REGULATING PRODUCT COMMUNICATION

Maarten C.W. Janssen and Santanu Roy

ONLINE APPENDIX

This online Appendix consists of three different parts: named Appendix A, B and C.

In Appendix A, we provide additional results in terms of the equilibrium characterization and proofs related to the analysis in Sections I - V of the main paper. We first provide some common properties of all equilibria and then establish in detail the equilibria that are not discussed in detail in the main body of the paper.

In Appendix B, we analyze an extension of the basic model in the main paper to more than two quality types as discussed in subsection VI.A of the main text. Much of the analysis here focuses on the case of three quality types and the results here are indicative of qualitative results one may obtain with a finite number of quality types. We also have some results to indicate the difficulties associated with the case of a continuum of types.

In Appendix C, we provide more detail in support of the discussion on quantity distortions contained in section VI.B of the main text.

Appendix A
Additional results and proofs related to Sections I - V of the main paper

The first Lemma in this online Appendix shows that some of the properties regarding the equilibrium structure of the pure price signaling equilibrium can be generalized to hold for all equilibria in our model. We use superscripts D and ND to indicate whether prices are accompanied by disclosure or not.

Lemma 1 *The following properties hold in equilibrium:*

(a) If a high quality type's equilibrium strategy places a strictly positive probability mass on charging price $\widehat{p} > c_H$ and message $\widehat{m} \in \{0, 1\}$, then a low quality type must be indifferent between following its equilibrium strategy and imitating price $\widehat{p}$ and message $\widehat{m}$;

(b) If a high quality type does not claim high quality, then it charges a deterministic price p_H^{ND} and sells only if the rival firm is of high quality; low quality firms randomize over a set of prices with upper bound $\overline{p}_L$ satisfying $c_L < \overline{p}_L \leq p_H^{ND} - \Delta V$. If, in addition, a high quality type claims high quality with strictly positive probability, then it charges lower prices when it discloses i.e., $\overline{p}_H^D \leq p_H^{ND}$ (where $\overline{p}_H^D$ is the upper bound of high quality prices under direct communication);

(c) The price distribution of a high quality firm claiming high quality can have a probability mass point only at its lower bound (in which case the lower bound is an isolated point);

(d) If a low quality type makes strictly positive profit, then it must randomize over prices without a mass point.

Part (a) of Lemma 1 follows directly from Proposition 1 as there is no point for the low quality type to reveal itself by incurring a cost of communication. Part (b) can be understood using the same logic as we discussed above for the pure price signaling equilibrium: the D1 equilibrium selects the most competitive revealing equilibrium in which the low quality type is indifferent between its equilibrium actions and imitating an action to which the high quality assigns a strictly positive probability mass. Also, like in the pure price signaling equilibrium, if a high quality firm does not disclose, it sets a price that is so high that a low quality competitor undercuts by at least ΔV so that the non-disclosing high quality only sells if the other firm is also of high quality. This is necessary not to give the low quality firm an incentive to imitate the high quality price. Part (d) of Lemma 1 is also similar to the pure price signaling equilibrium: if there would be a mass point in the price distribution, low quality firms would have an incentive to undercut this mass point. Undercutting would always be profitable as consumers would not have worse beliefs about product quality after observing such a deviation (even if it is out of equilibrium) than when charging the mass point.

Part (c) of Lemma 1 is also akin to the phenomenon in the pure price signaling equilibrium that the high quality firm can have a mass point. The difference with the low quality firm is that out-of-equilibrium beliefs can prevent the high quality firm from undercutting a mass point. does not disclose, a low quality firm makes positive profit and must randomize over prices. This also implies existence of a consumption distortion as low quality firms must make positive sales when they set a price equal to the upper bound of the price distribution and in that case they sell only if the rival is a high quality firm. When high quality firms randomize between disclosing and not disclosing, they must sell more when they disclose; otherwise they cannot be indifferent between the two actions. not only does the low quality type not have an incentive to imitate the high quality firm's actions, but in a D1 equilibrium the low quality type is actually indifferent between its equilibrium actions and imitating an action to which the high quality type assigns a strictly positive probability mass.

Proof of Lemma 1 : (a) Suppose that a high quality type's equilibrium strategy places a strictly positive probability mass on price $\widehat{p} > c_H$ and a message $\widehat{m} \in \{0,1\}$. Incentive compatibility requires that an L-type's equilibrium profit $\pi_L^* \geq \pi_L^d$, where $\pi_L^d = (\widehat{p} - c_L)q_H^* - (D+f)I_{\{\widehat{m}=1\}}$is L-type's profit from deviation to $(\widehat{p}, \widehat{m})$, q_H^* is the expected quantity sold by the H type at $(\widehat{p}, \widehat{m})$ and I is an indicator function. Note that π_H^*, the equilibrium profit of H type, is given by $\pi_H^* = (\widehat{p} - c_H)q_H^* - DI_{\{\widehat{m}=1\}}$so that

$$\frac{(\widehat{p} - c_L)}{(\widehat{p} - c_H)} = \frac{\pi_L^d + (D+f)I_{\{\widehat{m}=1\}}}{\pi_H^* + DI_{\{\widehat{m}=1\}}}. \tag{1}$$

Using symmetry of equilibrium, prices slightly below $\widehat{p}$ cannot be in the support of $H-$type's equilibrium strategy in the event that it chooses message $\widehat{m}$ as such a price would undercut rival H type when the latter chooses $(\widehat{p}, \widehat{m})$ and therefore yield higher payoff. Further, to deter the $H-$type from deviating to an out-of-equilibrium price $\widehat{p} - \epsilon$ (for $\epsilon > 0$ small enough) while sending message $\widehat{m}$, beliefs must assign sufficiently high probability that the deviating firm is of $L-$type. We now claim that if

$$\pi_L^* > \pi_L^d, \tag{2}$$

then the D1 criterion implies that after observing a deviation to a price $\widehat{p} - \epsilon$ (for $\epsilon > 0$ small enough) with message $\widehat{m}$, buyers must believe that the deviating firm is an H type with probability 1, a contradiction. To establish this claim, let $q_H(\widehat{p} - \epsilon)$, $q_L(\widehat{p} - \epsilon)$ be the expected quantity that an H and an L type firm must sell respectively in order to be indifferent between this deviation and their equilibrium strategies:

$$\begin{aligned} (\widehat{p} - \epsilon - c_H)q_H(\widehat{p} - \epsilon) - DI_{\{\widehat{m}=1\}} &= \pi_H^*, \\ (\widehat{p} - \epsilon - c_L)q_L(\widehat{p} - \epsilon) - (D+f)I_{\{\widehat{m}=1\}} &= \pi_L^* \end{aligned}$$

Using the D1 criterion, it is sufficient to show that $q_H(\widehat{p} - \epsilon) < q_L(\widehat{p} - \epsilon)$ for ϵ small enough. Note that $q_H(\widehat{p} - \epsilon) \geq q_L(\widehat{p} - \epsilon)$ if, and only if,

$$\frac{\widehat{p} - \epsilon - c_L}{\widehat{p} - \epsilon - c_H} \geq \frac{\pi_L^* + (D+f)I_{\{\widehat{m}=1\}}}{\pi_H^* + DI_{\{\widehat{m}=1\}}} \tag{3}$$

and as the left hand side of (3) is continuous and strictly increasing in ϵ, (3) holds for all ϵ arbitrarily close to 0, if, and only if,

$$\frac{\widehat{p} - c_L}{\widehat{p} - c_H} \geq \frac{\pi_L^* + (D+f)I_{\{\widehat{m}=1\}}}{\pi_H^* + DI_{\{\widehat{m}=1\}}} \tag{4}$$

and using (2) in (4) we obtain a contradiction to (1). Thus, $\pi_L^* = \pi_L^d$.

(b) Consider an equilibrium where $H-$types disclose with probability $\gamma_H \in [0,1)$. Let $\overline{p}_H^D, \underline{p}_H^D$ ($\overline{p}_H^{ND}, \underline{p}_H^{ND}$) be the supremum and the infimum of the support of prices charged by an $H-$type firm when it discloses (does not disclose). We first show that if $\gamma_H > 0$, $\overline{p}_H^D \leq \overline{p}_H^{ND}$. Suppose to the contrary that $\overline{p}_H^D > \overline{p}_H^{ND}$. As $D > 0$, in order to cover the disclosure cost, an $H-$type firm must sell strictly positive expected quantity at price $\overline{p}_H^D$ and (therefore) at price $\overline{p}_H^{ND} < \overline{p}_H^D$. As $L-$type firm can always imitate price $\overline{p}_H^{ND} \geq c_H > c_L$ without disclosing, it follows that L-type's equilibrium profit $\pi_L^* > 0$. Thus $\overline{p}_L > c_L$. Standard undercutting arguments (using symmetry of equilibrium) can be used to show that there is no probability mass point at $\overline{p}_L$ and so, at $\overline{p}_L$ the (limiting) expected quantity sold by $L-$type firm in the event where rival is of $H-$type is strictly positive.

As $\overline{p}_H^D$ is the upper bound of high quality prices, $\overline{p}_L \leq \overline{p}_H^D - \Delta V$. Thus, at price $\overline{p}_H^D$ an $H-$type firm that discloses sells zero when its rival is of $L-$type. Therefore, the only way $H-$type can sell strictly positive expected quantity at $\overline{p}_H^D$ is if there is a strictly positive probability mass $\sigma > 0$ at price $\overline{p}_H^D$ and the equilibrium profit of H-type must be $\pi_H^* = (\overline{p}_H^D - c_H)\frac{\alpha\sigma}{2} - D$ and further, using part (b) of this lemma, $L-$type must be indifferent between its equilibrium strategy and deviating to disclosing and charging $\overline{p}_H^D$ i.e.,

$$\begin{aligned} \pi_L^* &= (\overline{p}_H^D - c_L)\frac{\alpha\sigma}{2} - D - f = \pi_H^* + \Delta c\frac{\alpha\sigma}{2} - f \\ &= (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}) + \Delta c\frac{\alpha\sigma}{2} - f \end{aligned} \tag{5}$$

where $q(\underline{p}_H^{ND})$ is the expected quantity sold by the high quality type at price $\underline{p}_H^{ND}$ when it does not disclose. Observe that as $\underline{p}_H^{ND} < \overline{p}_H^D, q(\underline{p}_H^{ND}) \geq \alpha\sigma$. Further, the incentive constraint of an $L-$type implies:

$$\pi_L^* \geq (\underline{p}_H^{ND} - c_L)q(\underline{p}_H^{ND}) = (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}) + \Delta c q(\underline{p}_H^{ND}) \geq (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}) + \Delta c\alpha\sigma.$$

Thus, from (5)

$$\pi_L^* = (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}) + \Delta c\frac{\alpha\sigma}{2} - f \leq \pi_L^* - \Delta c\frac{\alpha\sigma}{2} - f,$$

a contradiction. Thus, if $\gamma_H > 0$, $\overline{p}_H^D \leq \overline{p}_H^{ND}$.

Next, we show that if $\gamma_H \in [0,1)$, then $\overline{p}_H^{ND} = \underline{p}_H^{ND}$. Suppose not. Then, $\overline{p}_H^{ND} > \underline{p}_H^{ND}$; this implies that $H-$type sells strictly positive expected quantity at price $\underline{p}_H^{ND} \geq c_H > c_L$ (for instance, when rival is of $H-$type and does not disclose) and so $\pi_H^* > 0$ (note $\overline{p}_H^{ND} > \underline{p}_H^{ND} \geq c_H$). To deter imitation of $H-$type's non-disclosure action, $\pi_L^* > 0$. Then, using identical arguments as above, $\overline{p}_L \leq \overline{p}_H^{ND} - \Delta V$ and there is no probability mass point at $\overline{p}_L$ and at price $\overline{p}_H^{ND}$, the high quality type sells zero where rival is $L-$type. Let $q(p, ND)$ and $q(p, D)$ denote respectively the expected quantity sold by the high quality type at p when it does not disclose and when it discloses. As $\pi_H^* > 0$, $q(\overline{p}_H^{ND}, ND) > 0$. Let $\xi_{ND} \geq 0$ and $\xi_D \geq 0$ be the respective probability probability masses, if any, placed by the high quality type at the price $\overline{p}_H^{ND}$ in the states where it does not disclose and where it discloses ($\xi_D = 0$ if $\overline{p}_H^D < \overline{p}_H^{ND}$). Using symmetry of the equilibrium, there exists $\widehat{\beta} \in [0,1]$ such that

$$q(\overline{p}_H^{ND}, ND) = \alpha\left[\frac{1-\gamma_H}{2}\xi_{ND} + \widehat{\beta}\gamma_H\xi_D\right], q(\overline{p}_H^{ND}, D) = \alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\widehat{\beta})(1-\gamma_H)\xi_{ND}\right].$$

Then

$$\pi_H^* = (\overline{p}_H^{ND} - c_H)\alpha\left[\frac{1-\gamma_H}{2}\xi_{ND} + \beta\gamma_H\xi_D\right] = (\underline{p}_H^{NA} - c_H)q(\underline{p}_H^{NA}, NA). \tag{6}$$

Note that as $\underline{p}_H^{ND} < \overline{p}_H^{ND}$,

$$q(\underline{p}_H^{ND}, ND) \geq \alpha((1-\gamma_H)\xi_{ND} + \gamma_H\xi_D). \tag{7}$$

Consider the case where $\gamma_H\xi_D > 0$. This implies that $\overline{p}_H^D = \overline{p}_H^{ND}$ and $\xi_D > 0$. Using part (b) of this lemma, $L-$type is then indifferent between its equilibrium strategy and deviating to disclosing (falsely) and charging $\overline{p}_H^{ND}$:

$$\begin{aligned}
\pi_L^* &= (\overline{p}_H^{ND} - c_L)\alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\beta)(1-\gamma_H)\xi_{ND}\right] - D - f \\
&= \pi_H^* - D + \Delta c\alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\beta)(1-\gamma_H)\xi_{ND}\right] - f, \text{ using (6)} \\
&= (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}, ND) + \Delta c\alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\beta)(1-\gamma_H)\xi_{ND}\right] - f \\
&\leq (\underline{p}_H^{ND} - c_L)q(\underline{p}_H^{ND}, ND) - \Delta c\alpha((1-\gamma_H)\xi_{ND} + \gamma_H\xi_D) \\
&\quad + \Delta c\alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\beta)(1-\gamma_H)\xi_{ND}\right] - f, \text{ using (7)} \\
&\leq \pi_L^* - \Delta c\alpha(\frac{1}{2}\gamma_H\xi_D + (1-\gamma_H)\beta\xi_{ND}) - f,
\end{aligned}$$

a contradiction (the last inequality uses the incentive constraint of the low quality type to not imitate $\underline{p}_H^{ND}$ without disclosing). Now, consider the case where $\gamma_H\xi_D = 0$. As $q(\overline{p}_H^{ND}, ND) > 0$, we have $\xi_{ND} > 0$ and $q(\overline{p}_H^{ND}, ND) = \frac{\alpha}{2}(1-\gamma_H)\xi_{ND}$ and

$$q(\underline{p}_H^{ND}, ND) > \frac{\alpha}{2}(1-\gamma_H)\xi_{ND} . \tag{8}$$

Using part (b) of this lemma, $L-$type must be indifferent between its equilibrium strategy and deviating to $\overline{p}_H^{ND}$ without disclosing:

$$\begin{aligned}
\pi_L^* &= (\overline{p}_H^{ND} - c_L)\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} = (\overline{p}_H^{ND} - c_H)\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} + \Delta c\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} \\
&= (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}, ND) + \Delta c\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} \\
&= (\underline{p}_H^{ND} - c_L)q(\underline{p}_H^{ND}, ND) - \Delta c q(\underline{p}_H^{ND}, ND) + \Delta c\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} \\
&\leq \pi_L^* - \Delta c\left[q(\underline{p}_H^{ND}, ND) - \frac{\alpha(1-\gamma_H)\xi_{ND}}{2}\right] < \pi_L^*
\end{aligned}$$

a contradiction (the last inequality follows from the incentive constraint of $L-$quality type to not imitate $\underline{p}_H^{ND}$ and (8)). Thus, if $\gamma_H \in [0,1)$, then $\overline{p}_H^{ND} = \underline{p}_H^{ND}$ and when it discloses, $H-$type charges a deterministic price $p_H^{ND} \geq \overline{p}_H^D$.

As this is a symmetric equilibrium, at price p_H^{ND} (and without disclosing), the high quality firm sells in the state where rival is of high quality and does not disclose and as $p_H^{ND} \geq c_H > c_L$, and to deter imitation by $L-$type of this action, $\pi_L^* > 0$. Using very similar arguments as above, $L-$types must randomize over prices and there cannot be a mass point at $\overline{p}_L$ and at that price L-type must sell with strictly positive probability in the state where rival is $H-$type; as p_H^{ND} is the upper bound of the support of high quality prices,

$$\overline{p}_L \leq p_H^{ND} - \Delta V. \tag{9}$$

Therefore, at price p_H^{ND}, $H-$type sells zero expected quantity in the event that rival is $L-$type. Note that $\pi_L^* > 0$ implies that $\overline{p}_L > c_L$ so that $p_H^{ND} \geq c_L + \Delta V > c_H$ so that $\pi_H^* > 0$.

(c) We now show that if an $H-$type charges a price $\widetilde{p}$ with strictly positive probability in the event that it discloses, then $\widetilde{p} = \underline{p}_H^D$. From above, discussion we know that $\underline{p}_H^{ND} \geq \widetilde{p}$. Suppose to the contrary that $\underline{p}_H^D < \widetilde{p}$. Then there exists $\widehat{p} \in [\underline{p}_H^D, \widetilde{p})$ such that

$$\pi_H^* = (\widehat{p} - c_H)q(\widehat{p}, D) - D = (\widetilde{p} - c_H)q(\widetilde{p}, D) - D. \tag{10}$$

As $\widehat{p} < \widetilde{p}$, $q(\widehat{p}, D) > q(\widetilde{p}, D)$. As there is a strictly positive probability mass at $\widetilde{p}$, out-of-equilibrium beliefs must deter H-type from undercutting $\widetilde{p}$ by assigning sufficient probability to the deviant being of L-type and using part (b) of this lemma

$$\begin{aligned} \pi_L^* &= (\widetilde{p} - c_L)q(\widetilde{p}, D) - D - f \\ &= (\widehat{p} - c_L)q(\widehat{p}, D) - D + \Delta c(q(\widetilde{p}, D) - q(\widehat{p}, D)) - f, \text{using (10)} \\ &< (\widehat{p} - c_L)q(\widehat{p}, D) - D \leq \pi_L^* \end{aligned}$$

a contradiction (the last inequality follows from the incentive constraint of an $L-$type).

(d) This follows readily from standard undercutting arguments (using the symmetry of the equilibrium and the fact that in a revealing equilibrium, adverse beliefs cannot deter an L-type firm from undercutting its rival).

Next, we present the proof of Proposition 6 in the main paper.

Proposition 6 of the main text. *A pure disclosure equilibrium (where high quality firms disclose with probability one) exists if, and only if,* $D \leq \widetilde{D}(f)$.

Proof of Proposition 6.

Define $\widetilde{D}(f)$ as

$$\widetilde{D}(f) = \begin{cases} \left(\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1\right) f \text{ , if } 0 \leq f \leq \frac{\alpha}{2}\Delta c \\ (\Delta V - \Delta c)\, f/\Delta c, \text{ if } \frac{\alpha}{2}\Delta c < f \leq (1 - \frac{\alpha}{2})\Delta c \\ (1 - \frac{\alpha}{2})\Delta V - f \text{ , if } (1 - \frac{\alpha}{2})\Delta c < f \leq \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c \\ (1 - \alpha)(\Delta V - \Delta c) \text{, if } f > \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c. \end{cases}$$

It is not difficult to see that $\widetilde{D}(f)$ is continuous in f. If $0 \leq f \leq \frac{\alpha}{2}\Delta c$ the value of $\widetilde{D}(f)$ follows from the characterization of the distortionary disclosure equilibrium in Proposition 3.[1] If $f > (1 - \frac{\alpha}{2})\Delta c$ the value of $\widetilde{D}(f)$ is determined by conditions for the non-distortionary disclosure equilibrium in Proposition 4 as $\max\left\{(1 - \frac{\alpha}{2})\Delta V - f, (1 - \alpha)(\Delta V - \Delta c)\right\}$. This yields the last two components of $\widetilde{D}(f)$.[2] If $\frac{\alpha}{2}\Delta c < f \leq \frac{\alpha}{2}\Delta V$ there are multiple full disclosure equilibria (the fully non-distortionary equilibrium of Proposition 4 and the partial disclosure equilibrium of Proposition 5).

Consider a pure disclosure equilibrium. There are two kinds of such equilibria: (i) Low quality type makes strictly positive profit; (ii) Low quality type makes zero profit. Consider an equilibrium of type (i). We first show that $H-$types must charge a deterministic price. Suppose to the contrary that $H-$type randomizes. Then, $L-$type must sell with strictly positive probability in the state where rival is of $H-$type and randomize its price between some lower bound $\underline{p}_L$ an upper bound $\overline{p}_L$.

[1] For these values of f, (6) implies (5) in the main text and $\widetilde{D}(f)$ is then the RHS of (6) in the main text.

[2] If $f < \frac{\alpha}{2}\Delta V + (1 - \alpha)\Delta c$ the first term is larger, whereas the second term is larger when f is larger.

