Online Appendix: Trust Building in Credence Goods Markets

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In this online appendix, we study a market with $n \geq 2$ experts and investigate how competition affects the monitoring technology for honesty and consumer welfare. We make minimal modifications to the monopoly model and retain its key elements, except for adding the following new features. First, consumers pay a small search cost $k > 0$ per visit, including the first visit. The assumption of a positive search cost captures the reality that search in expert markets often involves delay, which is costly. Second, we allow entry and exit to maintain a stable pool of active experts in each period. When an expert exits the market, he is replaced by a new expert. Allowing entry and exit is realistic and makes our model tractable. Third, following the existing literature,[1] we assume that an expert cannot identify whether a consumer has visited another expert in the past.

Events in period t unfold in the following sequence: Experts first simultaneously post treatment prices, which become public information. A consumer arrives in the market and consults an expert. The expert learns the consumer's problem and makes a recommendation. If treatment is prescribed, the consumer either follows the expert's recommendation or searches for another opinion. At the end of the period, experts' prices, the consumer's utility, and the recommendations made by all the experts she has visited become public information.

We focus on symmetric and stationary equilibria that maximize consumer welfare. There are two types of equilibria: (i) equilibria in which consumers search for second opinions, and (ii) equilibria in which consumers do not search for second opinions. We characterize each type of equilibria and compare consumer welfare.

We begin with equilibria in which consumers search for second opinions. We refer to these equilibria as "*search equilibria*" hereafter. Let

$$\beta^*_m(p;k) \equiv \frac{(1-\alpha)(p-l_m-k) - \sqrt{(1-\alpha)^2(p-l_m-k)^2 - 4k\alpha(1-\alpha)(p-l_m)}}{2(1-\alpha)(p-l_m)} \tag{1}$$

$$\underline{\delta}(p;k,n) \equiv \frac{n(p-c+\varepsilon)\beta^*_m(p;k)}{n(p-c+\varepsilon)\beta^*_m(p;k) + \alpha(1-\beta^*_m(p;k))(p-c)}, \tag{2}$$

$$\gamma^*(p;k,n) \equiv 1 - \frac{n(1-\delta)(p-c+\varepsilon)}{n(1-\delta)(p-c+\varepsilon)(1-\beta^*_m(p;k)) + \delta\alpha(1-\beta^*_m(p;k))(p-c)}. \tag{3}$$

Proposition 1 *There exists a continuum of search equilibria indexed by $p \in (c, c+k]$ if $\delta \geq \underline{\delta}(p;k,n)$ and*

[1] See Pesendorfer and Wolinsky (2003) and Wolinsky (1993).

k is low. In each period, experts post the same price p. They recommend the treatment for the serious problem with probability one and for the minor problem with probability $\beta_m^*(p;k) \in (0,1)$. A consumer randomly visits an expert when arriving at the marketplace. If recommended the treatment on her first visit, the consumer accepts it with probability $\gamma^*(p;k,\delta,n) \in (0,1)$ and searches for a second opinion with the complementary probability. If recommended the treatment again on her second visit, the consumer accepts the recommendation with probability one. Whenever the consumer is recommended no treatment, she exits the market. An expert loses all future consumers and exits the market if (i) he recommends the treatment to the consumer when her problem is minor and the consumer gets an honest second opinion, or (ii) he deviates in price.

To understand a search equilibrium indexed by p, we first analyze a consumer's incentives to solicit second opinions. When all of the experts lie with positive probabilities, the consumer may receive different recommendations from different experts and hence has incentives to search for honest second opinions. If the consumer accepts the treatment on her first visit, her problem will be fixed, and she receives a payoff $-p$. Alternatively, the consumer can pay the search cost to solicit a second opinion. If the second opinion recommends no treatment, the consumer infers that she must have the minor problem and hence will choose not to repair it. Therefore, the consumer has a net benefit of $p - l_m$ from search when the second opinion contradicts the first opinion. For the consumer to randomize between accepting the first treatment recommendation and soliciting a second opinion, it requires that the consumer's expected net benefit from search equals her search cost k, which gives

$$\Pr(\emptyset|p)(p - l_m) = k, \tag{4}$$

where $\Pr(\emptyset|p)$ is the probability that the second opinion recommends no treatment conditional on the first opinion recommending the treatment. When (4) holds, the consumer strictly prefers accepting the second treatment recommendation to soliciting a third opinion. This is because the likelihood that the third opinion recommends no treatment conditional on two treatment recommendations is smaller than $\Pr(\emptyset|p)$. Hence the consumer's expected net benefit from search is strictly less than the search cost.

