) = S(c) - S(p)$. The incumbent's net profit from each signed buyer is thus equal to

$$
\begin{aligned}
\hat{\Pi}^S(K, 1, p) &= \pi(p) - \hat{x}^*(p) \\
&= \pi(p) + S(p) - S(c),
\end{aligned}
$$

which is weakly negative for all $p \geq c$, and equal to zero if and only if $p = c$.

On the other hand, the incumbent can exploit from the unsigned buyers due to the inter-group externality that the signed buyers impose on the unsigned buyers. The incumbent charges the monopoly price p_m to each unsigned buyer when the entrant does not enter, and its expected payoff from each unsigned buyer is given by

$$\hat{\Pi}^U(K, 1) = (1 - G((N - K)\delta q(c)))\pi(p_m).$$

Therefore, the incumbent's problem in Period 1 under ED is to choose K and $p \in [c, v]$ to maximize the expected payoff

$$
\begin{aligned}
\hat{\Pi}(K, 1, p) &= K\hat{\Pi}^S(K, 1, p) + (N - K)\hat{\Pi}^U(K, 1) \\
&= K[\pi(p) + S(p) - S(c)] + (N - K)(1 - G((N - K)\delta q(c)))\pi(p_m).
\end{aligned}
$$

Since the incumbent's profit from the signed buyers is maximized at $p = c$ (which is equal to zero) while the incumbent's profit from the unsigned buyers is independent of p, it follows that the incumbent's optimal price must be $p = c$ under ED, and the incumbent's profit is given by

$$\hat{\Pi}(K, 1, c) = (N - K)(1 - G((N - K)\delta q(c)))\pi(p_m).$$

Maximizing this with respect to K, it follows that the optimal number of signed buyers, $\hat{K}^{ED}$, is independent of p. Moreover, when we normalize demand to make $q(c) = 1$, we can easily see that the solution $\hat{K}^{ED}$ is exactly the same as K^{ED} was in the case of inelastic demand.[40]

Q.E.D.

C. Increasing N

We have shown that increasing N does not affect the incumbent's maximized profit under ED. This implies that if $\tilde{K}$ denotes the optimal number of signed buyers before the increase in N, then $\tilde{K} + \Delta N$ will be the optimal number of signed buyers after the increase in N, and thus $N - \tilde{K}$ and the incumbent's constrained maximized profit under ED will also be unaffected. The intuition for this is simple. Under ED, the incumbent only earns profit from the unsigned buyers. After K optimally adjusts to the increase in N, the number of unsigned buyers does not change, nor is there any change in the probability of entry given that the number of uncommitted units is also unchanged. It follows that the incumbent will not benefit (or lose) from an increase in N.

Proposition A.2 *For a given δ, $v - c$, and distribution of entry costs, the incumbent's profit under ED depends only on the number of uncommitted units. Increases in N therefore have no effect on the incumbent's maximized profit under ED. As N increases, the incumbent simply adjusts the number of offers it makes in order to leave the number of uncommitted units unchanged. Although this increases the number of signed buyers, and thus the percentage of the market that is foreclosed to the entrant, it does not change the probability that the entrant will be deterred.*

Several implications follow from Proposition A.2. First, and most importantly, Proposition A.2 implies that the constraint that arises from K having to be an integer does *not* decrease in importance as the number of buyers increases, contrary to what one might have thought. To see this, suppose N increases from $N = 4$ to $N = 10$. If the flawed reasoning were correct (i.e., if the foreclosure percentage were fixed at 60%), the incumbent would offer ED to exactly six buyers and the integer constraint would no longer be binding. This is, however, clearly not optimal. Proposition A.2 implies that the incumbent should instead optimally foreclose 8.4 units in order to keep the number of uncommitted units at 1.6 units. The gap between the desired foreclosure level and the obtainable foreclosure level is therefore exactly the same as before. That is, it is equal to 0.4 units if ED is offered to $K = 8$ buyers, or 0.6 units if ED is offered to $K = 9$ buyers.