From the first proposition in this online appendix it follows that there is no mass point at $\overline{p}_L$ so that at price $\overline{p}_L$, $L-$type sells only in the state where rival is $H-$ type which implies that $\overline{p}_L \leq \overline{p}_H - \Delta V$. Also from the same proposition, $H-$type has no mass point at the upper bound $\overline{p}_H$ of the support of its price distribution and so at that price, it must sell strictly positive quantity (to cover disclosure cost) and as it sells only in the state where rival is L-type we must that $\overline{p}_H \leq \overline{p}_L + \Delta V$. Thus, $\overline{p}_H = \overline{p}_L + \Delta V$. As there is no mass point at $\overline{p}_H$ or $\overline{p}_L$, $L-$type is undercut with probability one at price $\overline{p}_L$ and must earn zero expected profit in equilibrium, a contradiction. Thus, $H-$type cannot randomize and must charge a deterministic price $\widetilde{p}_H = \overline{p}_L + \Delta V$. This is exactly the equilibrium outcome described in Proposition 3 and it exists if, and only if, conditions (7) and (8) of the main text hold; it is easy to check that these are equivalent to the following conditions:

$$0 < f \leq \frac{\alpha}{2}\Delta V \tag{11}$$

and

$$\begin{aligned} D &\leq \left[\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1\right] f \text{ , if } 0 \leq f < \frac{\alpha}{2}\Delta c & (12)\\ &\leq \frac{\alpha}{2}\Delta V - f \text{ , if } \frac{\alpha}{2}\Delta c \leq f \leq \frac{\alpha}{2}\Delta V. & (13)\end{aligned}$$

Now consider an equilibrium of type (ii) where $L-$types make zero profit. It easy to check (use symmetry), that $L-$types must charge a deterministic price c_L. The upper bound $\overline{p}_H$ of the support of the distribution of $H-$types prices must satisfy

$$\overline{p}_H \leq c_L + \Delta V \tag{14}$$

for otherwise, an $L-$type can deviate to a price above c_L and sell strictly positive expected quantity. There are two sub-cases: (ii.a) $L-$types sell strictly positive quantity in the state where rival is high quality; (ii.b) $L-$types sell only in the state where rival is low quality. Consider (ii.a). If $H-$types randomize over prices in such an equilibrium, then using part (c) of Lemma 1 in this online appendix there is no probability mass at $\overline{p}_H$ so that (14) implies that $L-$ type is undercut with probability one in the state where rival is $H-$type. Therefore, $H-$types must charge a deterministic price $\widetilde{p}_H$ $= c_L + \Delta V$. Buyers are then indifferent between buying high quality at price $\widetilde{p}_H$ and low quality at price c_L. In the state where one firm is of high quality and the other is of low quality, a fraction $\beta \in [0, 1)$ of buyers buy high quality and the rest buy low quality. This is identical to the equilibrium outcome described in Proposition 5 and such an equilibrium exists if, and only if, conditions (13) and (14) of the main text hold; it is easy to check that the latter conditions are equivalent to the following:

$$\frac{\alpha}{2}\Delta c \leq f \leq \left(1 - \frac{\alpha}{2}\right)\Delta V \tag{15}$$

and

$$\begin{aligned} \frac{\alpha}{2}\Delta V - f &\leq D \leq f(\lambda - 1), \text{ if } \frac{\alpha}{2}\Delta c \leq f \leq \left(1 - \frac{\alpha}{2}\right)\Delta c & (16)\\ \frac{\alpha}{2}\Delta V - f &\leq D < \left(1 - \frac{\alpha}{2}\right)\Delta V - f, \text{ if } \left(1 - \frac{\alpha}{2}\right)\Delta c \leq f < \left(1 - \frac{\alpha}{2}\right)\Delta V. & (17)\end{aligned}$$

Now, consider an equilibrium of type (ii.b) There are two further sub-cases: (ii.b.1) $H-$types charge a deterministic price; (ii.b.2) $H-$types randomize over prices. It is easy to check that an equilibrium of type (ii.b.1) must be essentially identical to a type (ii.a) equilibrium with $\beta = 1$ and using (12) and 13) of the main text such an equilibrium exists if and only if (15)-(16) hold and further,

$$D = \left(1 - \frac{\alpha}{2}\right)\Delta V - f, \ \ \left(1 - \frac{\alpha}{2}\right)\Delta c \le f \le \left(1 - \frac{\alpha}{2}\right)\Delta V. \tag{18}$$

Consider now an equilibrium of type (ii.b.2) We first show that the upper bound $\overline{p}_H$ of the support of the distribution of high quality prices must satisfy (14) with equality:

$$\overline{p}_H = c_L + \Delta V. \tag{19}$$

Suppose to the contrary that $\overline{p}_H < c_L + \Delta V$. Using Lemma 1, there is no mass point at price $\overline{p}_H$ and so at this price an H type can sell only if its rival is an L type. We now claim that D1 refinement implies that a firm disclosing and charging a price $\widehat{p} \in (\ \overline{p}_H, c_L + \Delta V)$ must be regarded as an $H-$type. To see this consider such a $\widehat{p}$. Let $q^{\tau}(\widehat{p})$ be the (expected) quantity that must be sold by a type τ at price $\widehat{p}$ to be indifferent between its equilibrium strategy and deviation to disclosing and charging $\widehat{p}.t$Then, $(\overline{p}_H - c_H)(1-\alpha) - A = (\widehat{p} - c_H)q^H(\widehat{p}) - D$ and $0 = (\widehat{p} - c_L)q^L(\widehat{p}) - D - f$ so that $\frac{q^H(\widehat{p})}{q^L(\widehat{p})} < 1$ if, and only if,.

$$(1-\alpha)((\overline{p}_H - c_H) < (D+f)\left(\frac{\widehat{p} - c_H}{\widehat{p} - c_L}\right). \tag{20}$$

As $\left(\frac{\widehat{p} - c_H}{\widehat{p} - c_L}\right)$ is strictly increasing in $\widehat{p}$, (20) must hold for some $\widehat{p} \in (\ \overline{p}_H, c_L + \Delta V)$ if $(1-\alpha)((\overline{p}_H - c_H) \le (D+f)\left(\frac{\overline{p}_H - c_H}{\overline{p}_H - c_L}\right)$ which holds as long as $(1-\alpha)((\overline{p}_H - c_L) - (D+f) \le 0$; this last inequality must hold in this equilibrium to ensure that $L-$type does not deviate to disclosing and imitating high quality price $\overline{p}_H$. Thus, $q^H(\widehat{p}) < q^L(\widehat{p})$ for some $\widehat{p} \in (\ \overline{p}_H, c_L + \Delta V)$ D1 criterion implies that out-of-equilibrium belief should regard any firm that discloses and charges such $\widehat{p}$ $H-$type with probability one. But this kind of belief makes deviation to price $\widehat{p}$ while disclosing strictly gainful for H type (sell the same expected quantity as at $\overline{p}_H$ and earn strictly higher profit). Hence, (19) holds. As there can only be a mass point at the lower bound of $H-$type's price distribution, it is easy to check that the $H-$type must randomize with a continuous distribution over an interval whose upper bound is $c_L + \Delta V$ with or without a positive mass point at an isolated price strictly below this interval. This is exactly the equilibrium outcome described in Proposition 4 and it exists if, and only if, (10) and (11) of the main text hold; it is easy to check that the latter conditions are equivalent to the following:

$$f \ge \left(1 - \frac{\alpha}{2}\right)\Delta c \tag{21}$$

and

$$\max\left\{\left((1-\alpha)\Delta V + \frac{\alpha \Delta c}{2} - f\right)\ ,0\right\} \le D \le (1-\alpha)(\Delta V - \Delta c). \tag{22}$$

We have now covered all possible pure disclosure equilibria. Using the necessary and sufficient conditions for all possible pure disclosure equilibria outlined above (in particular, conditions (11)-(13), (15)-(17), (18), (21)-(22)) one can check that a pure disclosure equilibrium exists if, and only if, $D \le \widetilde{D}(f)$. The details for various ranges of values of f are as follows:

(a) $0 \le f < \frac{\alpha}{2}\Delta c$: only an equilibrium of type (i) can hold and it does (for that range of f) if, and only if, (12) holds i.e., $D \le \left(\frac{2\alpha\lambda\Delta c}{\alpha\Delta c + 2f} - 1\right) f$

(b) $\frac{\alpha}{2}\Delta c \leq f < (1-\frac{\alpha}{2})\Delta c$: a type (i) equilibrium exists if, and only if (13) holds i.e., $D \leq \frac{\alpha}{2}\Delta V - f$ while a type (ii.a) equilibrium holds if and only if (16) holds i..e, $\frac{\alpha}{2}\Delta V - f \leq D \leq f(\lambda-1)$; a type (ii.b) equilibrium does not exist. As $f \geq \frac{\alpha}{2}\Delta c$ implies $\frac{\alpha}{2}\Delta V - f \leq f(\lambda-1)$, a pure disclosure equilibrium exists if, and only if, $D \leq f(\lambda-1)$.

(c) $(1-\frac{\alpha}{2})\Delta c \leq f \leq \frac{\alpha}{2}\Delta V$: a type (i) equilibrium exists if, and only if (13) holds i.e., $D \leq \frac{\alpha}{2}\Delta V - f$ and types (ii.a) or (ii.b.1) equilibria exist if and only if (17) holds i.e., $\frac{\alpha}{2}\Delta V - f \leq D \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$; thus, a pure disclosure equilibrium exists for all $D \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$. A type (ii.b.2) equilibrium exists, if and only if (22) holds i.e., $(1-\alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \leq D \leq (1-\alpha)(\Delta V - \Delta c)$; as $f \leq \frac{\alpha}{2}\Delta V$ implies $(1-\alpha)(\Delta V - \Delta c) \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$, this kind of equilibrium occurs only for $D \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$. Thus, a pure disclosure equilibrium exists if, and only if, $D \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$

(d) $\frac{\alpha}{2}\Delta V < f \leq \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$: a type (i) equilibrium does not exist, types (ii.a) or (ii.b.1) equilibria exist if and only if (17) holds which reduces to $D \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$; type (ii.b.2) equilibrium exists, if and only if (22) holds i.e., $(1-\alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \leq D \leq (1-\alpha)(\Delta V - \Delta c)$. As $f \leq \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$ implies $(1-\alpha)(\Delta V - \Delta c) \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$, this kind of equilibrium occurs only for $D \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$. Thus, a pure disclosure equilibrium exists if, and only if, $D \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$

(e) for $f > \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$: a type (i) equilibrium does not exist, types (ii.a) or (ii.b.1) equilibria exist if and only if (17) holds which reduces to $D \leq \left(1-\frac{\alpha}{2}\right)\Delta V - f$; while;type (ii.b.2) equilibrium exists, if and only if (22) holds i.e., $(1-\alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \leq D \leq (1-\alpha)(\Delta V - \Delta c)$. As for this range of f, $\left(1-\frac{\alpha}{2}\right)\Delta V - f < (1-\alpha)(\Delta V - \Delta c)$ we have that a pure disclosure equilibrium exists if, and only if, $D \leq (1-\alpha)(\Delta V - \Delta c)$.

This concludes the proof of Proposition 6.

We now state an important lemma that underlies all of the results in Section IV of the main text.

Lemma 2 *A partially distortionary mixed disclosure equilibrium exists if, and only if,*

$$D^*(f) > D > \widetilde{D}(f) \text{ and } f \in (\frac{\alpha}{2}\Delta c, (1-\frac{\alpha}{2})\Delta c] \tag{23}$$

or

$$D^*(f) > D > (1-\alpha)(\Delta V - \Delta c) \text{ and } f > (1-\frac{\alpha}{2})\Delta c. \tag{24}$$

All these equilibria generate lower welfare than the pure price signaling equilibrium. The expected profit earned by both low and high quality firms are lower than in the pure price signaling outcome.

Proof of Lemma 2. In a partially distortionary mixed disclosure equilibrium, high quality type must sell with positive probability in the state where its rival is of low quality type. From Lemma 1(b), we know that when it does not disclose, high quality type sells only in the state where rival is high quality type. Therefore, in a partially distortionary mixed disclosure equilibrium, high quality type must sell with strictly positive probability when it discloses and the rival firm is of low quality type. It follows then that $\underline{p}_H^D < p_H^{ND}$. As $\overline{p}_H^D \leq p_H^{ND}$, in the state where it discloses an H-type cannot have a probability mass point at $\overline{p}_H^D$ if it randomizes over prices (see Lemma 1(c)). Thus, with probability one all buyers buy from the disclosing high quality firm when both firms are of high quality and only one discloses. We now argue that either the disclosing high quality firm sets a deterministic price, or it serves the entire market if the rival firm produces low quality. To see this suppose that with strictly positive probability a disclosing high quality firm randomizes over prices but does not sell to all buyers when rival is low quality. Then, $\overline{p}_H^D > \underline{p}_L + \Delta V$ (no mass point at $\overline{p}_H^D$ conditional on disclosure). First, suppose that

$$\underline{p}_L + \Delta V < \overline{p}_H^D \leq \overline{p}_L + \Delta V.$$

Then, $\overline{p}_H^D - \Delta V \in (\underline{p}_L, \overline{p}_L]$. Further, there exists $\epsilon > 0$ such that $\overline{p}_H^D - \epsilon$ is the in the interior of the support of the distribution of high quality prices with disclosure and

$$\underline{p}_L < \overline{p}_H^D - \epsilon - \Delta V < \overline{p}_H^D - \Delta V \leq \overline{p}_L \tag{25}$$

The high quality equilibrium profit must be equalized at prices $\overline{p}_H^D - \epsilon$ and $\overline{p}_H^D$

$$\begin{aligned} & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\overline{p}_H^D - \epsilon - c_H) \\ = & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - c_H) \end{aligned}$$

However, this implies that

$$\begin{aligned} & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\overline{p}_H^D - \epsilon - \Delta V - c_L) \\ = & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\overline{p}_H^D - \epsilon - c_H) \\ & - \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\Delta V - \Delta c) \\ = & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - c_H) \\ & - \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\Delta V - \Delta c) \\ = & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - \Delta V - c_L) \\ & + \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\Delta V - \Delta c) \\ & - \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\Delta V - \Delta c) \\ = & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - \Delta V - c_L) \\ & - \left[(1-\alpha)(F_L(\overline{p}_H^D - \Delta V) - F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\Delta V - \Delta c) \\ < & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - \Delta V - c_L)] \end{aligned}$$

so that low quality type strictly prefers to charge $\overline{p}_H^D - \Delta V$ than $\overline{p}_H^D - \epsilon - \Delta V$ which contradicts (25). Next, suppose that $\overline{p}_H^D > \overline{p}_L + \Delta V$. The the disclosing high quality firm's profit at any $p \in (\overline{p}_L + \Delta V, \overline{p}_H^D)$ is

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(p)]\,(p - c_H) - D = \alpha(1-\gamma_H)(\overline{p}_H^D - c_H) - D \tag{26}$$

While setting a price $p - \Delta V > \overline{p}_L$ the low quality firm would make a profit of

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(p)]\,(p - \Delta V - c_L),$$

which using (26) can be rewritten as

$$\alpha(1-\gamma_H)(\overline{p}_H^D - c_H)\frac{p - \Delta V - c_L}{\overline{p}_H^D - c_H},$$

and this is increasing in p, implying low quality firm would gain by deviating to prices larger than $\overline{p}_L$, a contradiction. Thus, if the disclosing high quality firm randomizes, it serves the entire market when the rival firm produces low quality. We now consider two possibilities: the disclosing high quality firm sets a deterministic price or randomizes over prices.

First, consider a partially distortionary mixed disclosure equilibrium where high quality discloses and sets a deterministic price p_H^D. Then, as we have assumed $\underline{p}_H^D < p_H^{ND}$, it must the case that $p_H^D < p_H^{ND}$. As this equilibrium is partially distortionary, low quality type must sell to all buyers with probability one when rival is a non-disclosing high quality firm but share the market with strictly positive probability when rival is a disclosing high quality firm. It is easy to see that in that case the upper bound of low quality price distribution must be exactly equal $p_H^{ND} - \Delta V$ (if it is lower, low quality type would gain by increasing its price above the upper bound). Further, as the disclosing high quality firm facing a low quality rival sells with strictly positive probability, p_H^D $-\Delta V \in (\underline{p}_L, \overline{p}_L)$. As there would be discontinuity in expected quantity sold by low quality firm at price p_H^D $-\Delta V$, the support of low quality price distribution must consist of two disjoint intervals $[\underline{p}_L^1, \overline{p}_L^1]$ and $[\underline{p}_L^2, \overline{p}_L^2]$ with $\overline{p}_L^1 = p_H^D - \Delta V$ and $\overline{p}_L^2 = p_H^{ND} - \Delta V$ (note that if $\overline{p}_L^1 < p_H^D - \Delta V$, low quality firm can gain by deviating to a price slightly above $\overline{p}_L^1$). When a high quality firm discloses, he serves the entire market if the low quality rival sets prices in the upper interval, but not when low quality sets prices in the lower interval as for all $p, \widetilde{p}$ with $p < \overline{p}_L^1 < \widetilde{p}$, $V_L - p > V_H - p_H^D > V_L - \widetilde{p}$. Consider such a candidate equilibrium where high quality chooses p_H^{ND} with probability $1 - \gamma_H$ and price p_H^D with probability γ_H, where $p_H^D < p_H^{ND}$ and low quality randomizes over two disconnected sets $[\underline{p}_L^1, \overline{p}_L^1]$ and $[\underline{p}_L^2, \overline{p}_L^2]$ with probabilities γ_L and $1 - \gamma_L$, respectively. Let's call this an E1 equilibrium. We will show that:

R.1. An E1 equilibrium exists if, and only if,

$$\frac{\alpha}{2}\Delta c < f \leq (1 - \frac{\alpha}{2})\Delta c \tag{27}$$

and

$$\widetilde{D}(f) < D < D^*(f) \tag{28}$$

Next, consider a partially distortionary mixed disclosure equilibrium where when the high quality type discloses it randomizes its pricing decision. As argued above, such a disclosing high quality firm sells to the entire market if the competitor is of low quality. This requires $\overline{p}_H^D \leq \underline{p}_L + \Delta V$. If this behavior constitute part of an equilibrium, it must be the case that $\overline{p}_L = p_H^{ND} - \Delta V$ so that $\pi_L^* = \alpha(1 - \gamma_H)(\overline{p}_L - c_L)$ and $\pi_H^* = \frac{\alpha(1-\gamma_H)}{2}(p_H^{ND} - c_H)$. Let's call this E2 equilibrium. We will show that

R.2 An E2 equilibrium exists if, and only if,

$$f > \left[1 - \frac{\alpha}{2}\right]\Delta c. \tag{29}$$

$$(\lambda - 1)(1 - \alpha)\Delta c < D < \overline{D} = D^*(f) \tag{30}$$

Combining the conditions in **R.1** and **R.2,** we have Lemma 2. In the rest of the proof we prove **R.1** and **R.2.**

Proof of **R.1**

First note that for the range of values of f satisfying (27), $\widetilde{D}(f) = (\lambda - 1) f$ and $D^*(f) = \frac{\alpha}{2}\Delta V + (\lambda - 1) f$ so that (28) is equivalent to the following condition:

$$(\lambda - 1) f < D < \frac{\alpha}{2}\Delta V + (\lambda - 1) f \tag{31}$$

Consider the following partially distortionary mixed disclosure equilibrium where high quality type sets a deterministic price $p_H^D < p_H^{ND}$. A low quality type randomizes over two disconnected intervals of prices $[\underline{p}_L^1, \overline{p}_L^1]$ and $[\underline{p}_L^2, \overline{p}_L^2]$ with probabilities γ_L and $1-\gamma_L$ respectively and a continuous distribution over each interval, where $\overline{p}_L^1 < \underline{p}_L^2$ and $\gamma_L \in [0,1)$, ($\gamma_L < 1$ reflects the partially distortionary nature of this equilibrium). Further,

$$\overline{p}_L^1 = p_H^D - \Delta V,\ \overline{p}_L^2 = p_H^{ND} - \Delta V. \tag{32}$$

The equilibrium expected profits π_H^* and π_L^* then satisfy:

$$\pi_L^* = (\alpha + (1-\alpha)(1-\gamma_L))\,(\overline{p}_L^1 - c_L) = \alpha(1-\gamma_H)(\overline{p}_L^2 - c_L) \tag{33}$$

$$\pi_H^* = \left(\frac{\alpha\gamma_H}{2} + \alpha(1-\gamma_H) + (1-\alpha)(1-\gamma_L)\right)(p_H^D - c_H) - D = \frac{\alpha(1-\gamma_H)}{2}(p_H^{ND} - c_H) \tag{34}$$

Out of equilibrium beliefs regard any firm charging price $\in (\overline{p}_L^2, p_H^{ND})$ without disclosure or price $\in (\overline{p}_L^1, p_H^{ND})$ with disclosure as being of of low type. From Lemma 1(a), low quality type is indifferent between following its equilibrium strategy and deviating to disclosing and charging p_H^D or not disclosing and charging p_H^{ND} :

$$\pi_L^* = \left(\frac{\alpha\gamma_H}{2} + \alpha(1-\gamma_H) + (1-\alpha)(1-\gamma_L)\right)(p_H^D - c_L) - D - f = \frac{\alpha(1-\gamma_H)}{2}(p_H^{ND} - c_L) \tag{35}$$