For the consumer to be willing to enter the market, the following participation constraint must hold

$$p + k \leq \Pr(l_s|p)l_s + \Pr(l_m|p)l_m, \tag{5}$$

where $\Pr(l_i|p)$, $i = m, s$, is the probability that the consumer's problem is i conditional on the first treatment recommendation. If the second opinion also recommends the treatment, the consumer updates her belief of having the serious problem upward and expects a greater loss from the problem. Hence, the participation constraint (5) implies that the consumer strictly prefers to accept the treatment on her second visit.

Now, we turn to analyze experts' recommendation strategies. An expert is indifferent between whether or not to recommend the treatment to the minor problem when the following condition holds:

$$\overbrace{\delta V \underbrace{(1-\Pr(\mathrm{e}|l_m))}_{\substack{\text{prob that the}\\ \text{consumer is}\\ \text{on her first visits}}} \underbrace{(1-\gamma)(1-\beta_m)}_{\substack{\text{prob that the}\\ \text{consumer receives}\\ \text{an honest } 2^{nd} \text{ opinion}}}}^{\text{future loss}} = \overbrace{(p-c+\varepsilon)\underbrace{[\Pr(\mathrm{e}|l_m) + (1-\Pr(\mathrm{e}|l_m))\gamma]}_{\substack{\text{prob of acceptance}\\ \text{of the treatment}}}}^{\text{current gain}}, \tag{6}$$

where $\Pr(\mathrm{e}|l_m) \equiv \dfrac{\beta_m(1-\gamma)}{1+\beta_m(1-\gamma)}$ is the probability that the consumer is on her second visit conditional on her problem being minor. V is the expert's continuation profit when he is active in the market. Under the assumption that an exiting expert is replaced with a new entrant, the number of active experts is constant, and hence V is also constant across periods. When an expert recommends the treatment for the minor problem, the treatment is accepted if the consumer is on her second visit or on her first visit and chooses to accept the treatment, which happens with probability γ. The expert's profit margin from fixing the minor problem is $p - c + \varepsilon$. Therefore, his expected current gain from recommending the treatment for the minor problem is on the right-hand side of (6). On the other hand, the expert risks losing all future business if he is caught lying. This happens when the consumer is on her first visit and decides to solicit a second opinion, and the second opinion happens to be honest. The expert's discounted expected future loss from lying is therefore given by the left-hand side of (6). When (6) holds, it is the expert's best response to randomize between recommending and not recommending treatment for the minor problem.

Finally, we construct consumers' off-the-equilibrium-path beliefs to prevent a price deviation. There are two types of consumers: inexperienced consumers who never visit any experts and experienced consumers who have visited some experts. First, we consider inexperienced consumers' responses to the price deviation.

In our equilibrium specification, a price deviation results in no business in the future. As a result, a deviating expert will behave in the same way as a short-run player, always recommending the treatment as long as the price is above c. For the same argument as in the static game in the main text and anticipating not to learn any information about her condition, the inexperienced consumer will not visit the expert. The expert can potentially offer a low price p' to attract experienced consumers who search for second opinions, but $p \leq c + k$ prevents such a price deviation. To see this, because an inexperienced consumer expects the deviating expert to always recommend the treatment at p', she prefers seeking treatment from the deviant to accepting the first treatment recommendation if and only if $p' < p - k$. However, since $p \leq c + k$, $p' < c$. Because the deviating expert only has short-term incentives, he will not post a price less than c. Hence, for $p \in (c, c + k]$, there does not exist a price deviation that can profitably attract experienced or inexperienced consumers.

For a given price $p \in (c, c + k]$, there exists a search equilibrium which satisfies conditions (4), (5) and (6) when k is sufficiently low and $\delta \geq \underline{\delta}(p; k, n)$. To see that the discount factor must be high enough to sustain the search equilibrium, suppose δ is zero. Then, the expert does not bear any future loss from lying. Nevertheless, he has a positive current gain from lying because there is a positive probability that the consumer is on her second visit and will accept the treatment recommendation with probability one. As a result, the expert will lie with probability one when he does not care much about future business.