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[40]Note that the constant term $\pi(p_m)$ does not affect $\hat{K}^{ED}$.

Second, the fact that the gap between the incumbent's desired foreclosure level and its obtainable foreclosure level under ED is independent of N implies that there is just as much of a need to fine tune the level of foreclosure after an increase in N as there was before the increase. This implies that from the benefit side of foreclosure, increases in N favor neither ED nor MS.

Third, from a policy perspective, Proposition A.2 implies that judging the relative harm from exclusion in any given case by focusing on the percentage of the market that is foreclosed, as courts are often inclined to do, is at best misleading. In our stylized examples, the incumbent would ideally like to foreclose 60% of market when $N = 4$, leaving the entrant with only 1.6 uncommitted units, whereas, when $N = 8$, the incumbent would ideally like to foreclose 80% of market (because then $K^{ED} = 6.4$). Although the latter percentage is significantly higher than the former percentage, the number of uncommitted units is the same in the two cases, and therefore it follows that the entrant will neither be better or worse off in one case or the other.

Cost savings and the dilution effect

Our finding that increases in N need not affect the number of uncommitted units under ED also applies to MS. Under MS, the incumbent controls both K and s, and it can always adjust them in such a way as to keep the benefit from foreclosure the same. To see this, note that if $N - \hat{K}\hat{s}$ is the initial number of uncommitted units under MS before the increase in N, and N increases by ΔN, then the incumbent can always choose $K = \hat{K} + \Delta N$ and $s \equiv s^* = \frac{\hat{K}\hat{s} + \Delta N}{\hat{K} + \Delta N} > \hat{s}$ after the increase to achieve the same number of uncommitted units as before.[41] Since the benefit from foreclosure under MS, as it was under ED, depends only on the number of uncommitted units, it follows that the benefit from foreclosure need not change under MS when N increases.

Under ED, this finding, along with showing that the incumbent cannot do better, was sufficient to establish that the incumbent's maximized profit under ED is independent of N. It is not the end of the story under MS, however, because an increase in N impacts MS differently from ED. In addition to the benefit from foreclosure, there is also the cost of foreclosure to consider. Under ED, this cost is always zero. Under MS, however, this cost depends on both K and s.

Continuing with our example, where $K = \hat{K}$, $s = \hat{s}$, and f is distributed such that $x^*(\hat{s}, v) = x_{\Omega+1}(\hat{s}, v)$, the incumbent's cost of foreclosure under MS before the increase in N is

$$\hat{K}\left(x^*(\hat{s}, v) - s(v - c)\right)$$
$$= \hat{K}(1 - \hat{s})(1 - \alpha_{\Omega+1}(\hat{s}))(v - c). \tag{A.1}$$

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[41]Note that $N + \Delta N - \left(\hat{K} + \Delta N\right)s^* = N + \Delta N - \left(\hat{K}\hat{s} + \Delta N\right) = N - \hat{K}\hat{s}$.

After the increase in N, and after K and s adjust to keep the benefit from foreclosure the same (i.e., $K = \hat{K} + \Delta N$ and $s = s^*$), the incumbent's cost of foreclosure under MS is

$$(\hat{K} + \Delta N)(1 - s^*)(1 - \hat{\alpha}_{\Omega+1+\Delta N}(s^*))(v - c),$$

where $\hat{\alpha}_n(s) \equiv G((N + \Delta N - ns)\delta)$ denotes the probability of entry when n out of $N + \Delta N$ buyers sign the incumbent's contract. Substituting in for s^*, and rearranging terms, yields

$$\hat{K}(1 - \hat{s})(1 - \hat{\alpha}_{\Omega+1+\Delta N}(s^*))(v - c). \tag{A.2}$$