From (32), (33) and (35) we obtain

$$p_H^{ND} = c_L + 2\Delta V, \overline{p}_L^2 = c_L + \Delta V \tag{36}$$

Using (34), (35) reduces to

$$\gamma_L = \frac{1 - \frac{\alpha}{2} - \frac{f}{\Delta c}}{1-\alpha} \tag{37}$$

and this lies in $[0,1)$ if and only if condition (27) holds. From (33) and (34), we obtain:

$$p_H^D - c_H = \Delta V - \Delta c + \frac{\alpha(1-\gamma_H)\Delta V}{\frac{\alpha}{2} + \frac{f}{\Delta c}} \tag{38}$$

and

$$p_H^D - c_H = \frac{2D + \alpha(1-\gamma_H)\,(2\Delta V - \Delta c)}{2\left(\frac{\alpha(1-\gamma_H)}{2} + \frac{f}{\Delta c}\right)} \tag{39}$$

that simultaneously determine p_H^D and γ_H.These yield:

$$\Delta V\,(\alpha(1-\gamma_H))^2 + \left(\frac{f}{\Delta c} - \frac{\alpha}{2}\right)\Delta V \alpha(1-\gamma_H) + 2\left(\frac{\alpha}{2} + \frac{f}{\Delta c}\right)((\lambda - 1)\,f - D) = 0$$

It is easy to check that for $D = (\lambda-1)\,f, \gamma_H = 1$, that γ_H is decreasing in D, and that at $D = (\lambda-1)\,f + \frac{\alpha\Delta V}{2}\gamma_H = 0$. Thus, (31) or equivalently, condition (28), is necessary and sufficient to ensure that there is some $\gamma_H \in (0,1)$ and therefore (using (38) or (39)) there is some $p_H^D > c_H$ that meets the conditions for an equilibrium. Note that (38) implies that p_H^D is strictly decreasing in γ_H so we have

$$p_H^D \leq c_L + \Delta V + \frac{\alpha\Delta V}{\frac{\alpha}{2} + \frac{f}{\Delta c}} < c_L + 2\Delta V = p_H^{ND}$$

using the first inequality in condition (27). Using (32), we obtain $\overline{p}_L^1$. The values of $\underline{p}_L^1$ and $\underline{p}_L^2$ are determined by:

$$(\alpha + (1-\alpha)(1-\gamma_L))\,(\overline{p}_L^1 - c_L) = (\underline{p}_L^1 - c_L)$$

$$\alpha(1-\gamma_H)(\overline{p}_L^2 - c_L) = ((1-\alpha)(1-\gamma_L) + \alpha(1-\gamma_H))(\underline{p}_L^2 - c_L)$$

and using the previous equations one can check that $\overline{p}_L^1 < \underline{p}_L^2$. The distribution of low quality prices over the two segments $[\underline{p}_L^1, \overline{p}_L^1]$ and $[\underline{p}_L^2, \overline{p}_L^2]$ can now be determined in the usual manner by equalizing the expected profit earned at various prices and it can be shown that the distribution is continuous over each interval. Finally, the out of equilibrium beliefs can be used to show that no type of any firm has a unilateral incentive to deviate. This completes the proof of **R.1.**

Proof of **R.2**

We first consider a version of this equilibrium where the disclosing high quality firm randomizes with no probability mass point. Let $\gamma_H \in (0,1)$ denote the probability of disclosure by a high quality type.As in any mixed disclosure equilibrium, when it does not disclose, the high quality firm charges p_H^{ND} and at this price it only sells in the state where the rival is H type and does not disclose i.e., it sells with probability $\frac{\alpha(1-\gamma_H)}{2}$. Further, a low quality type does not disclose and sells in the state where rival is low quality as well as the state in which rival is high quality and charges p_H^N.In the specific equilibrium we construct, the low quality firm randomizes over an interval $[\underline{p}_L, \overline{p}_L]$ where $\overline{p}_L = p_H^{ND} - \Delta V$. When it discloses, the high quality firm randomizes prices over an interval $[\underline{p}_H^D, \overline{p}_H^D]$ where $\overline{p}_H^D = \underline{p}_L + \Delta V < p_H^{ND}$ i.e., buyers are indifferent between buying low quality at the lower bound of low quality prices $\underline{p}_L$ and the upper bound of high quality prices when the firm discloses. It is easy to see that $\overline{p}_H^D > \overline{p}_L$. At price $\overline{p}_L$ a low quality firm sells with probability $\alpha(1-\gamma_H)$ i.e., only when rival is high quality but does not disclose. At price $\underline{p}_L$ a low quality firm sells with probability $\alpha(1-\gamma_H) + (1-\alpha) = 1 - \alpha\gamma_H$. When it discloses and charges price $\overline{p}_H^D$, a high quality firm also sells with probability $1 - \alpha\gamma_H$,and it sells with probability 1 when it charges $\underline{p}_H^D$. The only restriction on out of equilibrium beliefs is that a firm that does not disclose and charges any price below p_H^{ND} is deemed to be low quality with probability one. Using Lemma 1(a), the low quality firm must be indifferent between charging p_H^{ND} (without disclosing) and sticking to its equilibrium strategy i.e.., $(p_H^{ND} - c_L)\frac{\alpha(1-\gamma_H)}{2} = (\overline{p}_L - c_L)\alpha(1-\gamma_H)$ and this yields:

$$p_H^{ND} = 2\Delta V + c_L \tag{40}$$

The equilibrium profit of the high quality firm is therefore:

$$\pi_H^* = (2\Delta V - \Delta c)\frac{\alpha(1-\gamma_H)}{2} \tag{41}$$

Further:

$$\overline{p}_L = p_H^{ND} - \Delta V = \Delta V + c_L \tag{42}$$

and therefore, the equilibrium profit of the low quality firm is

$$\pi_L^* = \Delta V \alpha(1-\gamma_H) \tag{43}$$

Further, as

$$(\underline{p}_L - c_L)(1 - \alpha\gamma) = \pi_L^* \tag{44}$$

we have

$$\underline{p}_L = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right]\Delta V + c_L \tag{45}$$

The upper bound of prices for a high quality firm that discloses is now:

$$\overline{p}_H^D = \underline{p}_L + \Delta V = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H} + 1\right]\Delta V + c_L \tag{46}$$

which is decreasing in γ_H and converges to $\overline{p}_L$ as $\gamma \to 1$. The profit of the high quality firm when it discloses and charges price $\overline{p}_H^D$ is given by

$$(\overline{p}_H^D - c_H)(1-\alpha\gamma_H) - D \tag{47}$$

and this is equal to π_H^* if, and only if,

$$\gamma_H = \frac{1}{\alpha}\left[\frac{\Delta V - \Delta c(1-\frac{\alpha}{2}) - D}{\Delta V - \frac{\Delta c}{2}}\right] \tag{48}$$

It can be checked that condition (30) is exactly what is needed to ensure that $\gamma_H \in (0,1)$. Indeed, $\gamma_H \to 0$ as $D \to \left[\lambda - (1-\frac{\alpha}{2})\right]\Delta c$ and $\gamma_H \to 1$ as $D \to (\lambda-1)(1-\alpha)\Delta c$. The lower bound $\underline{p}_H^D$ for high quality price when the firm discloses satisfies:

$$(\underline{p}_H^D - c_H) = (\overline{p}_H^D - c_H)(1-\alpha\gamma_H) = \pi_H^* + D \tag{49}$$

and this yields:

$$\begin{aligned}\underline{p}_H^D &= \left[\left(\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}+1\right)\Delta V - \Delta c\right](1-\alpha\gamma_H) + c_H \\ &= 2D + c_H - (1-\alpha)(\lambda-1)\Delta c \end{aligned}\tag{50}$$

The distribution function $F(.)$ for low quality price satisfies:

$$(p_L - c_L)[\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_L))] = \pi_L^* = \alpha(1-\gamma_H)\Delta V, p_L \in [\underline{p}_L, \overline{p}_L] \tag{51}$$

The distribution function $G(.)$ for high quality price when the firm discloses satisfies:

$$\begin{aligned} & (p_H^D - c_H)[(1-\alpha\gamma_H) + \alpha\gamma_H(1-G(p_H^D))] \\ = \; & \pi_H^* + D \qquad (52) \\ = \; & \Delta V(1+\alpha-2\alpha\gamma_H) - \Delta c(1-\alpha\gamma), p_H^D \in [\underline{p}_H^D, \overline{p}_H^D] \qquad (53)\end{aligned}$$

This completes the description of the equilibrium. Next, we show that there is no incentive to deviate from this equilibrium. It is easy to check that given the out of equilibrium belief, no high quality firm can strictly gain by deviating from its equilibrium strategy without disclosing. As the high quality firm gets the entire market at price $\underline{p}_H^D$ when it discloses, it has no incentive to disclose and charge price below $\underline{p}_H^D$. Nor can it gain by charging price above p_H^{ND} (sells zero). It remains to check that a high quality firm cannot gain by disclosing and charging an out of equilibrium price $p_H \in (\overline{p}_H^D, p_H^{ND})$.For any such deviation price p_H, there exists $p_L = p_H - \Delta V \in (\underline{p}_L, \overline{p}_L)$. The deviation profit is given by:

$$\begin{aligned}
& [\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_H - \Delta V))](p_H - c_H) - D \\
= \; & [\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_L))](p_L + \Delta V - c_H) - D \\
= \; & \left[\frac{p_L + \Delta V - c_H}{p_L - c_L}\right] \pi_L^* - D, \text{ using (139)},
\end{aligned}$$

and since $\frac{p_L + \Delta V - c_H}{p_L - c_L}$ is strictly decreasing in p_L (as $\Delta V > \Delta c$) this is

$$\begin{aligned}
\leq \; & \left[\frac{\underline{p}_L + \Delta V - c_H}{\underline{p}_L - c_L}\right] \pi_L^* - D = \left[\underline{p}_L + \Delta V - c_H\right](1 - \alpha\gamma_H) - D \\
= \; & \left[\overline{p}_H^D - c_H\right](1 - \alpha\gamma_H) - D = \pi_H^*
\end{aligned}$$

using (135) and (48). Therefore, the deviation cannot be strictly gainful. We now look at the incentive of a low quality firm to deviate. Whether or not it discloses, the firm will sell zero if it charges price above p_H^{ND} (even if it is thought of as a high quality firm). Given the out of equilibrium beliefs, if a low quality firm deviates without disclosing and charges price $\in (\overline{p}_L, p_H^{ND})$ it will be thought of as a low quality firm and will sell zero. If it charges price $p_L < \underline{p}_L$ (without disclosing) it will be perceived as a low quality firm but may be able to attract more buyers in the state where rival is high quality and discloses; without loss of generality, consider deviation to $p_L \in [\underline{p}_H^D - \Delta V, \underline{p}_L)$. The deviation profit is then given by

$$\begin{aligned}
& [(1-\alpha\gamma_H) + \alpha\gamma_H(1 - G(p_L + \Delta V))](p_L - c_L) \\
= \; & [(1-\alpha\gamma_H) + \alpha\gamma_H(1 - G(p_H^D))](p_H^D - \Delta V - c_L) \text{ where } p_H^D = p_L + \Delta V \\
= \; & \left[\frac{p_H^D - \Delta V - c_L}{p_H^D - c_H}\right](\pi_H^* + D), \text{using (140)}
\end{aligned}$$

and since $\left[\frac{p_H^D - \Delta V - c_L}{p_H^D - c_H}\right]$ is strictly increasing in p_H^D (as $\Delta V > \Delta c$) this is

$$\begin{aligned}
\leq \; & \left[\frac{\overline{p}_H^D - \Delta V - c_L}{\overline{p}_H^D - c_H}\right](\pi_H^* + D) \\
= \; & \left[\overline{p}_H^D - \Delta V - c_L\right](1 - \alpha\gamma_H) \quad \text{(use (135) and (48))} \\
= \; & \Delta V \alpha (1 - \gamma_H), \text{ using (46)} \\
= \; & \pi_L^* \quad \text{(see (131))}
\end{aligned}$$

and thus the deviation is not strictly gainful. We now consider deviation by a low quality firm where it discloses (falsely). If it does so, it cannot gain by charging price below $\underline{p}_H^D$ as it sells to the entire market at that price. So, consider deviation price $p_H^D \in [\underline{p}_H^D, \overline{p}_H^D]$ with disclosure. The deviation profit is given by

$$\begin{aligned}
& [(1-\alpha\gamma_H) + \alpha\gamma_H(1 - G(p_H^D))](p_H^D - c_L) - D - f \\
= \; & \left[\frac{p_H^D - c_L}{p_H^D - c_H}\right](\pi_H^* + D) - D - f, \text{ using (140)}
\end{aligned}$$

and as $\left[\frac{p_H^D - c_L}{p_H^D - c_H}\right]$ is strictly decreasing in p_H^D, this is

$$
\begin{aligned}
&\leq \left[\frac{\underline{p}_H^D - c_L}{\underline{p}_H^D - c_H}\right](\pi_H^* + D) - D - f \\
&= \underline{p}_H^D - c_L - D - f, \text{ using (137)} \\
&= (\underline{p}_H^D - c_H) + \Delta c - D - f = \pi_H^* + D + \Delta c - D - f \\
&= (2\Delta V - \Delta c)\frac{\alpha(1-\gamma_H)}{2} + \Delta c - f = \Delta V \alpha(1-\gamma_H) + \Delta c(1 - \frac{\alpha(1-\gamma_H)}{2}) - f \\
&= \pi_L^* + \Delta c(1 - \frac{\alpha(1-\gamma_H)}{2}) - f
\end{aligned}
\tag{54}
$$

which is $\leq \pi_L^*$ if $f \geq \Delta c(1 - \frac{\alpha}{2} + \frac{\alpha\gamma_H}{2})$ and the latter (using (48)) holds if, and only if :

$$
f \geq \left[1 - \frac{\alpha}{2} + \frac{\lambda - (1-\frac{\alpha}{2})}{2\lambda - 1}\right]\Delta c - \frac{D}{2\lambda - 1} \tag{55}
$$

Thus, under (55), deviation by a low quality type to advertising and charging price in $[\underline{p}_H^D, \overline{p}_H^D]$ is not gainful. Finally, consider deviation by the same firm to disclosing and setting price $p \in (\overline{p}_H^D, p_H^{ND})$. The maximum possible deviation profit (i.e., even if the firm is perceived as high quality with probability 1) is given by:

$$
\begin{aligned}
&[\alpha(1-\gamma_H) + (1-\alpha)(1 - F(p - \Delta V))](p - c_L) - D - f \\
&= [\alpha(1-\gamma_H) + (1-\alpha)(1 - F(p_L))](p_L + \Delta V - c_L) - D - f, \text{ where } p_L = p - \Delta V \\
&= \left[\frac{p_L + \Delta V - c_L}{p_L - c_L}\right]\pi_L^* - D - f, \text{ using (139)},
\end{aligned}
$$

and as $\left[\frac{p_L + \Delta V - c_L}{p_L - c_L}\right]$ is strictly decreasing in p_L, this is

$$
\begin{aligned}
&\leq \left[\frac{\underline{p}_L + \Delta V - c_L}{\underline{p}_L - c_L}\right]\pi_L^* - D - f \\
&= \left[\underline{p}_L + \Delta V - c_L\right](1 - \alpha\gamma) - D - f, \text{ using (132)} \\
&= \left[\overline{p}_H^D - c_L\right](1 - \alpha\gamma) - D - f = \left[\overline{p}_H^D - c_H\right](1 - \alpha\gamma) + \Delta c - D - f \\
&= (\underline{p}_H^D - c_H) + \Delta c - D - f = (\underline{p}_H^D - c_L) - D - f
\end{aligned}
$$

$\leq \pi_L^*$ under condition (55) as was shown above (see arguments following (54)). Thus, we have shown that under (55) is necessary and sufficient for ruling out any incentive to deviate.

Next, we consider a variation of the above equilibrium with the only difference that when a high quality firm discloses, it randomizes prices over an interval $[\underline{p}_H^D, \overline{p}_H^D]$ with probability $1 - \kappa \in (0, 1)$ and chooses a price $\widetilde{p}_H^D \in (c_H, \underline{p}_H^D)$ with probability κ. As before, $\overline{p}_H^D = \underline{p}_L + \Delta V < p_H^{ND}$. Also, as before, at price $\overline{p}_L$ a low quality firm sells with probability $\alpha(1-\gamma_H)$ i.e., only when rival is high quality but does not disclose. At price $\underline{p}_L$ a low quality firm sells with probability$1 - \alpha\gamma_H$. When it discloses and charges price $\overline{p}_H^D$, a high quality firm also sells with probability $1 - \alpha\gamma_H$,and it sells with probability $1-\alpha\kappa\gamma_H$ when it charges $\underline{p}_H^D$. When it discloses and charges $\widetilde{p}_H^D$, the high quality firm sells with probability $\left(1-\frac{\alpha\kappa\gamma_H}{2}\right)$. Only restrictions on out of equilibrium beliefs are that : (a) a firm that does not disclose and charges any price below p_H^{ND} is deemed to be low quality with probability one and (b) any firm disclosing and charging price below $\widetilde{p}_H$ is deemed to be low quality with probability one. From Lemma 1(a), a low quality firm should be indifferent between charging p_H^{ND} without disclosing and sticking to its equilibrium strategy which yields the same expressions for

$$p_H^{ND} = 2\Delta V + c_L. \tag{56}$$

and the equilibrium profit of the high quality firm :

$$\pi_H^* = (2\Delta V - \Delta c)\frac{\alpha(1-\gamma_H)}{2} \tag{57}$$

Further, as before,

$$\overline{p}_L = p_H^N - \Delta V = \Delta V + c_L \tag{58}$$

$$\pi_L^* = \Delta V \alpha(1-\gamma_H) \tag{59}$$

$$\underline{p}_L = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right]\Delta V + c_L \tag{60}$$

$$\overline{p}_H^D = \underline{p}_L + \Delta V = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}+1\right]\Delta V + c_L \tag{61}$$

$$\gamma_H = \frac{1}{\alpha}\left[\frac{\Delta V - \Delta c(1-\frac{\alpha}{2}) - D}{\Delta V - \frac{\Delta c}{2}}\right] \tag{62}$$

and condition (30) is necessary and sufficient for $\gamma_H \in (0,1)$. $\underline{p}_H^D$ satisfies:

$$(\underline{p}_H^D - c_H)(1-\alpha\kappa\gamma_H) = (\overline{p}_H^D - c_H)(1-\alpha\gamma_H) = \pi_H^* + D \tag{63}$$

and this yields:

$$\begin{aligned}\underline{p}_H^D &= [\Delta V(1+\alpha-2\alpha\gamma_H) - \Delta c(1-\alpha\gamma_H)]\frac{1}{(1-\alpha\kappa\gamma_H)} + c_H \\ &= \frac{\pi_H^*+D}{1-\alpha\kappa\gamma_H} + c_H \end{aligned} \tag{64}$$

As before, the distribution function $F(.)$ for low quality price satisfies:

$$(p_L - c_L)[\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_L))] = \pi_L^* = \alpha(1-\gamma_H)\Delta V, p_L \in [\underline{p}_L, \overline{p}_L] \tag{65}$$

The distribution function $G(.)$ for high quality price on the interval $[\underline{p}_H^D, \overline{p}_H^D]$ when the firm discloses satisfies:

$$\begin{aligned}&(p_H^D - c_H)[(1-\alpha\gamma_H) + \alpha\gamma_H(1-\kappa)(1-G(p_H^D))] \\ =\ & \pi_H^* + D \qquad (66)\\ =\ & \Delta V(1+\alpha-2\alpha\gamma_H) - \Delta c(1-\alpha\gamma_H), p_H^D \in [\underline{p}_H^D, \overline{p}_H^D] \qquad (67)\end{aligned}$$

Lemma 2(a) implies that a low quality firm should be indifferent between deviating to direct communication and charging $\widetilde{p}_H^D$ and sticking to its equilibrium strategy i.e..,

$$(\widetilde{p}_H^D - c_L)\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) - D - f = \pi_L^* = \Delta V \alpha(1-\gamma_H) \tag{68}$$

Further, high quality type must be indifferent between choosing $\widetilde{p}_H^D$ while disclosing and other actions in the support of its equilibrium strategy which requires:

$$\begin{aligned}(\widetilde{p}_H^D - c_H)\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) &= \pi_H^* + D & (69)\\ &= (2\Delta V - \Delta c)\frac{\alpha(1-\gamma_H)}{2} + D & (70)\\ &= \Delta V(1+\alpha-2\alpha\gamma_H) - \Delta c(1-\alpha\gamma_H) & (71)\end{aligned}$$

Comparing (69) and (63), we can see that $\widetilde{p}_H^D < \underline{p}_H^D$. From (68)

$$\begin{aligned}\pi_L^* + D + f &= (\widetilde{p}_H^D - c_L)\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right)\\ &= (\widetilde{p}_H^D - c_H)\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) + \Delta c\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right)\\ &= \pi_H^* + D + \Delta c\left(1 - \frac{\alpha\kappa\gamma}{2}\right),\ \text{using (69)}\end{aligned}$$

so that

$$\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) = \frac{1}{\Delta c}(\pi_L^* - \pi_H^* + f) \tag{72}$$

which yields:

$$\kappa = \frac{2}{\alpha\gamma_H}\left[1 - \frac{1}{\Delta c}(\pi_L^* - \pi_H^* + f)\right] \tag{73}$$