There are two notable properties of the search equilibrium. First, the search equilibrium involves small but pervasive cheating among experts. This is because some cheating is necessary to induce consumers to search. In other words, full honesty is incompatible with the search for second opinions: If experts all make honest recommendations, consumers will not pay the search cost to sample more opinions. Second, consumer search can eliminate undertreatment, but overtreatment still arises in the case when both the first and second opinions recommend the treatment for the minor problem.

In a one-shot game, Wolinsky (1993) and Dulleck and Kerschbamer (2006) identify a class of equilibria in which experts randomize between whether or not to lie, and consumers randomize between whether or not to seek second opinions. Despite the similar equilibrium behavior in their models and ours, the equilibria's driving forces are very different.[2] Because we want to focus on to what extend reputation concern can affect expert conduct and inefficiency arising from over- and undertreatment, we adopt the assumption that there is only one treatment which is efficient to use for the serious problem but not for the minor problem. Under this assumption, the search equilibrium collapses for short-lived experts.

The following lemma studies the impact of the search cost on the search equilibrium. When k decreases, the upper bound of the equilibrium price also decreases. Hence, we focus on the equilibrium in which the equilibrium price is the upper bound $p = c + k$.

Lemma 1 *Consider the search equilibrium indexed by $p = c + k$.*

(i) $\beta^*_m(p + k, k)$ decreases in k.

(ii) As $k \to 0$, $\beta^*_m(p + k, k) \to 0$, $\underline{\delta} \to \underline{\delta}^{cs} \equiv \dfrac{n\alpha\varepsilon}{n\alpha\varepsilon + \alpha(1-\alpha)(c - l_m)}$, and consumer's utility converges to the surplus in the first best.

Lemma 1 shows that experts become more honest when the search cost is reduced. Note that as k diminishes, a consumer's gain from soliciting a second opinion must be reduced proportionally for condition (4) to hold. As a result, the probability that the second opinion contradicts the first opinion must be reduced, which requires the cheating probability β^*_m to decrease. When the search cost converges to zero, the probability that the consumer draws conflicting recommendations must also converge to zero, which implies that the cheating probability converges to zero.

The cutoff discount factor $\underline{\delta}(p; k, n)$ is affected by two opposing forces when k decreases. On the one hand, an expert's continuation profit V converges to zero as the price converges to c, so he has less to lose when losing future business. This increases $\underline{\delta}(p; k, n)$. On the other hand, $\beta^*_m \to 0$ suggests that cheating

[2] In Wolinsky (1993) and Dulleck and Kerschbamer (2006), short-lived experts have two treatments. It is efficient to use the expensive treatment for the serious problem and the inexpensive treatment for the minor problem. When deciding whether to recommend the expensive treatment for the minor problem, an expert trades off a high profit margin against a low acceptance rate because the expensive treatment is sometimes rejected and the inexpensive treatment is always accepted. By contrast, in our setting, experts have only one treatment and it is a dominant strategy for them to recommend the treatment for the minor problem in a static setting. Hence, the search equilibrium is not sustainable for short-lived experts. A long-lived expert trades off a current gain from recommending unnecessary services against a future loss of business. Because an expert's degree of patience and the number of experts in each period affect the value of the expert's future business, they also affect consumers' equilibrium search frequency $\gamma^*(p; k, \delta, n)$.

will be caught almost surely because the second opinion is most likely to be honest. This reduces $\underline{\delta}(p; k, n)$. In the limit, the minimum discount factor necessary to support the search equilibrium converges to $\underline{\delta}^{cs}$.

A consumer's utility in a search equilibrium indexed by p is

$$\begin{aligned} u^s(p) &= \alpha\underbrace{(-p-(1-\gamma^*)k)}_{u_s}+ \\ &\quad (1-\alpha)\underbrace{\{\beta_m^*(\gamma^*+(1-\gamma^*)\beta_m^*)(-p))+((1-\beta_m^*)+\beta_m^*(1-\gamma^*)(1-\beta_m^*))(-l_m)-\beta_m^*(1-\gamma^*)k\}}_{u_m}. \end{aligned}$$

In the above expression, the item labeled as u_s is the consumer's expected utility conditional on the problem being serious. According to experts and the consumer's equilibrium strategies, the serious problem will be resolved with probability one, yielding the consumer a payoff $-p$. In addition, the consumer pays the search cost if she rejects the first treatment offer, which happens with probability $1-\gamma^*$. The item labeled as u_m is the consumer's expected utility conditional on the problem being minor. In this case, the consumer's payoff is $-p$ if her problem is resolved and is $-l_m$ if her problem is not treated. The former event happens with probability $\beta_m^*(\gamma^*+(1-\gamma^*)\beta_m^*)$ and the latter event happens with probability $(1-\beta_m^*)+\beta_m^*(1-\gamma^*)(1-\beta_m^*)$. In addition, the consumer pays the search cost k with probability $\beta_m^*(1-\gamma^*)$.