Comparing the cost of foreclosure in (A.2) with the cost of foreclosure in (A.1), we can see that the cost of foreclosure in (A.2) will be lower if and only if the probability of entry when the incumbent signs up the first effective buyer is higher after the increase in N than it was before. That is, the incumbent's cost of foreclosure will be lower in (A.2) than in (A.1) if and only if

$$\hat{\alpha}_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s}). \tag{A.3}$$

Fortunately, this relationship turns out to be relatively easy to establish because the same adjustments in K and s that keep the benefit from foreclosure the same imply that the actual probability of entry before and after the increase in N will be the same, and thus we know that

$$\hat{\alpha}_{\hat{K}+\Delta N}(s^*) = \alpha_{\hat{K}}(\hat{s}). \tag{A.4}$$

Going from the equality in (A.4) to establishing that the inequality in (A.3) holds then follows straightforwardly once it is recognized that after the increase in N, the difference between the actual number of uncommitted units available to the entrant and the number of uncommitted units that are available to the entrant after the first effective buyer is signed is greater than the corresponding difference before the increase in N. After the increase in N, the difference is $s^*(\hat{K} - (\Omega + 1))$ units, whereas before the increase, the difference is only $\hat{s}(\hat{K} - (\Omega + 1))$ units.

The intuition is that the incumbent must sign at least $\Omega + 1$ buyers before the increase in N if it is to have any effect on lowering the probability of entry, whereas after the increase in N, it must sign this many plus an additional ΔN more buyers if it is to have any effect. Each signing buyer's contribution to the initial reduction in the probability of entry is thereby effectively diluted when the number of buyers increases. We call this the *dilution effect*, and it implies that signing buyers do not have to be compensated as much for their contribution toward exclusion.

We can illustrate this effect with the help of our example in which $N = 4$ and $K^{ED} = 2.4$. Under MS, the incumbent can realize the optimal foreclosure level of 2.4 units by offering MS to

three buyers with $\hat{s} = 0.8$. Signing the first buyer in this case reduces the number of uncommitted units to $4 - 0.8 = 3.2$ units. Suppose now that $N = 7$. Here, the incumbent can realize the optimal foreclosure level of 5.4 units by offering MS to six buyers with $s^* = 0.9$ (so that the number of uncommitted units remains at 1.6). Signing the fourth buyer reduces the number of uncommitted units to $7 - 4 \times 0.9 = 3.4$, which is greater than it was before the increase in N.

We have just shown that while an increase in the number of buyers need not affect the incumbent's benefit from foreclosure, it would be expected to reduce its cost of foreclosure. It follows that we would expect the incumbent to strictly gain from an increase in N under MS.

Proposition A.3 *Because of the dilution effect, the incumbent need not compensate each signing buyer as much for its contribution toward exclusion after an increase in N as it did before the increase in N. The incumbent's maximized profit under MS is thus strictly increasing in N.*

Proof: We need to show that $\alpha_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s})$, and thus the cost of foreclosure is decreasing when N increases. The rest of the proposition has already been shown. Note that

$$
\begin{aligned}
\alpha_{\Omega+1+\Delta N}(s^*) &= G\left((N + \Delta N - (\Omega + 1 + \Delta N)\, s^*)\,\delta\right), \\
\alpha_{\Omega+1}(\hat{s}) &= G\left((N - (\Omega + 1)\,\hat{s})\,\delta\right),
\end{aligned}
$$

then $\alpha_{\Omega+1+\Delta N}(s^*) > \alpha_{\Omega+1}(\hat{s})$ if and only if $N + \Delta N - (\Omega + 1 + \Delta N)\, s^* > N - (\Omega + 1)\,\hat{s}$. To see this, note that the incumbent chooses s^* such that the optimal amount of uncommitted purchases is exactly the same as before N increases, that is,

$$
N + \Delta N - \left(\hat{K} + \Delta N\right) s^* = N - \hat{K}\hat{s}. \tag{A.5}
$$