Further, using (72) in (69) we have:

$$\widetilde{p}_H^D = \left[\frac{\pi_H^* + D}{\pi_L^* - \pi_H^* + f}\right]\Delta c + c_H \tag{74}$$

We need to ensure that $\kappa \in (0,1)$ which is satisfied as long as

$$(1 - \frac{\alpha\gamma_H}{2})\Delta c - (\pi_L^* - \pi_H^*) < f < \Delta c - (\pi_L^* - \pi_H^*) \tag{75}$$

Note that

$$(\pi_L^* - \pi_H^*) = \frac{\alpha(1-\gamma_H)}{2}\Delta c < \Delta c \tag{76}$$

and (75) is satisfied as long as

$$\left[1 - \frac{\alpha}{2}\right]\Delta c < f < \left[1 - \frac{\alpha}{2} + \frac{\alpha\gamma_H}{2}\right]\Delta c \tag{77}$$

which (using (62)) reduces to

$$\left[1 - \frac{\alpha}{2}\right]\Delta c < f < \left[1 - \frac{\alpha}{2} + \frac{\lambda - (1-\frac{\alpha}{2})}{2\lambda - 1}\right]\Delta c - \frac{D}{2\lambda - 1} \tag{78}$$

Observe that the inequalities in (78) can be written as

$$D + f(2\lambda - 1) < [(2-\alpha)(\lambda - 1) + \lambda]\Delta c \quad (79)$$

$$f > \left[1 - \frac{\alpha}{2}\right]\Delta c \quad (80)$$

This completes the description of the equilibrium. Next, we show that there is no incentive to deviate from this equilibrium. Observe that a high quality firm can never strictly gain by disclosing and choosing a price $p \in (\widetilde{p}_H^D, \underline{p}_H^D)$ as it sells the same expected quantity in that case as it would at $\underline{p}_H^D$. As the high quality firm gets the entire market at price $\widetilde{p}_H^D$ when it discloses, it has no incentive to disclose and charge price below $\widetilde{p}_H^D$. Using identical arguments to that in the first part of the proof, one can check that there is no other gainful deviation for a high quality type. We now look at the incentive of a low quality firm to deviate. Whether or not it discloses, the firm will sell zero if it charges price above p_H^{ND} (even if it is thought of as a high quality firm). Given the out of equilibrium beliefs, if a low quality firm deviates without disclosing and charges price $\in (\overline{p}_L, p_H^{ND})$ it will be thought of as a low quality firm and will sell zero. If it charges price $p_L < \underline{p}_L$ (without advertising) it will be perceived as a low quality firm but may be able to attract more buyers in the state where rival is high quality and discloses. First, consider deviation to $p_L \in [\underline{p}_H^D - \Delta V, \underline{p}_L)$. The deviation profit is then given by

$$\begin{aligned}
& [(1-\alpha\gamma_H) + \alpha\gamma_H(1-\kappa)(1 - G(p_L + \Delta V))](p_L - c_L) \\
= \; & [(1-\alpha\gamma_H) + \alpha\gamma_H(1-\kappa)(1 - G(p_H^D))](p_H^D - \Delta V - c_L) \text{ where } p_H^D = p_L + \Delta V \\
= \; & \left[\frac{p_H^D - \Delta V - c_L}{p_H^D - c_H}\right](\pi_H^* + D) \text{ (using (66))} \\
\leq \; & \left[\frac{\overline{p}_H^D - \Delta V - c_L}{\overline{p}_H^D - c_H}\right](\pi_H^* + D) \text{ (as } \left[\frac{p_H^D - \Delta V - c_L}{p_H^D - c_H}\right] \text{ is strictly increasing in } p_H^D) \\
= \; & \left[\overline{p}_H^D - \Delta V - c_L\right](1-\alpha\gamma_H) \text{ (using (61) and (62))} \\
= \; & \Delta V \alpha(1-\gamma_H) = \pi_L^*, \text{ using (61) and (59)}
\end{aligned}$$

and thus the deviation is not strictly gainful. We now consider deviation by a low quality firm where it discloses (falsely). Consider deviation to price $p_H^D \in [\underline{p}_H^D, \overline{p}_H^D]$ with disclosure. The deviation profit is given by

$$\begin{aligned}
& [(1-\alpha\gamma_H) + \alpha\gamma_H(1-\kappa)(1 - G(p_H^D))](p_H^D - c_L) - D - f \\
= \; & \left[\frac{p_H^D - c_L}{p_H^D - c_H}\right](\pi_H^* + D) - D - f, \text{ using (66)}
\end{aligned}$$

and as $\left[\frac{p_H^D - c_L}{p_H^D - c_H}\right]$ is strictly decreasing in p_H^D, this is

$$
\begin{aligned}
&\leq \left[\frac{\underline{p}_H^D - c_L}{\underline{p}_H^D - c_H}\right](\pi_H^* + D) - D - f \\
&= (\underline{p}_H^D - c_L)(1 - \alpha\kappa\gamma_H) - D - f, \text{ using (62)} \\
&= (\underline{p}_H^D - c_H)(1 - \alpha\kappa\gamma_H) + \Delta c(1 - \alpha\kappa\gamma_H) - D - f \\
&= \pi_H^* + D + \Delta c(1 - \alpha\kappa\gamma_H) - D - f \\
&= (2\Delta V - \Delta c)\frac{\alpha(1-\gamma_H)}{2} + \Delta c(1 - \alpha\kappa\gamma_H) - f \\
&= \Delta V \alpha(1-\gamma_H) + \Delta c(1 - \alpha\kappa\gamma_H - \frac{\alpha(1-\gamma_H)}{2}) - f \\
&= \pi_L^* + \Delta c(1 - \alpha\kappa\gamma_H - \frac{\alpha(1-\gamma_H)}{2}) - f
\end{aligned} \tag{81}
$$

which is $\leq \pi_L^*$ if

$$
\begin{aligned}
f &\geq \Delta c(1 - \alpha\kappa\gamma_H - \frac{\alpha(1-\gamma_H)}{2}) \\
&= \Delta c\left[\frac{2}{\Delta c}\{\pi_L^* - \pi_H^* + f\} - 1 - \frac{\alpha(1-\gamma_H)}{2}\right], \text{ using (73)} \\
&= \Delta c\left[\frac{2}{\Delta c}\{\frac{\alpha(1-\gamma_H)}{2}\Delta c + f\} - 1 - \frac{\alpha(1-\gamma_H)}{2}\right], \text{ using (76)} \\
&= \Delta c\left[\frac{\alpha(1-\gamma_H)}{2} - 1\right] + 2f
\end{aligned}
$$

which reduces to $f \leq \left[1 - \frac{\alpha}{2} + \frac{\alpha\gamma_H}{2}\right]\Delta c$ and the latter follows from condition (78). It is obvious that deviation to disclosing and setting any price in the segment $(\widetilde{p}_H^D, \underline{p}_H^D)$ cannot be strictly gainful as the maximum amount it can sell (even if it is perceived as high quality) is identical to that at $\underline{p}_H^D$.Given restriction (b) on out of equilibrium beliefs, deviating to disclosing and charging a price below $\widetilde{p}_H^D$ will make buyers believe that it is a low quality firm and therefore the firm will and so the deviating firm will earn strictly less profit than it would if it did not disclose and charged the same price; we have already seen that the latter kind of deviation cannot be gainful. Finally, consider deviation by the low quality firm to disclosing and charging price $p_H^D \in (\overline{p}_H^D, p_H^{ND})$. The maximum possible deviation profit (i.e., even if the firm is perceived as high quality with probability 1) is given by:

$$
\begin{aligned}
& [\alpha(1-\gamma_H)+(1-\alpha)(1-F(p-\Delta V))](p-c_L)-D-f \\
= \; & [\alpha(1-\gamma_H)+(1-\alpha)(1-F(p_L))](p_L+\Delta V-c_L)-D-f, \text{ where } p_L=p-\Delta V \\
= \; & \left[\frac{p_L+\Delta V-c_L}{p_L-c_L}\right]\pi_L^*-D-f, \text{ using (65),} \\
\leq \; & \left[\frac{\underline{p}_L+\Delta V-c_L}{\underline{p}_L-c_L}\right]\pi_L^*-D-f, \text{ (as } \left[\frac{p_L+\Delta V-c_L}{p_L-c_L}\right] \text{ is strictly decreasing in } p_L \\
= \; & \left[\underline{p}_L+\Delta V-c_L\right](1-\alpha\gamma_H)-D-f, \text{ using (59) and (60)} \\
= \; & \left[\overline{p}_H^D-c_L\right](1-\alpha\gamma_H)-D-f=\left[\overline{p}_H^D-c_H\right](1-\alpha\gamma_H)+\Delta c(1-\alpha\gamma_H)-D-f \\
= \; & \left[\underline{p}_H^D-c_H\right](1-\alpha\kappa\gamma_H)+\Delta c(1-\alpha\gamma_H)-D-f \\
\leq \; & \left[\underline{p}_H^D-c_H\right](1-\alpha\kappa\gamma_H)+\Delta c(1-\alpha\kappa\gamma_H)-D-f, \text{ as } \kappa\in(0,1) \\
= \; & \left[\underline{p}_H^D-c_L\right](1-\alpha\kappa\gamma_H)-D-f
\end{aligned}
$$

which is $\leq \pi_L^*$ under condition (78) as shown above (see arguments following (81)). Thus, condition (78) is necessary and sufficient for ruling out any incentive to deviate. Finally, note that (29) implies that either (55) or (78) holds. This completes the proof **R.2.** Finally, one can easily show that for both types of equilibria analyzed above, the profit of a high quality firms is not larger than $\frac{\alpha}{2}(1-\gamma_H)(p_H^{ND}-c_H)=\alpha(1-\gamma_H)(\Delta V-\Delta c/2)$, while the profit of a low quality firm is not larger than $\alpha(1-\gamma_H)(\overline{p}_L-c_L)=\alpha(1-\gamma_H)\Delta V$. This completes the proof.

Appendix B: The model with more than two qualities (Section VI.A of main paper)

In this appendix, we analyze extension of the basic model in the main paper to more than two quality types as discussed in Section 7.1 of the main text. Much of the analysis here focuses on the case of three quality types and the results here are indicative of qualitative results one may obtain with finite number of quality types. We also have some results to indicate the difficulties associated with the case of a continuum of types.

1 Three Quality types

Consider a market with two firms, $i = 1, 2$, where each firm's product may be of either high (H), medium (M) or low (L) quality. The true product quality is a firm's type and a firm's type is pure private information: it is not known to the rival firm or to consumers. It is common knowledge that the *ex ante* distribution of quality is independent and identically distributed across firms. The probability that a firm's product is of quality τ is $\alpha_\tau \in (0,1), \tau = H, M, L$ where $\alpha_H + \alpha_M + \alpha_L = 1$. The products of the firms are not differentiated in any dimension other than quality. Firms supply their output at constant unit cost c_s that depends on quality $s \in \{H, M, L\}$, where $c_H > c_M > c_L \geq 0$.

There is a unit mass of identical consumers; each consumer has unit demand. A consumer's valuation of a product of quality s is given by V_s, where

$$V_H > V_M > V_L, V_s > c_s, s = L, M, H.$$

We focus on the more interesting case where higher quality creates more social surplus than low quality, i.e.,

$$V_H - c_H > V_M - c_M > V_L - c_L.$$

A *consumption distortion* is said to occur if consumers buy lower quality even if a higher quality is provided by some firm in the market.

Firms may send direct messages about their product quality. The message is a claim about the true product quality of the firm and specifies a set of qualities in which it lies. Thus, the message sent by a firm is one of the following

$$\{H\}, \{M\}, \{L\}, \{H, M\}, \{M, L\}, \{H, L\}, \{H, M, L\}.$$

A firm may, of course, also abstain from sending a message. As in the main text, there is a direct communication cost D of sending a message (a fixed cost) that is independent of the message sent as well a the true type of the firm. Due to regulation, there is an additional (fixed) cost of lying f that a firm incurs if its true type is not an element of the set of qualities claimed in the message. For example, if a firm sends a message $\{H, M\}$ i.e., "My quality is at least M", then it incurs a direct communication cost D; in addition, if this message is sent by a low quality firm, then the firm incurs a lying cost f i.e., a total cost of $D + f$ for sending such a message; on the other hand if this message is sent by a high or medium quality firm, the total cost is D as their true quality is an element of the message.

As in the main text, we make an assumption to ensure that the market is fully covered. In this case, this requires that

$$V_H - V_M \leq (V_H - V_M)/2. \tag{82}$$

This assumption is very similar to condition (2) in the main text and plays a similar role in assuring that a consumer will always buy.

The extensive form is identical to that outlined in the main text and we focus on symmetric Perfect Bayesian equilibria satisfying the D1 criterion.

Virtually identical arguments as in the main text of the paper rule out the existence of pooling as well as a partially pooling equilibria that satisfy the D1 criterion. In particular, there is no partial pooling partial price signaling equilibrium that satisfies D1 where some types pool and others separate, and consumers are able to make some inferences on quality. The reason is exactly the same as described in the main text, namely the highest of the types that pool will have more incentives to deviate and charge a slightly higher price than the lowest of the types that pool and therefore, consumers will infer that it is the highest of the types that has deviated and will buy at the deviation price if the other firm sticks to its equilibrium strategy making the deviation profitable.

We show that our qualitative main results derived in the main text under the assumption of two quality types continue to hold with three types. In particular:

- Pure price signaling equilibria that satisfy the D1 criterion exist if f is small and/or D is large, and they do not exist when f is large and D is small enough.
- A fully non-distortionary (pure disclosure) equilibrium exists if f is large (strong regulation) and D is small enough. This equilibrium generates higher welfare than a pure price signaling outcome.
- There exists $f_0 > 0$ and for all $f < f_0$, there exists a $D_0(f) > 0$ such that for all $D < D_0(f)$, there exists a pure disclosure equilibrium with full distortion. The welfare generated in such an equilibrium is lower than the pure price signaling outcome (obtained under no or low regulation).
- For large enough f a mixed disclosure equilibrium exists for intermediate values of D. Such an equilibrium generates lower surplus than the worst possible pure price signaling equilibrium if D is close to, but smaller than $(\alpha_L + \alpha_M)(V_M - c_M - (V_L - c_L)) + \frac{\alpha_M}{2}(c_M - c_L)$ and α_H is close to 1 or $V_H - c_H$ is close to $V_M - c_M$.

Together these results show that if D is small, welfare is non-monotonic in f and total surplus is maximized when f is large. In addition, when D is intermediate mixed disclosure equilibria exist that are worse than the pure price signaling equilibria. From the proofs of these results it is intuitive that similar results continue to hold for any finite number of quality grades, but that the characterization is likely to be increasingly cumbersome. We will now discuss these results in more detail and also discuss where the three quality types extension differs from the two quality model.

1.1 Price signaling

To understand the role of direct communication and what regulation can achieve by making false statements more costly, it is important to understand how price signaling works and under what conditions firms abstain from direct communication. To do this, we first focus on a version of the model where firms do not have the option of sending any messages and construct the pure price signaling equilibrium in that framework.

It is clear that (as in the main text) the lowest quality type should randomize as out-of-equilibrium beliefs cannot prevent this type from undercutting. Thus, a low quality firms should randomize their prices over an interval $[\underline{p}_L, \overline{p}_L]$ without mass point.As in the main paper, we construct a pure price signaling equilibrium where consumers always buy low quality if it is around and where a high quality firm will choose a pure strategy. It is easy to see that this implies that $\overline{p}_L = c_L + V_M - V_L$ and $\underline{p}_L = (1-\alpha_L)\overline{p}_L + \alpha_L c_L$ and a distribution function F_L where

$$F_L(p) = 1 - \frac{1-\alpha_L}{\alpha_L}\left(\frac{V_M - V_L}{p - c_L} - 1\right) = \frac{1}{\alpha_L} - \frac{(1-\alpha_L)(V_M - V_L)}{\alpha_L(p - c_L)},$$

while the D1 requirements implies that a medium quality firm sets prices larger than or equal to $\overline{p}_L + V_M - V_L$.Given these considerations, there are a few possibilities left. First, medium quality chooses a pure strategy and consumers buy medium quality if both medium and high quality are around. Second, medium quality randomizes and consumers buy medium quality if both medium and high quality are around. Third, medium quality chooses a pure strategy and a fraction of consumers buy high quality if both medium and high quality are around. The next proposition shows that in a world where firms cannot directly communicate their quality (or it is too costly to do so) a pure price equilibrium always exists. The proof is constructive and shows that the equilibrium can be of any of the three possibilities mentioned above. Different from the two quality case a pure price signaling equilibrium does not need to be unique and for certain parameter values, multiple pure price signaling equilibrium exist.

Proposition 3 *A symmetric pure price signaling equilibrium exists if firms cannot directly communicate their quality. In such an equilibrium, consumers always buy low quality if a low quality firm is present, a majority of consumers buy from a medium quality firm when its rival is high quality, low quality firms randomize over prices and high quality firms choose a deterministic price.*

Proof. Let p_τ^{ND} denote the price charged by type τ in an equilibrium where type τ follows a pure strategy. The proof is constructive. A high type firm chooses a deterministic price p_H^{ND}. As mentioned above, a low quality firms should randomize their prices over an interval $[\underline{p}_L, \overline{p}_L]$ without mass point. We start showing the condition under which a pure price signaling equilibrium exists where medium quality chooses a pure strategy p_M^{ND} and consumers buy medium quality if both medium and high quality are around. The latter requires

$$p_M^{ND} < p_H^{ND} - (V_H - V_M). \tag{83}$$

To satisfy the D1 requirement and have that consumers believe quality to be medium if a price $p \in (p_M^{ND}, p_H^{ND})$ is observed a medium quality firm has to be indifferent between charging p_M^{ND} and p_H^{ND}. This implies

$$\left(\frac{\alpha_M}{2} + \alpha_H\right)\left(p_M^{ND} - c_M\right) = \frac{\alpha_H}{2}\left(p_H^{ND} - c_M\right) \tag{84}$$

which yields

$$p_H^{ND} = c_M + \frac{\alpha_M + 2\alpha_H}{\alpha_H}\left(p_M^{ND} - c_M\right) \tag{85}$$

For $p > p_H^{ND} - (V_H - V_M)$ consumers will not buy. For other deviations to prices $p \in (p_M^{ND}, p_H^{ND})$ *not* to be profitable for high and medium quality types, it is required that $\frac{\alpha_H}{2}\left(p_H^{ND} - c_H\right) \geq \alpha_H\,(p - c_H)$ and $\left(\frac{\alpha_M}{2} + \alpha_H\right)\left(p_M^{ND} - c_M\right) \geq \alpha_H\,(p - c_M)$ for all $p_M^{ND} < p < p_H^{ND} - (V_H - V_M)$ which hold iff

$$\left(\frac{\alpha_M}{2} + \alpha_H\right)\left(p_M^{ND} - c_M\right) \geq \alpha_H\left(p_H^{ND} - (V_H - V_M) - c_M\right) \tag{86}$$

and

$$\frac{\alpha_H}{2}\left(p_H^{ND}-c_H\right) \geq \alpha_H\left(p_H^{ND}-(V_H-V_M)-c_H\right) \tag{87}$$

Using (84), (86) can be written as

$$\frac{\alpha_H}{2}\left(p_H^{ND}-c_M\right) \geq \alpha_H\left(p_H^{ND}-(V_H-V_M)-c_M\right) \tag{88}$$

Observe that (88) implies (87). So, we only need to ensure that (88) holds to rule out deviations to $p \in (p_M^{ND}, p_H^{ND})$ and (88) reduces to

$$p_H^{ND} \leq 2(V_H-V_M)+c_M. \tag{89}$$

Note that if this condition holds (82) guarantees that $p_H^{ND} \leq V_H$. Requirement (83) then also implies that $p_M^{ND} \leq V_M$.