Consider the case in which $p = c + k$. If $k \to 0$, $p \to c$, $\beta_m^* \to 0$ and $u^s(p) \to -\alpha c - (1-\alpha)l_m$. Note that $-\alpha c - (1-\alpha)l_m$ is the surplus in the first best, which prescribes that the serious problem is treated and the minor problem is left untreated. When the search cost decreases, experts become more honest, and the price is closer to the cost c. In the limiting case, the consumer's utility converges to the first best. Our analysis highlights the importance of policies aiming at reducing consumers' search costs.

Next, We analyze the consumer-optimal symmetric and stationary equilibria in which consumers do not search. If consumers do not search in equilibrium, each expert will behave like a local monopolist, and the mechanism for monitoring experts' honesty is the same as in the monopoly market. Our main model has shown that monitoring of experts' honesty involves consumer rejection. Because a consumer has the same expected payoff from accepting and rejecting the treatment, she does not gain from visiting an expert whether or not the expert recommends the treatment. As a result, the consumer's surplus from trade is zero. It follows that if an equilibrium not involving search gives the consumer a positive surplus from trade, consumers must accept the treatment with probability one. Hence, the equilibrium has the feature of the one-price-fixes-all equilibrium in the monopoly market. The following proposition characterizes the consumer-optimal one-price-fixes-all equilibrium.

Let $\underline{\delta}^{co} \equiv \frac{n(c-E(l))}{n(c-E(l))+E(l)-c+\varepsilon(1-\alpha)}$.

Proposition 2 *If* $\widetilde{\alpha} < \alpha < \widehat{\alpha}$*, for a given* $\delta > \underline{\delta}^{co}$*, the consumer-optimal one-price-fixes-all equilibrium has the following features: Experts post the same price* $p^{co}(\delta) = c - \frac{\varepsilon\delta(1-\alpha)}{n(1-\delta)+\delta}$ *in each period and always recommends the treatment at* $p^{co}(\delta)$*. A consumer randomly selects an expert to visit and accepts* $p^{co}(\delta)$ *with probability one. If an expert rejects any consumer or posts a price different from* $p^{co}(\delta)$*, he loses all future consumers and exits the market.*

Recall that the cutoffs $\widetilde{\alpha}$ and $\widehat{\alpha}$ are defined in the main model, and $\widetilde{\alpha} < \alpha < \widehat{\alpha}$ is the same condition required for the existence of the one-price-fixes-all equilibrium in the monopoly market (Proposition 2 of the main model). For a given discount factor higher than $\underline{\delta}^{co}$, there exists a continuum of one-price-fixes-all equilibria indexed by price $p \in [p^{co}(\delta), E(l)]$. Both problems are treated with probability one in all the one-price-fixes-all equilibria. Hence, the consumer-optimal one-price-fixes-all equilibrium is the one indexed with the lowest price $p^{co}(\delta)$.

To see that the consumer-optimal one-price-fixes-all equilibrium yields the consumer a payoff higher than her reservation utility,

$$\begin{aligned} u^o &= -p^{co}(\delta) \\ &= \frac{\varepsilon\delta(1-\alpha)}{n(1-\delta)+\delta} - c \\ &> \frac{\varepsilon\underline{\delta}^{co}(1-\alpha)}{n(1-\underline{\delta}^{co})+\underline{\delta}^{co}} - c \\ &= -E(l), \end{aligned}$$

where the inequality follows since $\frac{\varepsilon\delta(1-\alpha)}{n(1-\delta)+\delta}$ increases in δ.