When N increases by ΔN, the incumbent must sign up $(\Omega + 1 + \Delta N)\, s^*$ units to reduce the probability of entry effectively, and sign up further $\left(\hat{K} - (\Omega + 1)\right) s^*$ units to reduce the uncommitted purchases to the optimal level. Thus,

$$
\begin{aligned}
&N + \Delta N - (\Omega + 1 + \Delta N)\, s^* \\
=\ & N + \Delta N - \left(\hat{K} + \Delta N\right) s^* + \left(\hat{K} - (\Omega + 1)\right) s^* \\
=\ & N - \hat{K}\hat{s} + \left(\hat{K} - (\Omega + 1)\right) s^* \\
>\ & N - \hat{K}\hat{s} + \left(\hat{K} - (\Omega + 1)\right) \hat{s} \\
=\ & N - (\Omega + 1)\,\hat{s},
\end{aligned}
$$

where we have used $(A.5)$ to get the third line and the inequality comes from the fact that $s^* > \hat{s}$.

Q.E.D.

Proposition A.3 in conjunction with our earlier finding that the incumbent's maximized profit under ED is independent of N, implies that an increase in N expands the number of settings in which the incumbent will choose MS over ED. This is the exact opposite of what one might have expected, and it implies that an increase in N benefits MS relative to ED in the sense that (i) if initial conditions are such that MS is more profitable than ED, then MS will continue to be more profitable when the number of buyers increases; and (ii) if initial conditions are such that ED is more profitable than MS, then for a given increase in the number of buyers, the gap will narrow, and it is possible that MS could even overtake ED ex-post and become more profitable.

As we have seen, one way to think about why increases in N favor MS over ED is that, for a given benefit from foreclosure, the cost of foreclosure under ED is independent of N, whereas the cost of foreclosure under MS is decreasing in N. However, another way to think about the relative effects of an increase in N is to note that while an increase in N leads to a corresponding increase in the number of signed buyers under both ED and MS, this increase does not help the incumbent under ED because full compensation must be offered to all signed buyers and therefore the incumbent cannot profit from them. But the increase in the number of signed buyers does help the incumbent under MS because of the intra-group externality that signed buyers impose on each other. In fact, one can show that this externality only gets stronger as N increases, which is what generates the cost savings and thus allows increases in N to favor MS.

Numerical Example

We now construct an example in which ED dominates MS when there are only three buyers, but in which MS dominates ED when the number of buyers increases to five or more.

Consider the same set-up as in Example 2, but suppose now that $N = 5$. In this case, the incumbent earns its maximized expected profit of 900 under ED by increasing the number of signed buyers from two to four. Under MS, the incumbent can achieve its desired foreclosure level of 3.5 units by offering $s^* = .875$ to four buyers. This reduces the actual probability of entry to $\hat{\alpha}_4\left(s^*\right) = .35$, but increases the probability of entry when the $\Omega + 1$'st buyer signs to[42]

$$\hat{\alpha}_3\left(s^*\right) = G\left(\left(5 - 3 \times .875\right) 100\right) = G\left(237.5\right) = 0.9.$$

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[42]In contrast, when $N = 3$, this probability was $\alpha_1(.75) = .8$. It is easy to check that $x_3\left(s^*\right) > x_4\left(s^*\right)$.

The cost of foreclosure when N increases to $N = 5$ is thus reduced from 100 when $N = 3$ to

$$4 \left(1 - s^*\right) \left(1 - \hat{\alpha}_3 \left(s^*\right)\right) \left(v - c\right) = 500 \times 0.1 = 50.$$

Thus, the profit under MS is now equal to $975 - 50 = 925$, which is higher than that under ED.

D. Mixed ED with MS

The following example shows that offering ED to the first $\underline{K}^{ED}$ buyers and MS only to the last buyer can be worse than offering MS to all $\overline{K}^{ED}$ buyers. The example has the same set-up as in Example 1 in the main appendix, except that there are five buyers instead of three buyers.