For a low quality firm not to have an incentive to imitate medium quality (and for consumers to believe that quality is low if they observe a price $p \in (\overline{p}_L, p_M^{ND})$), we should have $(\alpha_M+\alpha_H)(\overline{p}_L-c_L) = \left(\frac{\alpha_M}{2}+\alpha_H\right)\left(p_M^{ND}-c_L\right)$ and $p_M^{ND} = \overline{p}_L+V_M-V_L$. Together they imply that

$$\overline{p}_L = c_L+\left(\frac{\alpha_M+2\alpha_H}{\alpha_M}\right)(V_M-V_L) \tag{90}$$

so that

$$p_M^{ND} = c_L+2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L). \tag{91}$$

As $p_M^{ND} \leq V_M$, it follows that $\overline{p}_L \leq V_L$. From (85) and (91):

$$p_H^{ND} = c_M+\frac{\alpha_M+2\alpha_H}{\alpha_H}\left(2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right) \tag{92}$$

One can derive $\underline{p}_L$ in the usual manner

$$\underline{p}_L = (1-(\alpha_M+\alpha_H))c_L+(\alpha_M+\alpha_H)\overline{p}_L$$

it is clear that $c_L < \underline{p}_L < \overline{p}_L$; further, $p_M^{ND} > c_M$ and $p_H^{ND} > c_H$. We conclude that the above is a pure price signaling equilibrium if both (89) and (83) hold. Given the price expressions derived in (91) - (92) and using (85), these two conditions can be expressed in terms of conditions on the exogenous parameters as

$$\frac{\alpha_M+\alpha_H}{\alpha_H}\left(2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right) \geq (V_H-V_M) \tag{93}$$

and

$$\frac{\alpha_M+2\alpha_H}{\alpha_H}\left(2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right) \leq 2(V_H-V_M) \tag{94}$$

which can be combined into

$$\frac{\alpha_H(V_H-V_M)}{\alpha_M+\alpha_H} \leq \left(2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right) \leq \frac{2\alpha_H(V_H-V_M)}{\alpha_M+2\alpha_H}. \tag{95}$$

We now consider alternative pure price signaling equilibria and see for which values of the exogenous parameters they hold. In particular, let us next consider that consumers are indifferent between buying medium and high quality if both are around and some consumers buy low, while other buy high quality. Thus, we should have that

$$p_H^{ND} = p_M^{ND} + (V_H - V_M),$$

while to make a medium quality firm being indifferent between charging p_M^{ND} and p_H^{ND} we should have that a fraction β of consumers buys high quality if both medium and high quality are around, where β solves

$$\left(\frac{\alpha_M}{2} + \alpha_H(1-\beta)\right)\left(p_M^{ND} - c_M\right) = \left(\frac{\alpha_H}{2} + \alpha_M\beta\right)\left(p_M^{ND} + (V_H - V_M) - c_M\right),$$

or

$$(\alpha_M + \alpha_H)\left(\frac{1}{2} - \beta\right)\left(p_M^{ND} - c_M\right) = \left(\frac{\alpha_H}{2} + \alpha_M\beta\right)(V_H - V_M). \tag{96}$$

Note that there cannot be an equilibrium where β is larger than or equal to 0.5 as in that case demand is larger at p_H^{ND} and as $p_H^{ND} > p_M^{ND}$ firms would not have an incentive to set p_M^{ND}. Thus, even if the consumption distortion is smaller in this second type of pure price signaling equilibrium, it is still substantial.

Of course, the change in consumer behavior will also affect the relationship between $\overline{p}_L$ and p_M^{ND}. For a low quality firm not to have an incentive to imitate medium quality (and for consumers to believe that quality is low if they observe a price $p \in (\overline{p}_L, p_M^{ND})$), we now should have that $(\alpha_M + \alpha_H)(\overline{p}_L - c_L) = \left(\frac{\alpha_M}{2} + \alpha_H(1-\beta)\right)\left(p_M^{ND} - c_L\right)$, which because $p_M^{ND} = \overline{p}_L + V_M - V_L$ remains to hold, implies that

$$\overline{p}_L = c_L + \left(\frac{\alpha_M + 2(1-\beta)\alpha_H}{\alpha_M + 2\beta\alpha_H}\right)(V_M - V_L)$$

and

$$p_M^{ND} = c_L + 2\left(\frac{\alpha_M + \alpha_H}{\alpha_M + 2\beta\alpha_H}\right)(V_M - V_L).$$

As (89) continues to hold, (82) continues to imply that $p_H^{ND} \leq V_H$. As the construction uses the fact that $p_H^{ND} = p_M^{ND} + (V_H - V_M)$ and $p_M^{ND} = \overline{p}_L + V_M - V_L$ it is also implies that $p_M^{ND} \leq V_M$ and $\overline{p}_L \leq V_L$. In addition, like before, it is easy to see that all types charge prices above their respective marginal cost.

The only constraint that should hold is that $\beta \geq 0$. ($\beta \leq 1$ is guaranteed by (96) and the discussion below that equation). As p_M^{ND} is decreasing in β, the LHS of (96) is decreasing in β, while the RHS is increasing in β. Thus, (96) has a solution $\beta \geq 0$ if, and only if at $\beta = 0$ the LHS of (96) is larger than the RHS. This is the case if, and only if,

$$(\alpha_M + \alpha_H)\left(2\left(\frac{\alpha_M + \alpha_H}{\alpha_M}\right)(V_M - V_L) - (c_M - c_L)\right) > \alpha_H(V_H - V_M),$$

which is exactly the same condition as (93). Moreover, we do not need a condition like (89) or in terms of exogenous parameters (94), as by construction we have that $p_H^{ND} = p_M^{ND} + (V_H - V_M)$ so that there are no prices $p \in (p_M^{ND}, p_H^{ND})$ where consumers will buy if they believe quality is high.

Thus, we conclude that this equilibrium exists whenever the first type of equilibrium exists, but that the range of parameter values is larger.

Finally, consider a pure price signaling equilibrium where medium quality randomizes and consumers buy medium quality if both medium and high quality are around. So, suppose that medium quality chooses p_M^{ND} with probability γ and that with the remaining probability they randomize over an interval with $p_H^{ND} - (V_H - V_M)$ as an upper bound. (Note that (89) is derived from the consideration that in the first equilibrium we constructed a medium type should not have an incentive to deviate to $p_H^{ND} - (V_H - V_M)$, but if (89) he certainly will have such an incentive.) To make that consumers believe quality to be medium for all out-of-equilibrium prices $p \in (p_M^{ND}, p_H^{ND})$ and have that the medium type does not want to mimic the high quality price, the usual indifference condition now writes as

$$\left(\frac{\alpha_M \gamma}{2} + \alpha_M(1-\gamma) + \alpha_H\right)\left(p_M^{ND} - c_M\right) = \alpha_H[p_H^{ND} - (V_H - V_M) - c_M] = \frac{\alpha_H}{2}\left(p_H^{ND} - c_M\right).$$

From this it follows that

$$p_H^{ND} = 2(V_H - V_M) + c_M$$

and

$$p_M^{ND} = c_M + \frac{\alpha_H(V_H - V_M)}{\alpha_H + \alpha_M(1-\gamma/2)}.$$

Given (82) it follows from the equilibrium construction that $p_H^{ND} \leq V_H$ and that all prices that a medium quality type will possibly choose are smaller than V_M.

On the other hand, the usual indifference condition for the low quality type now can be written as

$$(\alpha_M + \alpha_H)\left(\overline{p}_L - c_L\right) = \left(\frac{\alpha_M \gamma}{2} + \alpha_M(1-\gamma) + \alpha_H\right)\left(p_M^{ND} - c_L\right),$$

which because $p_M^{ND} = \overline{p}_L + V_M - V_L$ continues to hold, implies that

$$\overline{p}_L = c_L + \frac{\alpha_M(2-\gamma) + 2\alpha_H}{\alpha_M \gamma}(V_M - V_L)$$

and

$$p_M^{ND} = c_L + 2\frac{\alpha_M + \alpha_H}{\alpha_M \gamma}(V_M - V_L).$$

It follows that γ is determined by

$$c_M + \frac{\alpha_H(V_H - V_M)}{\alpha_H + \alpha_M(1-\gamma/2)} = c_L + 2\frac{\alpha_M + \alpha_H}{\alpha_M \gamma}(V_M - V_L).$$

As (i) for γ close to 0 the LHS of this equation is smaller than the RHS and (ii) the LHS of this equation is increasing in γ, while the RHS of this equation is decreasing in γ, a solution to this equation exist for $0 < \gamma < 1$ if for $\gamma = 1$ the LHS is larger than the RHS, i.e., if

$$c_M + \frac{\alpha_H(V_H - V_M)}{\alpha_H + \alpha_M/2} > c_L + 2\frac{\alpha_M + \alpha_H}{\alpha_M}(V_M - V_L),$$

which can be rewritten as

$$\frac{\alpha_H + \alpha_M/2}{\alpha_H}(c_M - c_L) + (V_H - V_M) > \frac{\alpha_M + \alpha_H}{\alpha_M}\frac{2\alpha_H + \alpha_M}{\alpha_H}(V_M - V_L),$$

or

$$\frac{\alpha_M + 2\alpha_H}{2\alpha_H}\left(2\left(\frac{\alpha_M + \alpha_H}{\alpha_M}\right)(V_M - V_L) - (c_M - c_L)\right) < (V_H - V_M),$$

which is exactly equal to condition (94) of the first equilibrium. Again, as there are no other conditions to be satisfied (as it is easy to see that all prices are above their respective marginal costs), we conclude that this equilibrium exists whenever the first type of equilibrium exists, but that the range of parameter values is larger.

The three equilibria taken together cover all the possible parameter values. If both inequalities in (95) hold true, then all three types of pure price signaling equilibria exist; if the left inequality does not hold, then the second type of equilibrium exists, while the third type of equilibrium exists, if the right inequality of (95) does not hold. ■

It is clear that the above equilibrium construction can also be applied if there are more than three qualities. Depending on the parameters, for each pair of adjacent qualities one has to determine whether consumers always buy the lowest of these qualities if only these two qualities are around or that a fraction of consumers buy the highest of the two adjacent quality levels. This makes the actual equilibrium determination somewhat cumbersome, but the conceptual analysis is not different from the three qualities case considered here.

It is clear that the welfare loss in any pure price signaling equilibrium equals

$$\begin{aligned}&2\alpha_L\alpha_M(V_M - c_M - (V_L - c_L)) + 2\alpha_L\alpha_H(V_H - c_H - (V_L - c_L))\\ &+2\alpha_H\alpha_M(1-\beta)(V_H - c_H - (V_M - c_M)),\end{aligned}$$

where $\beta = 0$ in the equilibria where consumers always buy medium quality if medium and high quality are around.

Next, we allow for the possibility of direct communication as outlined in our model. For which values of f and D does the pure price signaling outcome outlined above continue to be an equilibrium?

It is easy to see that if D is large enough, no firm will send a direct message as any gain from doing so in terms of gaining market share will be outweighed by the direct cost of communication. In what what follows assume that D is below a critical level (like $\overline{D}$ derived explicitly in the main paper for the two types case). We can then follow a very similar analysis as in the proof of Proposition 2 of the main paper. Let us focus on the first type of pure price signaling equilibrium where consumers buy medium quality if both medium and high quality are around. The analysis is in three steps. We first focus on prices $\widehat{p} \in (\overline{p}_L, p_M^{ND})$ and consider the incentives of medium and low types to deviate by *directly communicating they sell medium quality* and charging prices in this interval. We follow the proof of Proposition 2 and determine the prices for which consumers should believe that the deviation should come from a medium quality firm and consider whether a medium quality firm then has an incentive to deviate. The second step is then to show that *for this interval of prices* there is no loss of generality in focussing on low and medium quality firms and not considering high quality. The third and final step is to focus on prices $\widehat{p} \in (p_M^{ND}, p_H^{ND})$ and consider the incentives of medium and high types to deviate by *directly communicating they sell high quality* and charging prices in this interval.

For the first step, let $q_M(\widehat{p}), q_L(\widehat{p})$ be the expected quantity that an M and an L type firm must sell respectively in order to be indifferent between this deviation and not deviating from their equilibrium strategies:

$$q_M(\widehat{p}) = \frac{\pi_M^* + D}{(\widehat{p} - c_M)}, q_L(\widehat{p}) = \frac{\pi_L^* + D + f}{(\widehat{p} - c_L)}$$

Note that $q_M(\widehat{p}) \geq q_L(\widehat{p})$ if, and only if,

$$\frac{\widehat{p} - c_L}{\widehat{p} - c_M} \geq \frac{\pi_L^* + D + f}{\pi_M^* + D} \tag{97}$$

and as the left hand side of (97) is continuous and strictly decreasing in $\widehat{p}$, (97) holds for all $\widehat{p} \in (\overline{p}_L, p_M^{ND})$ if, and only if, (97) holds for $\widehat{p} = p_M^{ND}$ which (using the fact that $\pi_M^* = \left(\frac{\alpha_M}{2} + \alpha_H\right)\left(p_M^{ND} - c_M\right)$, $p_M^{ND} = c_L + 2\left(\frac{\alpha_M + \alpha_H}{\alpha_M}\right)(V_M - V_L)$ .and $\pi_L^* = (\alpha_M + \alpha_H)\left(\frac{\alpha_M + 2\alpha_H}{\alpha_M}\right)(V_M - V_L)$) reduces to

$$f \leq \frac{D}{2\left(\frac{\alpha_M + \alpha_H}{\alpha_M}\right)\frac{V_M - V_L}{c_M - c_L} - 1}, \tag{98}$$

which is very similar to equation (16) in the proof of Proposition 2 in the main paper. Under (98), $q_H(\widehat{p}) \geq q_L(\widehat{p})$ so that the D1 refinement is consistent with beliefs associating direct communication and price $\widehat{p} \in (\overline{p}_L, p_H^{ND})$ as coming infinitely more likely from an $L-$type firm than from an $H-$type firm.

As a second step, consider whether a high quality firm has an incentive to set prices in this interval and communicate quality directly. Let $q_H(\widehat{p})$ be the expected quantity that an H type firm must sell in order to be indifferent between deviating to directly communicating quality and setting prices $\widehat{p} \in (\overline{p}_L, p_M^{ND})$ and not deviating from their equilibrium strategies, i.e.,

$$q_H(\widehat{p}) = \frac{\pi_H^* + D}{(\widehat{p} - c_H)}.$$

We will argue that whenever (83) holds, we have that $q_H(\widehat{p}) \geq q_M(\widehat{p})$ for all $\widehat{p} \in (\overline{p}_L, p_M^{ND})$ so that $q_H(\widehat{p}) \geq q_L(\widehat{p})$ if $q_M(\widehat{p}) \geq q_L(\widehat{p})$. It is easy to see that $\frac{\pi_H^* + D}{(\widehat{p} - c_H)} \geq \frac{\pi_M^* + D}{(\widehat{p} - c_M)}$ if, and only if,

$$(\widehat{p} - c_H)(\pi_H^* - \pi_M^*) + (c_H - c_M)D + (c_H - c_M)\,\pi_H^* \geq 0,$$

which using the expressions for equilibrium profits can be simplified to

$$-(\widehat{p} - c_H)\frac{\alpha_H}{2} + D + \frac{\alpha_H}{2}\left(p_H^{ND} - c_H\right) \geq 0$$

and

$$(p_H^{ND} - \widehat{p})\frac{\alpha_H}{2} + D \geq 0,$$

which is obviously the case. Thus, we conclude the first two steps by arguing that if (98) holds, both $q_M(\widehat{p}) \geq q_L(\widehat{p})$ and $q_H(\widehat{p}) \geq q_L(\widehat{p})$ so that the D1 refinement is consistent with beliefs associating direct communication and price $\widehat{p} \in (\overline{p}_L, p_H^{ND})$ as coming from an $L-$type firm with probability one.

Finally, consider the third and final step and prices $\widehat{p} \in (p_M^{ND}, p_H^{ND})$ together with the *communication they sell high quality.* The quantities $q_M(\widehat{p}), q_H(\widehat{p})$ to be indifferent between this deviation and not deviating from their equilibrium strategies are now given by:

$$q_M(\widehat{p}) = \frac{\pi_M^* + D + f}{(\widehat{p} - c_M)}, q_H(\widehat{p}) = \frac{\pi_H^* + D}{(\widehat{p} - c_H)}$$

Note that $q_H(\widehat{p}) \geq q_M(\widehat{p})$ if, and only if,

$$\frac{\widehat{p}-c_M}{\widehat{p}-c_H} \geq \frac{\pi_M^* + D + f}{\pi_H^* + D}$$

and as the left hand side is continuous and strictly decreasing in $\widehat{p}$, this inequality holds for all $\widehat{p} \in (p_M^{ND}, p_H^{ND})$ if, and only if, it holds for $\widehat{p} = p_H^{ND}$ which (using the fact that $\pi_M^* = \left(\frac{\alpha_M}{2} + \alpha_H\right)\left(p_M^{ND} - c_L\right)$, $p_M^{ND} = c_L + 2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M - V_L)$ and $\pi_H^* = \frac{\alpha_H}{2}\left(p_H^{ND} - c_H\right)$ can be rewritten as

$$\frac{p_H^{ND}-c_M}{p_H^{ND}-c_H} \geq \frac{\frac{\alpha_H}{2}\left(p_H^{ND}-c_M\right)+D+f}{\frac{\alpha_H}{2}\left(p_H^{ND}-c_H\right)+D},$$

which reduces to $D\left(c_H - c_M\right) \geq \left(p_H^{ND} - c_H\right) f$ and

$$f \leq \frac{D}{\frac{\alpha_M+2\alpha_H}{\alpha_H}\left[2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)\frac{V_M-V_L}{c_H-c_M} - \frac{c_M-c_L}{c_H-c_M}\right]-1}, \tag{99}$$

which (again) is similar to and condition outlined in the proof of Proposition 2 in the main paper. If (99) holds, consumers do not buy at prices $\widehat{p} \in (p_M^{ND}, p_H^{ND})$ even if they are communicating by claiming high quality as consumers believe they are more likely to be chosen by medium quality firms, we do not need to check whether it is even more likely that they are set by low quality firms. We conclude that for any given D (below a prohibitive level), if f is small enough a pure price signaling equilibrium exist. Though we do not work it out explicitly, for small enough D it is easy to see that a pure price signaling equilibrium will not exist if f is large enough as high and/or medium type firms will have an incentive to communicate directly and truthfully and gain market share from the rival firm. To summarize:

Proposition 4 *There exists $\overline{D}$ such that a pure price signaling equilibrium (as outlined in the previous proposition) exists if and only if (i) $D \geq \overline{D}$ or (ii) $D < \overline{D}$ and f is relatively small.*

1.2 Direct communication with Strong Regulation: High f, Low D.

We now consider the case of strong regulation where the penalty for lying is high enough and the cost of sending a truthful message is small. We construct an equilibrium where high and medium quality types send different but truthful messages (for instance, high quality sends $\{H\}$ and medium sends either $\{M\}$ or $\{H, M\}$). If f is large enough, no firm wants to imitate the message of the other types. We will look for the conditions that needs to be satisfied for elimination of the consumption distortion: consumers buy the highest quality that is provided in the market.

It is clear that in such an equilibrium both medium and high quality must make some profit, while low quality earns zero profits. Thus, it must be that $p_L = c_L$ and $\overline{p}_M = c_L + V_M - V_L$. It follows that the medium quality type randomizes over the price interval $[\underline{p}_M, \overline{p}_M]$, where $\underline{p}_M$ is such that $(\alpha_L + \alpha_M)\left(\underline{p}_M - c_M\right) = \alpha_L\left(\overline{p}_M - c_M\right)$, or $\underline{p}_M = \frac{\alpha_L(c_L+V_M-V_L)+\alpha_M c_M}{\alpha_L+\alpha_M}$.

Finally, we must determine the price region over which high quality can randomize. To have that consumers always buy high quality when it is around, we must have that

$$V_H - \overline{p}_H \geq \min\{V_M - \underline{p}_M, V_L - c_L\}.$$

using the definition of $\underline{p}_M$, it easily follows that $V_M - \underline{p}_M < V_L - c_L$. Thus, for an equilibrium to exist without consumption distortion we need $\overline{p}_H = \underline{p}_M + V_H - V_M$.

The high quality firm's profit is then given by $(\alpha_L + \alpha_M)(\underline{p}_M + V_H - V_M - c_H) - D$. Does a high quality firm wants to deviate to higher prices? Deviating to prices $p \in (\underline{p}_M + V_H - V_M, \overline{p}_M + V_H - V_M)$ gives a profit of $(\alpha_L + \alpha_M(1 - F_M(p - (V_H - V_M)))(p - c_H)$. From the indifference condition of the medium quality type over $p \in (\underline{p}_M + V_H - V_M, \overline{p}_M + V_H - V_M)$ it follows that $(\alpha_L + \alpha_M(1 - F_M(p - (V_H - V_M)))(p - c_M)$ is constant. As we can write

$$\begin{aligned} & (\alpha_L + \alpha_M(1 - F_M(p - (V_H - V_M)))(p - c_H) \\ = \; & (\alpha_L + \alpha_M(1 - F_M(p - (V_H - V_M)))(p - c_M) + (\alpha_L + \alpha_M(1 - F_M(p - (V_H - V_M)))(c_M - c_H) \end{aligned}$$

it follows that the high quality's profit is decreasing in p over this interval. For prices $p > \overline{p}_M + V_H - V_M = c_L + V_H - V_L$ the profit of the high quality firm equals 0. Thus, we have that the high quality firm wants to randomize over prices $[\underline{p}_H, \underline{p}_M + V_H - V_M]$, where

$$\begin{aligned} \underline{p}_H &= c_H + (\alpha_L + \alpha_M)(\underline{p}_M + V_H - V_M - c_H) \\ &= c_H + \alpha_L(c_L + V_M - V_L) + \alpha_M c_M + (\alpha_L + \alpha_M)(V_H - V_M - c_H) \\ &= (1 - \alpha_L)c_H + \alpha_L c_L + (\alpha_L + \alpha_M) V_H - \alpha_L V_L - \alpha_M V_M \\ &= c_H + \alpha_L(V_H - c_H - (V_L - c_L)) + \alpha_M(V_H - V_M). \end{aligned}$$

For which values of D and f does the above constitute an equilibrium? First, as the low quality firm has most incentives to deviate, consumers will believe that any out-of equilibrium deviation comes from a low quality firm and therefore will not buy from such a deviating firm. Thus, we only need to consider two constraints. First, medium and high quality firms must make positive profits. This implies that

$$D \leq \min\{\alpha_L(c_L + V_M - V_L - c_M), (\alpha_L + \alpha_M)\left(\frac{\alpha_L(c_L + V_M - V_L) + \alpha_M c_M}{\alpha_L + \alpha_M} + V_H - V_M - c_H\right)\}.$$

As the second term in brackets equals $\alpha_L(c_L + V_M - V_L) + \alpha_M c_M + (\alpha_L + \alpha_M)(V_H - V_M - c_H) = \alpha_L(V_H - c_H - (V_L - c_L)) + \alpha_M(V_H - c_H - (V_M - c_M))$ it follows that the first term, the medium quality's expected operating profits is the smallest of the two terms. Therefore if D starts increasing from small values, it is first the medium quality type that stops always directly communicating quality.