Next, we compare the consumer's utility in the consumer-optimal one-price-fixes-all equilibrium with that in the search equilibrium at a sufficiently low search cost. Rearrange terms in $p^{co}(\delta)$, we can rewrite the consumer's utility of the one-price-fixes-all equilibrium as

$$\begin{aligned}
u^o &= -\alpha c - (1-\alpha)\left(c - \frac{\varepsilon\delta}{n(1-\delta)+\delta}\right) \\
&< -\alpha c - (1-\alpha)\left(c - \varepsilon\right) \\
&< -\alpha c - (1-\alpha) l_m \\
&= \lim_{k \to 0} u^s.
\end{aligned}$$

The first inequality follows by setting $\delta = 1$. The second inequality holds under the assumption that $l_m < c - \varepsilon$. It can be verified that the consumer's utility in the one-price-fixes-all equilibrium is increasing in δ, and is bounded above by the consumer's utility in the search equilibrium when the search cost is sufficiently small. We summarize the result in the following proposition.

Proposition 3 *When δ is sufficiently high and k is sufficiently low, the search equilibrium is the consumer-optimal equilibrium among all the symmetric and stationary equilibria.*

Proof for Proposition 1: First, note that the expression under the square root in (1) is positive if $k \leq \frac{(c-l_m)(1-\sqrt{\alpha})}{1+\sqrt{\alpha}}$. Thus, $\beta_m^*(p;k) \in [0,1)$ for $k \leq \frac{(c-l_m)(1-\sqrt{\alpha})}{1+\sqrt{\alpha}}$.

Next, we show that consumers' strategy is a best response given experts' strategies. When a consumer receives a treatment recommendation on her first visit, the probability that the second opinion recommends no treatment is $\Pr(\emptyset|p) = \dfrac{(1-\alpha)\beta_m(1-\beta_m)}{\alpha+(1-\alpha)\beta_m}$. Substitute $\Pr(\emptyset|p)$ into the search condition (4), it becomes

$$\frac{(1-\alpha)\beta_m(1-\beta_m)(p-l_m)}{\alpha+(1-\alpha)\beta_m} = k. \tag{7}$$

It can be verified that (7) is satisfied at $\beta_m^*(p;k)$ defined in (1). Note that the item in the square root in $\beta_m^*(p;k)$ is positive and $\beta_m^*(p;k) \in (0,1)$ when k is sufficiently low.

Next, consider the consumer's participation constraint (5). Substitute $\Pr(l_s|p) = \frac{\alpha}{\alpha+(1-\alpha)\beta_m}$ and $\Pr(l_m|p) = \frac{(1-\alpha)\beta_m}{\alpha+(1-\alpha)\beta_m}$ into (5), we obtain

$$p + k \leq \frac{\alpha l_s + (1-\alpha)\beta_m l_m}{\alpha+(1-\alpha)\beta_m}. \tag{8}$$

When k converges to 0, $\beta_m^*(p;k)$ converges to 0. Therefore, the right hand side of (8) converges to l_s. Since $p \leq c+k$ and $c < l_s$, the participation constraint (8) is satisfied when k is sufficiently small. We conclude that given p and $\beta_m^*(p;k)$, it is the consumer's best response to randomize between accepting the treatment on her first visit and searching for a second opinion.

Next, we show that it is the consumer's best response to accept the second treatment recommendation with probability one. The probability that the third opinion recommends no treatment conditional on the first two opinions recommending the treatment is denoted by $\Pr(\emptyset|pp) = \dfrac{(1-\alpha)(\beta_m)^2(1-\beta_m)}{\alpha+(1-\alpha)(\beta_m)^2}$. Because $\Pr(\emptyset|pp) < \Pr(\emptyset|p)$, it follows that $\Pr(\emptyset|pp)(p-l_m) < k$ and hence the consumer strictly prefers accepting the second treatment recommendation to searching for the third opinion. Finally, the participation constraint (8) is satisfied when the consumer receives the second treatment recommendation. Hence, it is the consumer's best response to accept the second treatment recommendation with probability one.