Assume that $v - c = 1000$, $\delta = 100$, and $N = 5$. As in Example 1, suppose that f can take on one of five values, with the listed probabilities:

$$
\begin{array}{cccccc}
f : & 50, & 100, & 150, & 225, & 275 \\
g : & 0.1, & 0.1, & 0.6, & 0.1, & 0.1.
\end{array}
$$

In the absence of ED and MS, the entrant can earn a flow profit of 500 if it enters. Since this exceeds the maximum value of f, we would expect the entrant to enter with probability one in the absence of any exclusionary contracts. On the other hand, if, for instance, the incumbent can foreclose 3.5 units, then only 1.5 units will be available to the entrant, in which case the entrant's profit will be reduced to 150 and the probability of entry will be reduced to $\Pr\{f < 150\} = 0.2$.

If the incumbent offers ED only, it will be optimal give it to four buyers, in which case the incumbent can reduce the probability of entry to $\alpha_4 \left(1\right) = G \left(100\right) = 0.1$, and the incumbent earns an expected profit of $(5 - 4)(1 - 0.1)1000 = 900$.

Notice that the incumbent could have done even better if it could have signed up 3.5 buyers because then the probability of entry would have been reduced 0.2, and its expected profit would have been $(5 - 3.5)(1 - 0.2)1000 = 1200$. This maximum benefit from foreclosure can be achieved with MS, by offering $s = 0.875$ to four buyers. In order to induce all four buyers to accept MS with $s = 0.875$, however, the incumbent will have to give each buyer an inducement (here $\Omega = 2$) of $x^* \left(s\right) = x_{\Omega+1} \left(s\right) = \left(\alpha_2 - \alpha_3 \left(s\right) \left(1 - s\right)\right) \left(v - c\right)$. The total cost of foreclosure is then given by

$$C = 4 \left[x^* \left(s\right) - s \left(v - c\right)\right] = 4 \left[\left(\alpha_2 - \alpha_3 \left(s\right) \left(1 - s\right)\right) \left(v - c\right) - s \left(v - c\right)\right].$$

Note that signing up only two buyers does not reduce the probability of entry, that is, $\alpha_2 = 1$. On the other hand, signing up three buyers reduces the probability of entry effectively, with

9

$\alpha_3(s) = G((5 - 3 \times .875)\delta) = G(237.5) = 0.9$. Thus, the cost of foreclosure is given by

$$C = 4[1 - 0.9(1 - 0.875) - 0.875](v - c) = 0.05(v - c) = 50,$$

and the resulting profit is equal to $1200 - 50 = 1150$.

Suppose now the incumbent offers ED to three buyers and MS to the fourth buyer, with $s = 0.5$. The incumbent fully compensates the first three buyer with inducement $x^*(1) = v - c$, and the cost of foreclosure from these three buyers is zero. On the other hand, to sign up the fourth buyer, the incumbent will have to offer

$$x = x_4(s) = (\alpha_3 - \alpha_4(1 - s))(v - c).$$

Note that signing up the first three buyers with ED reduces the probability of entry to $\alpha_3 = G(200) = 0.8$, while signing up the fourth buyer further reduces the probability of entry to $\alpha_4 = G((5 - 3.5)\delta) = G(150) = 0.2$. Thus, the cost of signing up the fourth buyer is equal to

$$
\begin{aligned}
C &= x_4(s) - s(v - c) = (\alpha_3 - \alpha_4(1 - s) - s)(v - c) \\
&= (0.8 - 0.2 \times 0.5 - 0.5)(v - c) = 0.2(v - c) = 200,
\end{aligned}
$$

which is much higher than the cost of signing up four buyers with the uniform MS contract. As a result, the incumbent's profit, which is equal to $1200 - 200 = 1000$, is much lower. **Q.E.D.**