Second, lower quality types should not have an incentive to imitate higher quality types. As with two types, if lower quality types imitate higher quality types it is always best to imitate the lowest price in the support of the higher type they want to imitate. Thus, for the low quality firms we need that $D + f \geq \max\{(\alpha_L + \alpha_M)\left(\frac{\alpha_L(c_L + V_M - V_L) + \alpha_M c_M}{\alpha_L + \alpha_M} - c_L\right), c_H + \alpha_L(V_H - c_H - (V_L - c_L)) + \alpha_M(V_H - V_M) - c_L\}$, which (as the second term is larger than the first) reduces to

$$D + f \geq \alpha_L(V_H - V_L) + \alpha_M(V_H - V_M) + (1 - \alpha_L)(c_H - c_L).$$

For the medium quality firms we need that $f \geq c_H + \alpha_L(V_H - c_H - (V_L - c_L)) + \alpha_M(V_H - V_M) - c_M$, which is always satisfied if the condition for the low types is satisfied.

Thus, we conclude that the above strategies constitute an equilibrium if

$$D + f \geq \alpha_L(V_H - V_L) + \alpha_M(V_H - V_M) + (1 - \alpha_L)(c_H - c_L) \tag{100}$$

and

$$D \leq \alpha_L(V_M - c_M - (V_L - c_L)). \tag{101}$$

Note that if D is small enough so that (101) holds, then (100) is always satisfied for f large enough.

The welfare loss in this non-distortionary equilibrium is $2D\{(1-\alpha_L)\alpha_L+(1-\alpha_L)^2\} = 2(1-\alpha_L)D$.

The welfare loss in a pure price signaling equilibrium is at least

$$\begin{array}{l} 2\alpha_L\alpha_M(V_M - c_M - (V_L - c_L)) + 2\alpha_L\alpha_H(V_H - c_H - (V_L - c_L)) \\ +2\alpha_H\alpha_M(1-\beta)(V_H - c_H - (V_M - c_M)). \end{array}$$

It is easy to check that under (101), the welfare loss in the non-distortionary equilibrium is at most

$$\begin{array}{ll} & 2(1-\alpha_L)\alpha_L(V_M - c_M - (V_L - c_L)) \\ < & 2\alpha_L\alpha_M(V_M - c_M - (V_L - c_L)) + 2\alpha_L\alpha_H(V_H - c_H - (V_L - c_L)). \end{array}$$

Thus, the pure disclosure non-distortionary equilibrium welfare dominates any pure price signaling equilibrium.

To sum up:

Proposition 5 *If the cost of directly communicating truthful messages is small, sufficiently strong regulation (high f) generates an outcome where better quality types disclose truthfully, all consumption distortion is eliminated and net welfare is higher than under pure price signaling.*

1.3 Intermediate Regulation: Pure Disclosure with Full Distortion

Assume that the conditions for a fully distortionary pure price signaling equilibrium are satisfied.

We look for a fully distortionary D1 equilibrium where M, H types disclose for sure. L type does not; quantities sold are exactly identical to that in the fully distortionary pure price signaling equilibrium. We aim to outline conditions under which such an equilibrium exists for D small enough.

The H type of each firm charges a deterministic price p_H^D and sells with probability $\alpha_H/2$. The M type charges a deterministic price p_M^D where

$$p_H^D \geq (V_H - V_M) + p_M^D \tag{102}$$

and sells with probability $(\alpha_H + \frac{\alpha_M}{2})$. The L type of each firm randomizes in price over an interval $[\underline{p}, \overline{p}_L]$ as in the pure price signaling equilibrium where

$$\overline{p}_L = p_M^D - (V_M - V_L) \tag{103}$$

and at price $\overline{p}_L$, L type of each firm sells with probability $(\alpha_M + \alpha_H)$.

Out-of-equilibrium beliefs are such that any firm that does not send a message and charges price in $(\overline{p}_L, p_M^D]$ is of L type with probability one; any firm that does not send a message and charges price in $(p_M^D, p_H^D]$ is of H type with probability zero; any firm that sends a message and sets price in $(\overline{p}_L, p_M^D)$ is of L type with probability one, any firm that sends a message and sets price in (p_M^D, p_H^D) is of M type with probability one.

D1 requires that the L-type's incentive to imitate the $M-$type is binding: for a low quality firm to be indifferent to imitate medium quality, we should have

$$(\alpha_M+\alpha_H)(\overline{p}_L-c_L)=\left(\frac{\alpha_M}{2}+\alpha_H\right)(p_M^D-c_L)-(f+D)=\left(\frac{\alpha_M}{2}+\alpha_H\right)(\overline{p}_L+V_M-V_L-c_L)-(f+D), \tag{104}$$

or

$$\frac{\alpha_M}{2}(\overline{p}_L-c_L)=\left(\frac{\alpha_M}{2}+\alpha_H\right)(V_M-V_L)-(f+D),$$

or

$$\overline{p}_L=c_L+\left(\frac{\alpha_M+2\alpha_H}{\alpha_M}\right)(V_M-V_L)-\frac{2(f+D)}{\alpha_M} \tag{105}$$

so that

$$p_M^D=c_L+2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-\frac{2(f+D)}{\alpha_M} \tag{106}$$

In a D1 equilibrium an M-type's incentive to imitate H-type is also binding:

$$(p_M^D-c_M)\left(\frac{\alpha_M}{2}+\alpha_H\right)=(p_H^D-c_M)\frac{\alpha_H}{2}-f,$$

or

$$p_H^D=c_M+\frac{2}{\alpha_H}f+\left[2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-\frac{2(f+D)}{\alpha_M}-(c_M-c_L)\right]\left(\frac{\alpha_M+2\alpha_H}{\alpha_H}\right) \tag{107}$$

Also,

$$p_H^{ND}=c_M+\frac{\alpha_M+2\alpha_H}{\alpha_H}\left(2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right)$$

Comparing (106) and (107) with the expressions for p_M^{ND} and p_H^{ND}, the medium and high types' prices in the pure price signaling equilibrium with full distortion, we can see that $p_M^D\le p_M^{ND}$, $p_H^D\le p_H^{ND}$ and as the conditions for the existence of pure price signaling equilibrium with full distortion ensure that $p_M^{ND}\le V_M, p_H^{ND}\le V_H$ we have $p_M^D\le V_M, p_H^D\le V_H$. The equilibrium profit of L type is non-negative if

$$f+D\le\left(\frac{\alpha_M+2\alpha_H}{2}\right)(V_M-V_L) \tag{108}$$

The equilibrium profit of the H type is positive iff

$$D\le(p_H^D-c_H)\frac{\alpha_H}{2},$$

which reduces to

$$f+(\frac{\alpha_M+\alpha_H}{\alpha_H})D\le\left[\left(2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right)\left(\frac{\alpha_M+2\alpha_H}{\alpha_H}\right)-(c_H-c_M)\right]\frac{\alpha_M}{4}. \tag{109}$$

The equilibrium profit of the M type is positive (this is not ensured by (**??**)) if:

$$D\le(p_M^D-c_M)\left(\frac{\alpha_M}{2}+\alpha_H\right)=(p_H^D-c_M)\frac{\alpha_H}{2}-f$$

i.e.,

$$2D\left(\frac{\alpha_M+\alpha_H}{\alpha_M+2\alpha_H}\right)+f\le\left[2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right]\left(\frac{\alpha_M}{2}\right). \tag{110}$$

To ensure (102) we need that

$$\begin{aligned} V_H-V_M &\le p_H^D-p_M^D \\ &= \frac{2}{\alpha_H}f-\frac{2(f+D)}{\alpha_M}\left(\frac{\alpha_M+\alpha_H}{\alpha_H}\right)+\left[2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right]\left(\frac{\alpha_M+\alpha_H}{\alpha_H}\right) \end{aligned}$$

and this holds if

$$\left[\left(2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right)\left(\frac{\alpha_M+\alpha_H}{\alpha_H}\right)-(V_H-V_M)\right]\frac{\alpha_M}{2}\ge f+D\left(\frac{\alpha_M+\alpha_H}{\alpha_H}\right). \tag{111}$$

Note that $p_H^D-p_M^D\le p_H^{ND}-p_M^{ND}$ and (102) cannot be ensured here by simply appealing to the condition for it to hold under pure price signaling.

Given out-of-equilibrium beliefs and (103), no firm wants to deviate to sending a message and charging price in $(\overline{p}_L,p_M^D)$ as no buyer would buy. To ensure that no firm has an incentive to deviate to sending a message and setting a price in the interval (p_M^D,p_H^D), we need to rule out deviations to a price $p\in(p_M^D,p_H^D-(V_H-V_M))$; at all of these prices, buyers believe that the deviant is of an M type for sure and the firm can only sell in the state where the rival is of an H type so that the total quantity sold is α_H and the optimal deviation is to set $p=p_H^D-(V_H-V_M)$, the highest price in this range. Such a deviation is not gainful for the M type if:

$$(p_H^D-c_M)\frac{\alpha_H}{2}-f=(p_M^D-c_M)\left(\frac{\alpha_M}{2}+\alpha_H\right)\ge(p_H^D-(V_H-V_M)-c_M)\alpha_H-f$$

which reduces to

$$p_H^D\le 2(V_H-V_M)+c_M$$

and using (107)

$$f+D\left(\frac{\alpha_M+2\alpha_H}{2\alpha_H}\right)>\left[\left(2\left(\frac{\alpha_M+\alpha_H}{\alpha_M}\right)(V_M-V_L)-(c_M-c_L)\right)\left(\frac{\alpha_M+2\alpha_H}{2\alpha_H}\right)-(V_H-V_M)\right]\frac{\alpha_M}{2}. \tag{112}$$

The out-of-equilibrium beliefs specify any firm that does not send a message and charges price in $(\overline{p}_L,p_M^D]$ is of an L type with probability one; any firm that does not send a message and charges price in $(p_M^D,p_H^D]$ is of an H type with probability zero; these ensure that M and H types cannot gain from deviating to not sending a message. We need to ensure that these beliefs relating to deviations where a firm does not send a message satisfy the D1 criterion.

To ensure that the out-of-equilibrium beliefs corresponding to deviations to a price $\widehat{p}\in(p_M^D,p_H^D]$ with no message (or a truthful out-of-equilibrium message) satisfy D1 let $q_\tau(\widehat{p})$ be defined as above:

$$\begin{aligned} (\widehat{p}-c_L)q_L(\widehat{p}) &= (\alpha_H+\alpha_M)(\overline{p}_L-c_L) \\ (\widehat{p}-c_M)q_M(\widehat{p}) &= (\alpha_H+\frac{\alpha_M}{2})(p_M^D-c_M)-D \\ (\widehat{p}-c_H)q_H(\widehat{p}) &= \frac{\alpha_H}{2}(p_H^D-c_H)-D. \end{aligned}$$

It is sufficient to ensure that for all $\widehat{p} \in (p_M^D, p_H^D]$

$$q_M(\widehat{p}) \leq q_H(\widehat{p}),$$

i.e.,

$$\frac{(\widehat{p} - c_H)}{(\widehat{p} - c_M)} \leq \frac{\frac{\alpha_H}{2}(p_H^D - c_H) - D}{(\alpha_H + \frac{\alpha_M}{2})(p_M^D - c_M) - D},$$

i.e.,

$$\begin{aligned} \frac{(p_H^D - c_H)}{(p_H^D - c_M)} &\leq \frac{\frac{\alpha_H}{2}(p_H^D - c_H) - D}{(\alpha_H + \frac{\alpha_M}{2})(p_M^D - c_M) - D} \\ &= \frac{(p_H^D - c_H) - D\frac{2}{\alpha_H}}{(p_H^D - c_M) - (f + D)\frac{2}{\alpha_H}} \end{aligned}$$

and this always holds for D small enough.

Next, consider deviations to a price $\widehat{p} \in (\overline{p}_L, p_M^D]$ with no message (or a truthful out-of-equilibrium message) and denote by $q_\tau(\widehat{p})$ the post-deviation quantity that makes a type τ firm indifferent between such a deviation and its equilibrium strategy. Then,

$$\begin{aligned} (\widehat{p} - c_L)q_L(\widehat{p}) &= (\alpha_H + \alpha_M)(\overline{p}_L - c_L) \\ (\widehat{p} - c_M)q_M(\widehat{p}) &= (\alpha_H + \frac{\alpha_M}{2})(p_M^D - c_M) - D. \end{aligned}$$

Thus,

$$q_L(\widehat{p}) \leq q_M(\widehat{p})$$

iff

$$\frac{(\widehat{p} - c_M)}{(\widehat{p} - c_L)} \leq \frac{(\alpha_H + \frac{\alpha_M}{2})(p_M^D - c_M) - D}{(\alpha_H + \alpha_M)(\overline{p}_L - c_L)}.$$

This holds for all $\widehat{p} \in (\overline{p}_L, p_M^D]$ if it holds for $\widehat{p} = p_M^D$

$$\frac{(p_M^D - c_M)}{(p_M^D - c_L)} \leq \frac{(\alpha_H + \frac{\alpha_M}{2})(p_M^D - c_M) - D}{(\alpha_H + \alpha_M)(\overline{p}_L - c_L)} \tag{113}$$

and using (104) this reduces to

$$\begin{aligned} \frac{(p_M^D - c_M)}{(p_M^D - c_L)} &\leq \frac{(\alpha_H + \frac{\alpha_M}{2})(p_M^D - c_M) - D}{(\alpha_H + \frac{\alpha_M}{2})(p_M^D - c_L) - (D + f)} \\ &= \frac{(p_M^D - c_M) - D(\frac{2}{2\alpha_H + \alpha_M})}{(p_M^D - c_L) - (D + f)(\frac{2}{2\alpha_H + \alpha_M})} \end{aligned}$$

and this always holds as $D \to 0$.

We also need to ensure that $q_L(\widehat{p}) \leq q_H(\widehat{p})$ for all $\widehat{p} \in (\overline{p}_L, p_M^D)$ which holds if

$$\frac{(\widehat{p} - c_H)}{(\widehat{p} - c_L)} \leq \frac{\frac{\alpha_H}{2}(p_H^D - c_H) - D}{(\alpha_H + \alpha_M)(\overline{p}_L - c_L)}$$

and this holds for all $\widehat{p} \in (\overline{p}_L, p_M^D)$ if it holds for $\widehat{p} = p_M^D$ (unless $p_M^D \leq c_H$ in which case it is automatically satisfied; so suppose $p_M^D > c_H$)

$$\frac{(p_M^D - c_H)}{(p_M^D - c_L)} \leq \frac{\frac{\alpha_H}{2}(p_H^D - c_H) - D}{(\alpha_H + \alpha_M)(\overline{p}_L - c_L)} = \frac{\frac{\alpha_H}{2}(p_H^D - c_M) - \frac{\alpha_H}{2}(c_H - c_M) - D}{(\alpha_H + \alpha_M)(\overline{p}_L - c_L)}$$

*and this reduces to

$$\begin{aligned}
\frac{(p_M^D - c_H)}{(p_M^D - c_L)} &\leq \frac{(\frac{\alpha_M}{2} + \alpha_H)(p_M^D - c_M) - (D - f + \frac{\alpha_H}{2}(c_H - c_M))}{(\frac{\alpha_M}{2} + \alpha_H)(p_M^D - c_L) - (f + D)} \\
&= \frac{(\frac{\alpha_M}{2} + \alpha_H)(p_M^D - c_H) + (c_H - c_M)(\frac{\alpha_M + \alpha_H}{2}) - D + f}{(\frac{\alpha_M}{2} + \alpha_H)(p_M^D - c_L) - (f + D)} \\
&= \frac{(p_M^D - c_H) + ((c_H - c_M)(\frac{\alpha_M + \alpha_H}{2}) - D + f)\frac{2}{\alpha_M + 2\alpha_H}}{(p_M^D - c_L) - (f + D)\frac{2}{\alpha_M + 2\alpha_H}}
\end{aligned}$$

which certainly holds as long as

$$D < (c_H - c_M)(\frac{\alpha_M + \alpha_H}{2}) + f. \tag{114}$$

It is easy to see that the right hand side of the inequality in condition (108):

$$\left(\frac{\alpha_M + 2\alpha_H}{2}\right)(V_M - V_L)$$

is strictly positive. The right hand side of the inequality in (109) is strictly positive if

$$\left[\left(2\left(\frac{\alpha_M + \alpha_H}{\alpha_M}\right)(V_M - V_L) - (c_M - c_L)\right)\left(\frac{\alpha_M + 2\alpha_H}{\alpha_H}\right) - (c_H - c_M)\right] > 0$$

and this holds as long as in the full distortion pure price signaling equilibrium $p_H^{ND} > c_H$ i.e., high type earns strictly positive profit. The right hand side of the inequality in (110) is strictly positive if

$$2\left(\frac{\alpha_M + \alpha_H}{\alpha_M}\right)(V_M - V_L) - (c_M - c_L) > 0$$

and this follows from the fact that in the full distortion pure price signaling equilibrium $p_M^{ND} > c_M$ i.e., medium type must earn strictly positive profit. The right hand side of the inequality in (111) is strictly positive if

$$\left(2\left(\frac{\alpha_M + \alpha_H}{\alpha_M}\right)(V_M - V_L) - (c_M - c_L)\right)\left(\frac{\alpha_M + \alpha_H}{\alpha_H}\right) - (V_H - V_M) > 0$$

which holds as long as in the full distortion pure price signaling equilibrium $p_H^{ND} - p_M^{ND} > V_H - V_M$. Finally, note that the right hand side of the inequality in (112) does not bind at all if

$$\left(2\left(\frac{\alpha_M + \alpha_H}{\alpha_M}\right)(V_M - V_L) - (c_M - c_L)\right)\left(\frac{\alpha_M + 2\alpha_H}{2\alpha_H}\right) - (V_H - V_M) \leq 0$$

which follows from the condition for the medium type to not deviate to $p_H^{ND} - (V_H - V_M)$ in a full distortion pure price signaling equilibrium. It follows that the conditions for a full distortion pure price signaling equilibrium where high type earns strictly positive profit and buyers strictly prefer to buy from medium quality firm when rival has high quality imply that (108), (109), (110), (111), (114) as well as all other conditions described above are satisfied if f and D are small enough.

Proposition 6 *Suppose that under no regulation there exists a pure price signaling equilibrium with full distortion where the high type earn strictly positive profit and prices are such that buyers strictly prefer to buy from the medium quality firm when the other firm sells high quality. Then, there exists $f_0 > 0$ and $D_0(f) > 0$ for all $f \in [0,\ f_0]$, such that for every $f < f_0, D < D_0(f)$, there exists a pure disclosure equilibrium with full distortion. The welfare generated in such an equilibrium is lower than the pure price signaling outcome (obtained under no or low regulation).*

1.4 Mixed Communication with Strong Regulation: High f, Intermediate D.

We now characterize a mixed disclosure equilibrium where (101) does not hold i.e., D is larger than $\alpha_L(V_H - c_H - (V_L - c_L)) + \alpha_M(V_H - c_H - (V_M - c_M))$, but not too large for disclosure to be prohibitively costly. The above analysis suggests that in this region the medium quality firm may randomize between direct communication and relying on price signaling, while the high quality firm continues to directly communicate for sure.

Similar to the analysis in the main body of the paper, we propose the following equilibrium. With probability γ the medium quality type sets a price p_M without communicating quality directly and with probability $1-\gamma$ randomizes over an interval $[\underline{p}_M, \overline{p}_M]$ combined with directly communicating quality. Medium quality cedes the market to low quality when it sets p_M, while it takes the market from low quality when it randomizes over $[\underline{p}_M, \overline{p}_M]$. It follows that low quality randomizes over the interval $[\underline{p}_L, \overline{p}_L]$, where $\overline{p}_L = p_M - (V_M - V_L)$ and $\underline{p}_L = \overline{p}_M - (V_M - V_L) > c_L$. To ensure that consumers always buy high quality when it is around, we must have that

$$V_H - \overline{p}_H \geq \min\{V_M - \underline{p}_M, V_L - \underline{p}_L\}.$$

To have that medium quality is indifferent between its different strategy parts, we must have that $\frac{\gamma\alpha_M}{2}(p_M - c_M) = (\alpha_L + \gamma\alpha_M)(\overline{p}_M - c_M) - D = (\alpha_L + \alpha_M)(\underline{p}_M - c_M) - D$. D1 requires that the low quality firm is indifferent between charging any price in its support and p_M, i.e., $\frac{\gamma\alpha_M}{2}(p_M - c_L) = \gamma\alpha_M(\overline{p}_L - c_L) = (\alpha_L + \gamma\alpha_M)(\underline{p}_L - c_L)$.