Now, we show that experts' strategies are their best response. We show that experts have no profitable deviation in their recommendation strategies. When the expert diagnoses that a consumer has the minor problem, the probability that she is on her second visit is

$$\Pr(\mathrm{e}|l_m) = \frac{(1-\alpha)\frac{1}{n}\beta_m(1-\gamma)}{(1-\alpha)[\frac{1}{n}+\frac{1}{n}\beta_m(1-\gamma)]} = \frac{\beta_m(1-\gamma)}{1+\beta_m(1-\gamma)}. \tag{9}$$

Let $\Pi \equiv \alpha(p-c)+(1-\alpha)(p-c+\varepsilon)(\gamma\beta_m+(1-\gamma)(\beta_m)^2)$ denote the industry profit in a period. The expected industry profit from treating the serious problem is $\alpha(p-c)$ because the serious problem is repaired with probability one. The minor problem is fixed with probability $\gamma\beta_m+(1-\gamma)(\beta_m)^2$, where $\gamma\beta_m$ is the probability that the minor problem is fixed on the consumer's first visit and $(1-\gamma)(\beta_m)^2$ is the probability that it is fixed on her second visit. Recall for tractability, we assume that once an expert exits the market, he is replaced by an entrant and hence the number of experts is stable overtime. Let V denote the present value of the expert's profit when he is active in the market. It follows that

$$V = \frac{\Pi}{n} + \delta[1-\frac{1}{n}(1-\alpha)\beta_m(1-\beta_m)(1-\gamma)]V. \tag{10}$$

If the expert is active in a period, he receives $\frac{1}{n}$ of the industry profit in that period. The expert will lose all future consumers if he is caught lying in the current period, which occurs with probability $\frac{1}{n}(1-\alpha)\beta_m(1-\beta_m)(1-\gamma)$. As a result, the expert will survive to the next period with the complementary probability and his expected continuation payoff is the second item in (10). Solving (10), we obtain the following equation:

$$V = \frac{\Pi}{n-\delta[n-(1-\alpha)\beta_m(1-\beta_m)(1-\gamma)]} = \frac{\alpha(p-c)+(1-\alpha)(p-c+\varepsilon)(\gamma\beta_m+(1-\gamma)(\beta_m)^2)}{n-\delta[n-(1-\alpha)\beta_m(1-\beta_m)(1-\gamma)]}. \tag{11}$$

Substituting V and (9) into (6), it follows that

$$\begin{aligned} & \delta(1-\gamma)(1-\beta_m)\frac{\alpha(p-c)+(1-\alpha)(p-c+\varepsilon)(\gamma\beta_m+(1-\gamma)(\beta_m)^2)}{n-\delta[n-(1-\alpha)\beta_m(1-\beta_m)(1-\gamma)]} \\ = \;& (p-c+\varepsilon)\left[\beta_m(1-\gamma)+\gamma\right]. \end{aligned} \tag{12}$$

The condition (12) pins down a unique solution $\gamma = 1-\frac{n(1-\delta)(p-c+\varepsilon)}{n(1-\delta)(p-c+\varepsilon)(1-\beta_m)+\delta\alpha(1-\beta_m)(p-c)}$. Given that all other experts choose the strategy $(p,\ \beta_m^*(p;k))$ and that the consumer chooses $\gamma = \gamma^*(p;k,n)$, $\beta_m^*(p;k)$ is the expert's best response. Note that for $\gamma^*(p;k,n) > 0$, it is necessary to have $\delta > \underline{\delta}(p;k,n)$. Q.E.D.

Proof for Lemma 1: Substituting $p = c+k$ into $\beta_m^*(p;k)$ defined in (1)

$$\beta_m^*(c+k;k) \equiv \frac{(1-\alpha)(c-l_m)-\sqrt{(1-\alpha)^2(c-l_m)^2-4k\alpha(1-\alpha)(c+k-l_m)}}{2(1-\alpha)(c+k-l_m)} \tag{13}$$

Taking the derivative,

$$\begin{aligned}\frac{\partial \beta_m^*}{\partial k} &= \psi \times \left[2\alpha(c+k-l_m) + (1-\alpha)(c-l_m) - \sqrt{(1-\alpha)^2(c-l_m)^2 - 4k\alpha(1-\alpha)(c+k-l_m)}\right] \\ &> \psi \times 2\alpha(c+k-l_m) \\ &> 0,\end{aligned}$$

where ψ is a positive term. The inequalities follow from $c+k-l_m > 0$.