Thus, we have $\frac{1}{2}(p_M - c_L) = p_M - (V_M - V_L) - c_L$ or $p_M = c_L + 2(V_M - V_L), \overline{p}_L = c_L + (V_M - V_L)$ $\underline{p}_L = \frac{\gamma\alpha_M\overline{p}_L + \alpha_L c_L}{\gamma\alpha_M + \alpha_L} = c_L + \frac{\gamma\alpha_M(V_M - V_L)}{\gamma\alpha_M + \alpha_L}, \overline{p}_M = c_L + \frac{\gamma\alpha_M(V_M - V_L)}{\gamma\alpha_M + \alpha_L} + (V_M - V_L)$ so that γ is determined by

$$\frac{\gamma\alpha_M}{2}(c_L + 2(V_M - V_L) - c_M) = (\alpha_L + \gamma\alpha_M)\left[c_L + \frac{\gamma\alpha_M(V_M - V_L)}{\gamma\alpha_M + \alpha_L} + (V_M - V_L) - c_M\right] - D$$

yielding $(\alpha_L + \gamma\alpha_M)((V_M - c_M - (V_L - c_L)) + \frac{\gamma\alpha_M}{2}(c_M - c_L) = D$, or

$$\gamma = \frac{D - \alpha_L(V_M - c_M - (V_L - c_L))}{\alpha_M((V_M - c_M - (V_L - c_L)) + \frac{\alpha_M}{2}(c_M - c_L)},$$

which is indeed positive for

$$D > \alpha_L(V_M - c_M - (V_L - c_L)). \tag{115}$$

Finally, $\underline{p}_M$ is such that

$$\underline{p}_M = c_M + \frac{\gamma\alpha_M}{2(\alpha_L+\alpha_M)}(c_L + 2(V_M - V_L) - c_M) + \frac{D}{\alpha_L+\alpha_M} = c_M + \frac{2D - \alpha_L(V_M - c_M - (V_L - c_L))}{\alpha_L+\alpha_M}$$

so that

$$\overline{p}_H = c_M + V_H - V_M + \frac{2D - \alpha_L(V_M - c_M - (V_L - c_L))}{\alpha_L+\alpha_M}.$$

The high quality firm's profit is then given by $(\alpha_L+\alpha_M)(\underline{p}_M + V_H - V_M - c_H)$.

Does a high quality firm wants to deviate to higher prices? Deviating to prices $p \in (\underline{p}_M + V_H - V_M, \overline{p}_M + V_H - V_M)$, while directly communicating quality, gives a profit of $(\alpha_L + \gamma\alpha_M + (1-\gamma)\alpha_M(1 - F_M(p - (V_H - V_M$ $c_H)$. From the indifference condition of the medium quality type over $p \in (\underline{p}_M, \overline{p}_M)$ it follows that $(\alpha_L + \gamma\alpha_M + (1-\gamma)\alpha_M(1 - F_M(p - (V_H - V_M)))(p - c_M)$ is constant. As we can write

$$\begin{aligned} & (\alpha_L + \gamma\alpha_M + (1-\gamma)\alpha_M(1 - F_M(p - (V_H - V_M)))(p - c_H) \\ = \; & (\alpha_L + \gamma\alpha_M + (1-\gamma)\alpha_M(1 - F_M(p - (V_H - V_M)))(p - c_M) \\ & + (\alpha_L + (1-\gamma)\alpha_M(1 - F_M(p - (V_H - V_M)))(c_M - c_H) \end{aligned}$$

it follows that the high quality's profit is decreasing in p over this interval. While directly communicating quality, at prices $\overline{p}_L + V_H - V_L = p_M > p > \overline{p}_M + V_H - V_M$ the operating profit of the high quality firm equals $(\alpha_L(1 - F_L(p - (V_H - V_L)) + \gamma\alpha_M)))(p - c_H)$, which can be rewritten as

$$\begin{aligned} & (\alpha_L(1 - F_L(p - (V_H - V_L)) + \gamma\alpha_M)))(p - c_L) \\ & + (\alpha_L(1 - F_L(p - (V_H - V_L)) + \gamma\alpha_M)))(c_L - c_H), \end{aligned}$$

which is again decreasing. Finally, at prices $p_M > p$ the operating profit of the high quality firm equals 0. Thus, a high quality firm does not have an incentive to deviate only in price.

One can also check that given the equilibrium strategy of the rival firm, a high quality firm cannot profitably deviate by not communicate directly; lower types have higher incentive to imitate any deviation to price at or below $\overline{p}_H$ with no direct communication if buyers believe the deviation comes from high type (and firm cannot sell at price above $\overline{p}_H$).

Other equilibrium conditions are such that $0 < \gamma \leq 1$ and $(\alpha_L+\alpha_M)(\underline{p}_M + V_H - V_M - c_H) - D \geq 0$, which guarantees that the high quality type makes positive profits. The latter condition is equivalent to $(\alpha_L+\alpha_M)(V_H - c_H - (V_M - c_M)) + D + \alpha_L(V_M - c_M - (V_L - c_L)) \geq 0$, which is always the case. The condition $\gamma \leq 1$ is equivalent to

$$D \leq (\alpha_L+\alpha_M)(V_M - c_M - (V_L - c_L)) + \frac{\alpha_M}{2}(c_M - c_L) \tag{116}$$

so that this equilibrium exists if (115) and (116) hold.

Let us then do the welfare analysis of this type of equilibrium and compare it with the welfare loss of the pure price signaling equilibrium. The welfare loss in this equilibrium is equal to

$$\begin{aligned} & \left[2(\alpha_H + (1-\gamma)\alpha_M)^2 + 2(\alpha_L + \gamma\alpha_M)(\alpha_H + (1-\gamma)\alpha_M)\right] D + 2\gamma\alpha_M\alpha_L(V_M - c_M - (V_L - c_L)) \\ = \; & 2(\alpha_H + (1-\gamma)\alpha_M)D + 2\gamma\alpha_M\alpha_L(V_M - c_M - (V_L - c_L)) \\ = \; & 2(1-\alpha_L)D + 2\frac{D - \alpha_L(V_M - c_M - (V_L - c_L))}{((V_M - c_M - (V_L - c_L)) + \frac{1}{2}(c_M - c_L)}[\alpha_L(V_M - c_M - (V_L - c_L)) - D]. \end{aligned}$$

If D is such that γ is close to 1 (which is the case if D is close to $(\alpha_L + \alpha_M)(V_M - c_M - (V_L - c_L)) + \frac{\alpha_M}{2}(c_M - c_L)$), this is approximately equal to

$$
\begin{aligned}
& 2(1-\alpha_L)[(1-\alpha_H)(V_M - c_M - (V_L - c_L)) + \frac{\alpha_M}{2}(c_M - c_L)] \\
& -2\alpha_M^2[(V_M - c_M - (V_L - c_L)) + \frac{1}{2}(c_M - c_L)] \\
= \quad & 2[\alpha_L(\alpha_M + \alpha_H) + \alpha_M\alpha_H](V_M - c_M - (V_L - c_L)) + \alpha_M\alpha_H(c_M - c_L).
\end{aligned}
$$

This is larger than the welfare loss in any pure price signaling equilibrium (even if β is equal to 0) if, and only if,

$$\alpha_M[(V_M - c_M - (V_L - c_L)) + \frac{c_M - c_L}{2}] \geq (1-\alpha_H)(V_H - c_H - (V_M - c_M)),$$

which could well be the case if the *additional* total surplus generated by consuming high quality is relatively small compared to the *additional* total surplus generated by consuming medium quality, or if α_H is close to 1.

To sum up:

Proposition 7 *For large enough f a mixed disclosure equilibrium exists for intermediate values of D. Such an equilibrium generates lower surplus than the worst possible pure price signaling equilibrium if D is close to, but smaller than $(\alpha_L + \alpha_M)(V_M - c_M - (V_L - c_L)) + \frac{\alpha_M}{2}(c_M - c_L)$ and α_H is close to 1 or $V_H - c_H$ is close to $V_M - c_M$.*

1.5 NO VAGUE MESSAGES

In this subsection, we discuss whether types can send the same correct message in a D1 equilibrium (for instance, M and H both send message $\{M, H\}$. If these two types also choose the same price, then these types would pool and standard arguments given in the main text would apply to show that this is not possible. (First, different types cannot randomize over the same price range, so, at least one type should choose a pure strategy and the other firm should choose this price with strictly positive probability. Second, the higher type can deviate and choose a slightly higher price and consumers should infer these prices are set by the highest of these types, making the deviation gainful).

Given that the equilibrium should be separating (at least in prices), it is clear that the lowest type L does not want to send a message as consumers anyway infer these prices are set by the lowest type. Low quality firms can thus economize on the cost of sending direct messages without affecting consumer choices. This also implies that low quality types cannot set any price $p_L > c_L$ with strictly positive probability.

So, consider an equilibrium with direct communication where two types M and H choose the same message, but different prices. Let $\overline{p}_M$ and $\underline{p}_H$ be the highest (resp. lowest) price that the lowest (resp. highest) type charges in equilibrium together with the direct communication message. It is clear that in any equilibrium it must be that $V_M - \overline{p}_M \geq V_H - \underline{p}_H$ implying that $\overline{p}_M < \underline{p}_H$. If this was not the case then the demand at $\overline{p}_M$ would be smaller than the demand at $\underline{p}_H$ implying that lower quality types would want to deviate to $\underline{p}_H$.

Also, it cannot be the case that M type chooses a price p with strictly positive probability. If this were the case, then the M type could profitably deviate by sending the same message but undercutting price p. As for f high enough, D1 (or the Intuitive Criterion for that matter) would imply that consumers believe this message is not set by a low quality firm and would buy at least in all cases where the consumer would buy at price p and would certainly buy if the rival is an M types that sets p. This makes the deviation profitable. Thus, the medium type must randomize in such a way that no price gets positive probability.

We distinguish two subcases: (i) $\underline{p}_H$ is set with strictly positive probability and (ii) $\underline{p}_H$ is set with zero probability.

In the first case, D1 implies that

$$\pi_M^* = (\overline{p}_M - c_M)q(\overline{p}_M) - D = (\underline{p}_H - c_M)q(\underline{p}_H) - D$$

with $q(\overline{p}_M) > q(\underline{p}_H)$. To see this, note that it must be that $\pi_L^* \geq (\overline{p}_M - c_L)q(\overline{p}_M) - D$, or alternatively $\pi_L^* \geq (\overline{p}_M - c_L)q(\overline{p}_M) - D - f$ in case the message sent by M and H does not include type L. If it were the case that $(\overline{p}_M - c_M)q(\overline{p}_M) - D > (\underline{p}_H - c_M)q(\underline{p}_H) - D$, then

$$\begin{aligned} \pi_L^* &\geq (\overline{p}_M - c_M)q(\overline{p}_M) - (c_L - c_M)q(\overline{p}_M) - D \\ &> (\underline{p}_H - c_M)q(\underline{p}_H) - (c_L - c_M)q(\overline{p}_M) - D \\ &= (\underline{p}_H - c_L)q(\underline{p}_H) - (c_L - c_M)(q(\overline{p}_M) - q(\underline{p}_H)) - D. \end{aligned}$$

Thus, if M strictly does not want to imitate H's behaviour, then the same applies to any smaller type.[3] If $\underline{p}_H$ is set with strictly positive probability, then there must be a left-neighborhood of $\underline{p}_H$ such that in equilibrium no firm sets prices in this neighborhood. D1 then implies that if a consumer would observe a price just below $\underline{p}_H$ it will infer that the quality is at least equal to H making a deviation to such a price gainful for type H.

Consider then a deviation where type H sends message "I am type H" and charges price just below $\underline{p}_H$. It is clear that because of the fee $f > 0$ no type smaller than H wants to imitate this behaviour. Therefore, consumers have to believe that this behaviour comes from a type that is at least as good as H. In addition, for the high type there is no additional cost of changing the message to being precise and truthful. Thus, this deviation is gainful. We conclude it cannot be the case that $\underline{p}_H$ is set with positive probability.

We will now argue that case (ii) where $\underline{p}_H$ is set with zero probability cannot arise. Suppose to the contrary that type H chooses prices over an interval $[\underline{p}_H, x]$ without mass points. If $\overline{p}_M \leq \underline{p}_H < \overline{p}_M + (V_H - V_M)$, then clearly $\underline{p}_H$ attracts more demand than $\overline{p}_M$ and the medium type would like to deviate to $\underline{p}_H(+\varepsilon)$. We will now argue that it cannot be that $\overline{p}_M > \underline{p}_H$ or $\underline{p}_H \geq \overline{p}_M + (V_H - V_M)$. Consider first the case where $\overline{p}_M > \underline{p}_H$ and denote by $q(\overline{p}_M)$, resp. $q(\underline{p}_H)$, the quantity sold at the respective prices. Given the price relation, it is clear that $q(\overline{p}_M) < q(\underline{p}_H)$. For the medium and high quality not to have an incentive to imitate the other type's price is must be that

$$q(\overline{p}_M)(\overline{p}_M - c_M) \geq q(\underline{p}_H)(\underline{p}_H - c_M)$$

and

$$q(\overline{p}_M)(\overline{p}_M - c_H) \leq q(\underline{p}_H)(\underline{p}_H - c_H).$$

As the first of these inequalities can be rewritten as

$$q(\overline{p}_M)(\overline{p}_M - c_H) + \left(q(\underline{p}_H) - q(\overline{p}_M)\right) c_M \geq q(\underline{p}_H)(\underline{p}_H - c_H)$$

[3] The same argument applies in case the message sent by M and H does not include type L in which case we have to include a cost f in all equations.

and thus

$$q(\overline{p}_M)(\overline{p}_M - c_H) > q(\underline{p}_H)(\underline{p}_H - c_H)$$

contradicting the second inequality.

Consider next the case where $\underline{p}_H \geq \overline{p}_M + (V_H - V_M)$. This implies that if both medium and high qualities are around, then consumers buy medium quality. This also implies that at $\overline{p}_H$ consumers should buy with strictly positive probability high quality if the other firm is a low quality type as otherwise the high quality type does not sell at all at $\overline{p}_H$ contradicting the supposition that he is randomizing (note as the equilibrium is symmetric, that there cannot be a mass point at $\overline{p}_H$). But, given that low quality types cannot set any price $p_L > c_L$ with strictly positive probability, this also implies that $\overline{p}_L = c_L$ as the low quality will not sell if $\overline{p}_L > c_L$ and that $\overline{p}_H \leq c_L + (V_H - V_L)$. Also, given $\underline{p}_H \geq \overline{p}_M + (V_H - V_M)$, this also implies that consumers buy medium quality if both high and medium quality are around. It follows that $q(\overline{p}_M) = q(\underline{p}_H)$ so that given that $\underline{p}_H > \overline{p}_M$ the medium quality has an incentive to deviate to $\underline{p}_H$.

To sum up:

Proposition 8 *There does not exist an equilibrium with pure disclosure where medium and high types send identical (correct) messages.*

2 CONTINUUM OF TYPES: NONEXISTENCE ISSUES

One may wonder whether a model with a continuum of types may be easier to work with. We show that equilibrium may not exist with a continuum of possible qualities.

Consider the duopoly model with continuum of (quality) types $\theta \in [\underline{\theta}, \overline{\theta}]$

$$0 < \underline{\theta} < \overline{\theta} < \infty.$$

Unit cost of production of type θ is identical to θ.

Let $\mathcal{F}$ be the class of distribution functions defined by :

$$\mathcal{F} = \{F : F \;\; : [\underline{\theta}, \overline{\theta}] \to [0,1] \text{ is continuous and strictly increasing, } F(\underline{\theta}) = 0, F(\overline{\theta}) = 1\}$$

We assume that the prior distribution of θ is in $\mathcal{F}$.

Buyer's valuation of quality θ is given by $V(\theta) > \theta$. We assume $(V(\theta) - \theta)$ is continuous and strictly increasing in θ on $[\underline{\theta}, \overline{\theta}]$.

We focus on the existence of a separating equilibrium where the price $p(\theta)$ set by each quality type fully reveals its type. Let $q(\theta) \in [0,1]$ be the probability that type θ sells in such an equilibrium.

The incentive compatibility constraint of sellers implies that for all $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$(p(\theta) - \theta)q(\theta) \geq (p(\theta') - \theta)q(\theta') \text{ for all } \theta' \neq \theta \tag{117}$$

Further, individual rationality requires:

$$p(\theta) \geq \theta \tag{118}$$

Note that (117) and (118) do not involve F, the prior distribution of θ. It is easy to check from (117) that in any separating equilibrium $p(\theta)$ is strictly increasing and $q(\theta)$ is decreasing in θ. Further, we claim that

$$q(\theta) \uparrow 1 \text{ as } \theta \downarrow \underline{\theta} \tag{119}$$

To see (119), suppose not. Then there is a decreasing sequence $\{\theta^n\}_{n=1}^{\infty} \downarrow \underline{\theta}$, such that $q(\theta^n) \leq 1-\gamma$ for some $\gamma > 0$. Note that $\{p(\theta^n)\}$ is a decreasing bounded sequence (bounded below by $p(\underline{\theta})$) and therefore converges to some $\underline{p} \geq p(\underline{\theta})$. If

$$\underline{p} = \lim_{n\to\infty} p(\theta^n) = \underline{\theta}$$

then

$$\underline{\theta} \leq p(\underline{\theta}) \leq \lim_{n\to\infty} p(\theta^n) = \underline{p} = \underline{\theta}$$

implies $p(\underline{\theta}) = \underline{\theta}$; to prevent gainful deviation by type $\underline{\theta}$ it must be the case that $q(\theta) = 0$ for all $\theta \in (\underline{\theta}, \overline{\theta}]$;but sellers of almost all types sell zero in a symmetric equilibrium only if $p(\theta) = V(\theta)$ for all $\theta \in (\underline{\theta}, \overline{\theta}]$ in which case $\lim_{n\to\infty} p(\theta^n) = V\underline{\theta} > \underline{\theta}$, a contradiction. Thus, $\underline{p} > \underline{\theta}$. Now, suppose type θ^n deviates to price $\underline{p}$ it can sell with probability one (undercuts rival with probability one) and its deviation profit

$$\begin{aligned}
\underline{p} - \theta^n &= (\underline{p} - \theta^n)(1-\gamma) + (\underline{p} - \theta^n)\gamma \\
&= (p(\theta^n) - \theta^n)(1-\gamma) - (p(\theta^n) - \underline{p})(1-\gamma) + (\underline{p} - \theta^n)\gamma \\
&\geq (p(\theta^n) - \theta^n)q(\theta^n) - (p(\theta^n) - \underline{p})(1-\gamma) + (\underline{p} - \theta^n)\gamma
\end{aligned}$$

i.e.,

$$(p(\theta^n) - \theta^n)q(\theta^n) - (\underline{p} - \theta^n) \leq (p(\theta^n) - \underline{p})(1-\gamma) - (\underline{p} - \theta^n)\gamma$$

and as the right hand side converges to $-(\underline{p} - \underline{\theta})\gamma < 0$ as $n \to \infty$ we have

$$(p(\theta^n) - \theta^n)q(\theta^n) < (\underline{p} - \theta^n)$$

for n sufficiently large, a contradiction. This establishes (119).

For any $(\theta, \theta') \in [\underline{\theta}, \overline{\theta}] \times [\underline{\theta}, \overline{\theta}]$,let $\beta(\theta, \theta') \in [0,1]$ denote the probability with buyer buys from a firm of type θ when its rival is of type θ' (in equilibrium).Obviously,

$$\beta(\theta, \theta') + \beta(\theta', \theta) \leq 1 \text{ for all } (\theta, \theta') \in [\underline{\theta}, \overline{\theta}] \times [\underline{\theta}, \overline{\theta}] \tag{120}$$

Note β could depend on the distribution of types in equilibrium. To be consistent with buyer's behavior, for all $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$q(\theta) = \int \beta(\theta, \theta')dF(\theta') = E_{\theta'}[\beta(\theta, \theta')] \tag{121}$$

We say that $q(.)$ cannot be rationalized by buyer's behavior for prior distribution F if (121) and (120) cannot be jointly satisfied by any admissible $\beta : [\underline{\theta}, \overline{\theta}] \times [\underline{\theta}, \overline{\theta}] \to [0,1]$.