As $k \to 0$, $\beta_m^*(c+k;k) \to 0$. At $p^* = c+k$, the cutoff discount factor is

$$\underline{\delta}(c+k;k,n) \equiv \frac{n(k+\varepsilon)\beta_m^*(c+k;k)}{n(k+\varepsilon)\beta_m^*(c+k;k) + \alpha(1-\beta_m^*(c+k;k))k}.$$

Since both the numerator and the denominator of $\underline{\delta}(c+k;k,n)$ converges to 0 as $k \to 0$, we apply L'Hospital's Rule

$$\begin{aligned}\lim_{k\to 0}\underline{\delta}(c+k;k,n) &= \lim_{k\to 0}\frac{n\beta_m^*(c+k;k) + n(k+\varepsilon)\partial\beta_m^*(c+k;k)/\partial k}{n\beta_m^*(c+k;k) + n(k+\varepsilon)\partial\beta_m^*(c+k;k)/\partial k + \alpha(1-\beta_m^*(c+k;k)) - \alpha k\partial\beta_m^*(c+k;k)/\partial k} \\ &= \frac{n\varepsilon\lim_{k\to 0}\partial\beta_m^*(c+k;k)/\partial k}{n\varepsilon\lim_{k\to 0}\partial\beta_m^*(c+k;k)/\partial k + \alpha}, \qquad (14)\end{aligned}$$

where the first equality follows from L'Hospital's Rule and the second equality follows from $\beta_m^*(c+k;k) \to 0$ as $k \to 0$.

Next, we derive $\lim_{k\to 0}\partial\beta_m^*(c+k;k)/\partial k$. Let $N(k)$ denote the numerator of $\beta_m^*(c+k;k)$ and $D(k)$ denote the denominator of $\beta_m^*(c+k;k)$. It follows that

$$\partial\beta_m^*(c+k;k)/\partial k = \frac{N'(k)D(k) - N(k)D'(k)}{(D(k))^2}. \qquad (15)$$

The derivatives $N'(k)$ and $D'(k)$ are

$$\begin{aligned}N'(k) &= \frac{2\alpha(1-\alpha)(c+2k-l_m)}{\sqrt{(1-\alpha)^2(c-l_m)^2 - 4k\alpha(1-\alpha)(c+k-l_m)}} \\ D'(k) &= 2(1-\alpha).\end{aligned}$$

Substitute $N'(k)$ and $D'(k)$ into (15) and take the limit,

$$\lim_{k\to 0}\partial\beta_m^*(c+k;k)/\partial k = \lim_{k\to 0}\frac{N'(k)D(k) - N(k)D'(k)}{(D(k))^2} = \frac{\alpha}{(1-\alpha)(c-l_m)}, \qquad (16)$$

which follows because $\lim_{k\to 0}D(k) = 2(1-\alpha)(c-l_m)$, $\lim_{k\to 0}D'(k) = 2(1-\alpha)$, $\lim_{k\to 0}N(k) = 0$, and $\lim_{k\to 0}N'(k) = 2\alpha$. Substitute (16) into (14), it follows that $\lim_{k\to 0}\underline{\delta}(c+k;k,n) = \frac{n\alpha\varepsilon}{n\alpha\varepsilon + \alpha(1-\alpha)(c-l_m)}$. Q.E.D.

Proof for Proposition 2: Consider a one-price-fixes-all equilibrium in which experts post price p. The expert's no deviation constraint conditional on l_s is

$$p - c + \frac{\delta \widehat{\pi}^c}{1-\delta} \geq 0,$$

where $\widehat{\pi}^c = \frac{p-(\alpha c+(1-\alpha)(c-\varepsilon))}{n}$. Substituting $\widehat{\pi}^c$ and rearranging terms, the no deviation constraint becomes

$$p \geq p^{co}(\delta) = c - \frac{\varepsilon\delta(1-\alpha)}{n(1-\delta)+\delta}.$$

By the same argument in the proof for Proposition 2**,** if the expert does not reject the consumer when her problem is serious, he will not reject the consumer when her problem is minor.

The consumer will accept $p^{co}(\delta)$ if and only if $p^{co}(\delta) \leq \alpha l_s + (1-\alpha) l_m$, which implies $\delta \geq \underline{\delta}^{co} \equiv \frac{n(c-E(l))}{n(c-E(l))+E(l)-c+\varepsilon(1-\alpha)}$. The discount factor $\underline{\delta}^{co} < 1$ if and only if $\alpha > \widetilde{\alpha} \equiv \dfrac{c-\varepsilon-l_m}{l_s-\varepsilon-l_m}$.

Because a price deviation will trigger the punishment phase and result in no trade, the deviating expert will behave like the monopolist in the static game. By the argument for Lemma 1 in the main text, the consumer will not visit the expert in the period in which he posts a different price. Thus, an expert does not have a profitable deviation in price. Q.E.D.

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