Proposition 9 *There is an open set $\mathcal{F}' \subset \mathcal{F}$of distribution functions such that no separating equilibrium exists if the prior distribution $F \in \mathcal{F}'$.*

Proof. Let

$$G = \{(p(.), q(.)) : p : [\underline{\theta}, \overline{\theta}] \to [\underline{\theta}, V(\overline{\theta})], q : [\underline{\theta}, \overline{\theta}] \to [\underline{\theta}, V(\overline{\theta})], (p(.), q(.)) \text{ satisfy (117) and (118)}\}$$

Note that G is a family of monotonic and uniformly bounded functions. For any $(p,q) \in G$, using (121) and (120), for any $\theta \in [\underline{\theta}, \overline{\theta}]$

$$\begin{aligned} q(\theta) &= E_{\theta'}[\beta(\theta,\theta')] \\ &\leq E_{\theta'}[1-\beta(\theta',\theta)] = 1 - E_{\theta'}\beta(\theta',\theta) \end{aligned}$$

and taking expectation with respect to θ

$$\begin{aligned} E_\theta(q(\theta)) &\leq 1 - E_\theta E_{\theta'}\beta(\theta',\theta) \\ &= 1 - E_{\theta'} E_\theta \beta(\theta',\theta) \\ &= 1 - E_{\theta'}(q(\theta')) \end{aligned}$$

which means

$$E_\theta(q(\theta)) \leq \frac{1}{2} \tag{122}$$

Fix $\epsilon \in (0, \frac{1}{2})$ sufficiently small. For each pair of functions $(p,q) \in G$, define

$$\widehat{\theta}(p,q,\epsilon) = \sup\{\theta : q(\theta) \geq \frac{1}{2} + \epsilon\}$$

Using (119)), $\widehat{\theta}(p,q,\epsilon) \in (\underline{\theta}, \overline{\theta})$. Define $\theta^*(\epsilon)$ by

$$\theta^*(\epsilon) = \inf_{(p,q)\in G} \widehat{\theta}(p,q,\epsilon)$$

Then there exists a sequence of functions $(p^n, q^n) \in G, n = 1, 2, ...,$ such that $\widehat{\theta}(p^n, q^n, \epsilon) \to \theta^*(\epsilon)$ as $n \to \infty$. Note that $\{p^n\}$ and $\{q^n\}$ are sequences of uniformly bounded monotonic functions on $[\underline{\theta}, \overline{\theta}]$; so the Helly Selection Theorem ensures there is a subsequence $\{(p^{n'}, q^{n'})\}$ of functions that converge pointwise; let $(\widetilde{p}, \widetilde{q})$ be that pointwise limit. As the (117) and (118) hold for $(p,q) = (p^{n'}, q^{n'})$ for every n', taking limit as $n' \to \infty$ one can check that they hold for $(p,q) = (\widetilde{p}, \widetilde{q})$ i.e., $(\widetilde{p}, \widetilde{q}) \in G$. However, this implies $\widehat{\theta}(\widetilde{p}, \widetilde{q}, \epsilon) > \underline{\theta}$. Further,

$$\widehat{\theta}(\widetilde{p}, \widetilde{q}, \epsilon) = \theta^*(\epsilon) \leq \widehat{\theta}(p,q,\epsilon) \text{ for all } (p,q) \in G$$

Define $\mathcal{F}' \subset \mathcal{F}$ by

$$\mathcal{F}' = \left\{ F \in \mathcal{F}\colon \ F(\theta^*(\epsilon)) > \frac{1}{1+2\epsilon} \right\}$$

Then for every $F \in \mathcal{F}'$

$$F(\widehat{\theta}(p,q,\epsilon)) \geq F(\theta^*(\epsilon)) > \frac{1}{1+2\epsilon} \text{for all } (p,q) \in G \tag{123}$$

For each $(p,q) \in G$,

$$\begin{aligned} & E_\theta(q(\theta)) \\ = \ & E_\theta\{q(\theta) \| \theta \leq \widehat{\theta}(p,q,\epsilon)\} F(\widehat{\theta}(p,q,\epsilon)) + E_\theta\{q(\theta) \| \theta > \widehat{\theta}(p,q,\epsilon)\}(1 - F(\widehat{\theta}(p,q,\epsilon))) \\ \geq \ & E_\theta\{q(\theta) \| \theta \leq \widehat{\theta}(p,q,\epsilon))\} F(\widehat{\theta}(p,q,\epsilon)) \\ \geq \ & \left(\frac{1}{2} + \epsilon\right) F(\widehat{\theta}((p,q,\epsilon)) \\ > \ & \frac{1}{2}, \text{ using (123)} \end{aligned}$$

which contradicts (122). Thus, for $F \in \mathcal{F}'$ implies there is no (p, q) satisfying (117) and (118) that can be rationalized by buyer's behavior. This completes the proof. ■

Appendix C: Quantity Distortions (Section VI.B of main paper)

This online appendix provides more details concerning the claims in subsection VI.B of the main paper concerning the extension on quantity distortions.

Consider the case where

$$2\left(\frac{V_L - c_L}{V_H - c_L}\right) < 1 \tag{124}$$

Then from Lemma 2 in Janssen and Roy (2010), a symmetric pure price signaling equilibrium must be one where $p_H = V_H$, $\overline{p}_L = V_L$, $\underline{p}_L = \alpha V_L + (1-\alpha)c_L$, and in the state where both firms are high quality, a fraction $\eta^S < 1$ of buyers buy (quantity distortion). Further, Proposition 3 in that paper and the proof of Proposition 3 establishes that the unique symmetric D1 outcome is one where the quantity distortion is minimized and, in particular

$$\eta^S = 2\left(\frac{V_L - c_L}{V_H - c_L}\right) \tag{125}$$

It is easy to check that for any $D > 0$ this is the unique equilibrium of the model with direct communication if f is small enough or equivalently, for any $f \geq 0$ if D is large enough.

The quantity distortion described above is directly related to the incentive constraint; as high quality price has hit its ceiling V_H it is no longer possible to raise low quality rent by increasing the high quality price and so the only way to reduce the incentive to mimic is by reducing the quantity sold by high types. The welfare loss in the pure price signaling equilibrium is given by:

$$WL^S = 2\alpha(1-\alpha)(\Delta V - \Delta c) + \alpha^2(1-\eta^S)(V_H - c_H) \tag{126}$$

Regulation may not only divert market share to high quality firms (when facing a low quality rival) but also reduce the quantity distortion. This makes regulation welfare improving for a larger section of the parameter space.

Observation 1:

One conclusion in the main text of the paper where we assume (124) does not hold is that:

$$D \leq (1-\alpha)(\Delta V - \Delta c) \tag{127}$$

implies that sufficiently high regulation is optimal; in particular, if

$$f \geq (1-\alpha)\Delta V + \frac{\alpha}{2}\Delta, c$$

then we have a pure disclosure equilibrium where low types charge their marginal cost, high types disclose with probability one and randomize over prices in the interval $[\underline{p}_H, \overline{p}_H]$ where $\overline{p}_H = c_L + \Delta V$ and high type serves the entire market with probability one when rival is of low type while. The conditions for this kind of an equilibrium are unaffected by and the conclusion continues to hold when (124) hold.

Observation 2:

The main text of the paper shows (again assuming (124) does not hold) that if direct communication cost is in an intermediate range

$$\Delta V - (1 - \frac{\alpha}{2})\Delta c > D > (1 - \alpha)(\Delta V - \Delta c) \quad (128)$$

then a fine f generates strictly lower welfare than no fine. In particular, for this range of the direct communication cost if regulation is high enough to make a difference to the outcome (i.e., leads to any kind of disclosure), then equilibrium is necessarily one with mixed disclosure where high types randomize between disclosing and not disclosing. In such an equilibrium, the maximum social gain from disclosure by a high type is the correction of the distortion (in the pure price signaling outcome) by switching buyers from low to high quality consumption (surplus gain $(\Delta V - \Delta c)$) in the state where the rival is of low type (which occurs with probability $1 - \alpha$) while the deadweight cost of disclosure incurred by the high type is given by D. So, $D > (1 - \alpha)(\Delta V - \Delta c)$ implies that disclosure with positive probability is suboptimal. Thus, no regulation is optimal under (128).

To explore how this result is modified when (124) holds assume that regulation is very high i.e., f is large enough. Consider the pure price signaling equilibrium under (124) described above. Let $\widehat{D}$ be the maximum gain to a high type firm that deviates from a pure price signaling equilibrium by disclosing and reducing to price to $\underline{p}_L + \Delta V$ at which it sells to all buyers with probability one (assuming f is large enough for buyers to D1 believe that such a deviation comes from a high type for sure). Then,

$$\begin{aligned}
\widehat{D} &= (\underline{p}_L + \Delta V - c_H) - \frac{\alpha\eta^S}{2}(V_H - c_H) \\
&= (\alpha V_L + (1 - \alpha)c_L + \Delta V - c_H) - \frac{\alpha\eta^S}{2}(V_H - c_H) \\
&= (1 - \alpha)(\Delta V - \Delta c) + \alpha[1 - \frac{\eta^S}{2}](V_H - c_H)
\end{aligned}$$

For $D \geq \widehat{D}$, the equilibrium outcome is necessarily one with pure price signaling. Note that if $\eta^S = 1, \widehat{D} = \overline{D}$ so that the model with and without quantity distortion smoothly transit into each other.

We will show that:

(a) As in the main text of the paper, regulation leads to a mixed disclosure equilibrium if the direct communication cost is in an intermediate range $((1 - \alpha)(\Delta V - \Delta c), \widehat{D})$

(b) Unlike the main text of the paper, these mixed disclosure equilibrium outcomes are better than the pure price signaling outcome if the direct communication cost is at the lower end of the range

(c) As in the main text of the paper, mixed disclosure equilibrium outcomes are worse than the pure price signaling outcome if the direct communication cost is at the upper end of the intermediate range (even if f is infinitely large)

Consider $D \in ((1-\alpha)(\Delta V - \Delta c), \widehat{D})$ and assume f is large enough. Then the (unique symmetric) D1 mixed disclosure equilibrium is as follows: high type discloses with probability $\gamma_H \in (0, 1)$

$$\gamma_H = 1 - \frac{D - (1 - \alpha)(\Delta V - \Delta c)}{\alpha(V_H - c_H)(1 - \frac{\eta^S}{2})}$$

where $\eta^S < 1$ is as defined in (125). When it does not disclose, the high type charges V_H and at this price it only sells in the state where the rival is H type; further, a fraction η^S of buyers buy in the state where both firms charge V_H without claiming high quality; thus a non-disclosing high quality type sells with probability $\frac{\alpha(1-\gamma_H)\eta^S}{2}$. A low quality type does not claim high quality and randomizes its price with a continuous distribution over an interval $[\underline{p}_L, \overline{p}_L]$, where $\overline{p}_L = V_L$. When it claims high quality, the high quality firm randomizes prices over an interval $[\underline{p}_H^D, \overline{p}_H^D]$, where $\overline{p}_H^D = \underline{p}_L + \Delta V < V_H$, i.e., buyers are indifferent between buying low quality at the lower bound of low quality prices $\underline{p}_L$ and the upper bound of high quality prices when the firm claims high quality. It is easy to see that $\overline{p}_H^D > \overline{p}_L$. At price $\overline{p}_L$ a low quality firm sells with probability $\alpha(1-\gamma_H)$, i.e., only when the rival is of high quality but does not claim high quality. At price $\underline{p}_L$ a low quality firm sells with probability $1-\alpha\gamma_H$. When it claims high quality and charges price $\overline{p}_H^D$, a high quality firm also sells with probability $1-\alpha\gamma_H$,and it sells with probability 1 when it charges $\underline{p}_H^D$. The only restriction on out-of-equilibrium beliefs is that a firm that does not claim high quality and charges any price below V_H is deemed to be low quality with probability one.

To derive this equilibrium, note that low quality firm must be indifferent between charging V_H without claiming high quality and sticking to its equilibrium strategy i.e.,

$$(V_H - c_L)\frac{\alpha(1-\gamma_H)\eta}{2} = (\overline{p}_L - c_L)\alpha(1-\gamma_H)$$

and this reduces to :

$$(V_H - c_L)\frac{\alpha\eta}{2} = (\overline{p}_L - c_L)\alpha, \tag{129}$$

which is exactly the binding incentive constraint used in the D1 pure price signaling to pin down the value of η in (125) so that $\eta = \eta^S$. The equilibrium profit of the high quality firm is therefore:

$$\pi_H^* = (V_H - c_H)\frac{\alpha\eta^S(1-\gamma_H)}{2} \tag{130}$$

and the equilibrium profit of the low quality firm is

$$\pi_L^* = (V_L - c_L)\alpha(1-\gamma_H) \tag{131}$$

Further, as

$$(\underline{p}_L - c_L)(1-\alpha\gamma_H) = \pi_L^* \tag{132}$$

we have

$$\underline{p}_L = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right](V_L - c_L) + c_L \tag{133}$$

The upper bound of prices for a high quality firm that discloses is now:

$$\overline{p}_H^D = \underline{p}_L + \Delta V = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right](V_L - c_L) + c_L + \Delta V \tag{134}$$

which is decreasing in γ_H and converges to $\overline{p}_L$ as $\gamma_H \to 1$. The profit of the high quality firm when it discloses and charges price $\overline{p}_H^D$ is given by

$$(\overline{p}_H^D - c_H)(1-\alpha\gamma_H) - D \tag{135}$$

$$= (1-\alpha\gamma_H)\left\{\left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right](V_L - c_L) + \Delta V - \Delta c\right\} - D \tag{136}$$

and this is equal to π_H^* if, and only if,

$$\{[\alpha(1-\gamma_H)](V_L-c_L)+(1-\alpha\gamma_H)(\Delta V-\Delta c)\}-D=(V_H-c_H)\frac{\alpha\eta^S(1-\gamma_H)}{2}$$

i.e.,

$$D=\alpha(1-\gamma_H)(1-\frac{\eta^S}{2})(V_H-c_H)+(1-\alpha)(\Delta V-\Delta c)$$

which yields the value of $\gamma_H \in (0,1)$ mentioned above. Indeed, γ_H is strictly decreasing in D, $\gamma_H \downarrow 0$ as $D \uparrow \widehat{D}$ and $\gamma_H \uparrow 1$ as $D \downarrow (1-\alpha)(\Delta V-\Delta c)$. The lower bound $\underline{p}_H^D$ for the high quality price when it discloses satisfies:

$$(\underline{p}_H^D-c_H)=(\overline{p}_H^D-c_H)(1-\alpha\gamma_H)=\pi_H^*+D \tag{137}$$

and this yields:

$$\underline{p}_H^D=\left[\left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right](V_L-c_L)+\Delta V-\Delta c\right](1-\alpha\gamma_H)+c_H. \tag{138}$$

The distribution function $F(.)$ for the low quality price satisfies:

$$(p_L-c_L)[\alpha(1-\gamma_H)+(1-\alpha)(1-F(p_L))]=\pi_L^*=\alpha(1-\gamma_H)(V_L-c_L), p_L\in[\underline{p}_L,\overline{p}_L]. \tag{139}$$

The distribution function $G(.)$ for the high quality price when the firm discloses satisfies:

$$\begin{aligned} & (p_H^D-c_H)[(1-\alpha\gamma_H)+\alpha\gamma_H(1-G(p_H^D))] \\ = {} & \pi_H^*+D \qquad (140) \\ = {} & \left[\left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right](V_L-c_L)+\Delta V-\Delta c\right](1-\alpha\gamma_H), p_H^D\in[\underline{p}_H^D,\overline{p}_H^D] \qquad (141) \end{aligned}$$

This completes the description of the equilibrium.

Next, we show that there is no incentive to deviate from this equilibrium. To begin note that if f is large enough, a low quality firm has no incentive to falsely claim high quality. It is easy to check that given the out-of-equilibrium belief, no high quality firm can strictly gain by deviating from its equilibrium strategy without disclosing. As the high quality firm gets the entire market at price $\underline{p}_H^D$ when it discloses, it has no incentive to disclose and charge price below $\underline{p}_H^D$. Nor can it gain by charging a price above V_H (sells zero). It remains to check that a high quality firm cannot gain by disclosing and charging an out-of-equilibrium price $p_H \in (\overline{p}_H^D, p_H^{ND}).t$For any such deviation price p_H, there exists $p_L = p_H - \Delta V \in (\underline{p}_L, \overline{p}_L)$. The deviation profit is given by:

$$\begin{aligned} & [\alpha(1-\gamma_H)+(1-\alpha)(1-F(p_H-\Delta V))](p_H-c_H)-D \\ = {} & [\alpha(1-\gamma_H)+(1-\alpha)(1-F(p_L))](p_L+\Delta V-c_H)-D \\ = {} & \left[\frac{p_L+\Delta V-c_H}{p_L-c_L}\right]\pi_L^*-D, \text{ using (139)}, \end{aligned}$$

and since $\frac{p_L+\Delta V-c_H}{p_L-c_L}$ is strictly decreasing in p_L (as $\Delta V > \Delta c$) this is

$$\begin{aligned} \leq {} & \left[\frac{\underline{p}_L+\Delta V-c_H}{\underline{p}_L-c_L}\right]\pi_L^*-D=\left[\underline{p}_L+\Delta V-c_H\right](1-\alpha\gamma_H)-D \\ = {} & \left[\overline{p}_H^D-c_H\right](1-\alpha\gamma_H)-D=\pi_H^* \end{aligned}$$

using (135) and (137). Therefore, the deviation cannot be strictly gainful. We now look at the incentive of a low quality firm to deviate. Whether or not it claims high quality, the firm will sell zero if it charges price above V_H (even if it is thought of as a high quality firm). Given the out-of-equilibrium beliefs, if a low quality firm deviates without claiming high quality and charging price $\in (\overline{p}_L, V_H)$ it will be thought of as a low quality firm and will sell zero. If it charges price $p_L < \underline{p}_L$ (without claiming high quality) it will be perceived as a low quality firm but may be able to attract more buyers in the state where the rival is high quality and claims high quality; without loss of generality, consider a deviation to $p_L \in [\underline{p}_H^D - \Delta V, \underline{p}_L)$. The deviation profit is then given by

$$\begin{aligned}
& [(1-\alpha\gamma_H) + \alpha\gamma_H(1 - G(p_L + \Delta V))](p_L - c_L) \\
= \; & [(1-\alpha\gamma_H) + \alpha\gamma_H(1 - G(p_H^D))](p_H^D - \Delta V - c_L) \text{ where } p_H^D = p_L + \Delta V \\
= \; & \left[\frac{p_H^D - \Delta V - c_L}{p_H^D - c_H}\right](\pi_H^* + D), \text{using (140)}
\end{aligned}$$

and since $\left[\frac{p_H^D - \Delta V - c_L}{p_H^D - c_H}\right]$ is strictly increasing in p_H^D (as $\Delta V > \Delta c$) this is

$$\begin{aligned}
\leq \; & \left[\frac{\overline{p}_H^D - \Delta V - c_L}{\overline{p}_H^D - c_H}\right](\pi_H^* + D) \\
= \; & \left[\overline{p}_H^D - \Delta V - c_L\right](1-\alpha\gamma_H) \\
= \; & (\underline{p}_L - c_L)(1-\alpha\gamma_H) = \pi_L^*
\end{aligned}$$

and thus the deviation is not strictly gainful. This establishes the mixed disclosure equilibrium.

The welfare loss in this mixed disclosure equilibrium:

$$WL^M = 2\alpha\gamma_H D + 2\alpha(1-\gamma_H)(1-\alpha)(\Delta V - \Delta c) + \alpha^2(1-\gamma_H)^2(1-\eta^S)(V_H - c_H) \tag{142}$$

and so using (126),

$$\begin{aligned}
& WL^M - WL^S \\
= \; & 2\alpha\gamma_H D - 2\alpha\gamma_H(1-\alpha)(\Delta V - \Delta c) - \alpha^2\gamma_H(2-\gamma_H)(1-\eta^S)(V_H - c_H) \\
\leq \; & 0
\end{aligned}$$

iff

$$D \leq (1-\alpha)(\Delta V - \Delta c) + \alpha(1 - \frac{\gamma_H}{2})(1-\eta^S)(V_H - c_H) \tag{143}$$

As $D \downarrow (1-\alpha)(\Delta V - \Delta c), \gamma_H \uparrow 1$ so that the right hand side of (143) decreases and converges to

$$\begin{aligned}
& (1-\alpha)(\Delta V - \Delta c) + (\frac{1}{2})\alpha(1-\eta^S)(V_H - c_H) \\
> \; & (1-\alpha)(\Delta V - \Delta c)
\end{aligned}$$

so that for the direct communication cost at the lower end of the interval $((1-\alpha)(\Delta V - \Delta c), D\hat{})$,

$$D < (1-\alpha)(\Delta V - \Delta c) + \alpha(1 - \frac{\gamma_H}{2})(1-\eta^S)(V_H - c_H)$$

which implies $WL^M < WL^S$ i.e., high regulation (and the associated mixed disclosure outcome) dominates no regulation in terms of social surplus. On the other hand, as $D \uparrow \widehat{D}$, $\gamma_H \downarrow 0$ so that the right hand side of (143) increases and converges to

$$\begin{aligned} & (1-\alpha)(\Delta V - \Delta c) + \alpha(1-\eta^S)(V_H - c_H) \\ < \quad & (1-\alpha)(\Delta V - \Delta c) + \alpha(1-\frac{\eta^S}{2})(V_H - c_H) = D\hat{} \end{aligned}$$

so that for disclosure cost D at the upper end of the interval $((1-\alpha)(\Delta V - \Delta c), D\hat{})$,

$$D > (1-\alpha)(\Delta V - \Delta c) + \alpha(1-\frac{\gamma_H}{2})(1-\eta^S)(V_H - c_H)$$

so that $WL^M > WL^S$, i.e., a high fine f (and the associated mixed disclosure outcome) generates lower social surplus than no fine (pure price signaling welfare dominates mixed disclosure). Note that the right hand side of (143) is strictly decreasing in γ_H.Thus, there exists a critical $D_0 \in ((1-\alpha)(\Delta V - \Delta c), \widehat{D})$ such that for "intermediate" direct communication costs $D \in (D_0, \widehat{D})$, arbitrarily high fines are strictly worse than no fine despite the added quantity distortion. On the other hand, the correction of quantity distortion through direct communication implies that mixed disclosure equilibria can welfare dominate no fine if the direct communication cost is below a critical level. The range of the direct communication cost for which high fines are desirable is broader than in the version of the model without quantity distortion.