Production Clustering and Offshoring

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Online Appendix

A. Proofs of Propositions

Proposition 2

PROOF:

Let's assume there is an optimal path A for τ_0, an optimal path B for τ_1, $\tau_0 > \tau_1$, and $MC(A, \tau_0, C) < MC(B, \tau_1, C)$, where C is a vector of the costs of production. Note that $MC(A, \tau_0, C) \geq MC(A, \tau_1, C)$ and by optimality of B: $MC(A, \tau_1, C) \geq MC(B, \tau_1, C)$. It follows that $MC(A, \tau_0, C) \geq MC(B, \tau_1, C)$, which contradicts the initial assumption.

Lemma 1

PROOF:

Let path A with transportation quantity $TQ(A)$ be chosen for $\tau = \tau_0$ and path B with transportation quantity $TQ(B)$ be chosen for $\tau = \tau_1$ and $\tau_0 > \tau_1$. Now assume that the transportation quantity is an increasing function of τ and hence $TQ(A) > TQ(B)$. Then given the choice that the firm made under τ_1: $NTMC(B) + TQ(B)\tau_1 < NTMC(A) + TQ(A)\tau_1$ and under τ_0: $NTMC(B) + TQ(B)\tau_0 > NTMC(A) + TQ(A)\tau_0$. Adding $TQ(B)(\tau_0 - \tau_1)$ to the first inequality, I get: $NTMC(B) + TQ(B)\tau_0 < NTMC(A) + TQ(A)\tau_1 + TQ(B)(\tau_0 - \tau_1) < NTMC(A) + TQ(A)\tau_1 + TQ(A)(\tau_0 - \tau_1) = NTMC(A) + TQ(A)\tau_0$ or $NTMC(B) + TQ(B)\tau_1 < NTMC(A) + TQ(A)\tau_1$, which contradicts the condition on optimality of A under τ_0.

Proposition 3

PROOF:

Let's assume $\tau_0 > \tau_1$. Let A be an optimal path for $\tau = \tau_0$ and transportation quantity $TQ(A) > 0$. Then by Proposition 2 $MC(A, \tau_0) > MC(A, \tau_1)$. Let B an optimal path for τ_1, then by definition of optimal path $MC(A, \tau_1) \geq MC(B, \tau_1)$, and hence $MC(A, \tau_0) > MC(B, \tau_1)$.

Proposition 4

PROOF:

Let A be an optimal path for τ_0, B an optimal path for τ_1, $\tau_0 > \tau_1$, and $A \neq B$. By definition of optimality and because of the uniqueness of optimal paths, $MC(A, \tau_0) < MC(B, \tau_0)$ and $MC(A, \tau_1) > MC(B, \tau_1)$. From Lemma 1 $\tau_1 TQ(A) < \tau_1 TQ(B)$. Assume $NTMC(A) < NTMC(B)$, then

$MC(A, \tau_1) = NTMC(A) + \tau_1 TQ(A) < NTMC(B) + \tau_1 TQ(B) = MC(B, \tau_1)$, which contradicts the optimality of B under τ_1.

B. *Incomplete Trees*

To write down the problem for an arbitrary tree, I need to enumerate production nodes. Every node has a unique index $\{i, b\}$ that represents at what stage i the part is produced and to what branch b it belongs. Production costs for a part from branch b, produced on stage i in country k, are then $a_{i,b,k}$.

Stage $i = 1$ corresponds to the most downstream stage of production and $i = N$ denotes the most upstream stage. In case two or more of the intermediate goods are assembled together, each of the corresponding nodes gets the same stage number i; in addition, each of these nodes gets branch index b, which was not previously assigned to another branch.

I define n_b as the last stage of branch b; I call n_b the length of branch b. In addition, for each stage i I introduce an assembly set $\Omega_{i,b}$, which is the set of branch indexes b of all parts produced on stage $i+1$, connected to stage $\{i, b\}$. $\nu_{i,b}$ is a branch of a part produced at stage $i-1$, a node that $\{i, b\}$ is connected to. B_i is a set of all branches present at stage i. I present an example of such enumeration in Figure .1.

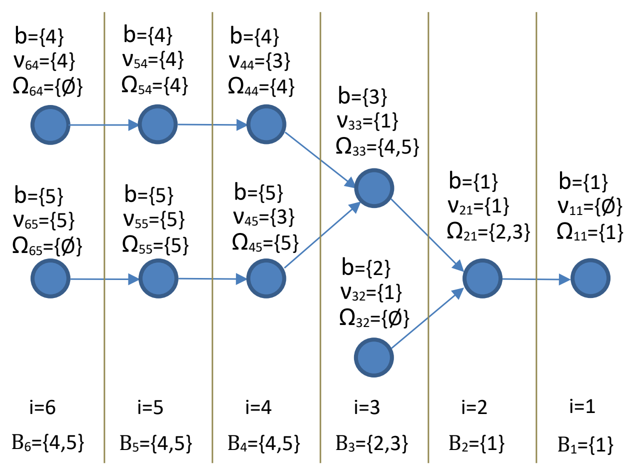

FIGURE .1. INCOMPLETE TREE NOTATION

$$MC = \min_{\{c_{i,b}\}} \sum_{i=1}^{\max\{n_b\}} \sum_{b \in B_i} \left(\sum_{k=1}^{K} \mathbf{1}\left(c_{i,b} = k\right) a_{i,b,k} + \tau T\left(c_{i,b}, c_{i-1\nu_{i,b}}\right) \right).$$

This expression differs from (4) due to more complex indexing structure of an incomplete tree. The corresponding Bellman equation is

$$V_{i,b}\left(c_{i,b}\right)=\min_{c_{i,b}\in K}\left\{\sum_{k=1}^{K}\mathbf{1}\left(c_{i,b}=k\right)a_{i,b,k}+\sum_{l\in\Omega_{i,b}}\left[\tau T\left(c_{i,b},c_{i+1,l}\right)+V_{i+1l,}\left(c_{i+1,l}\right)\right]\right\}.$$

C. *Clustering with Iceberg Trade Costs*

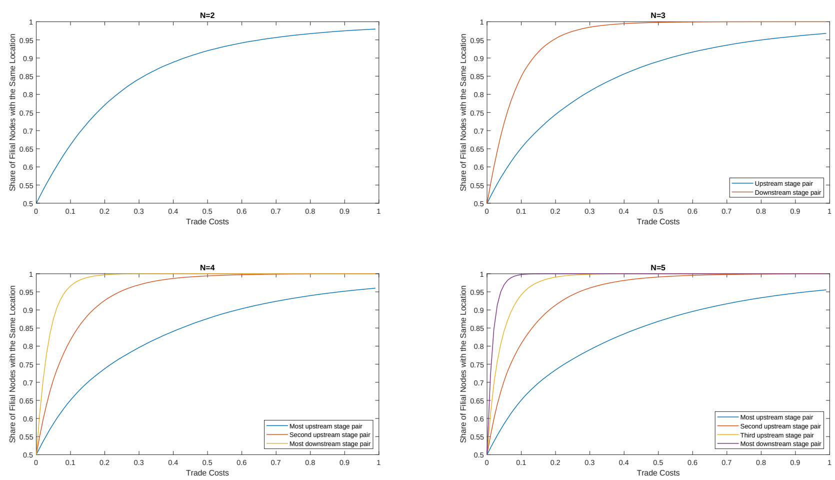

FIGURE .2. CLUSTERING AND TREE LENGTH: ICEBERG TRADE COSTS

D. *Elasticities*

E. *Endogenous Wages*

The problem presented above is the model of absolute advantage as there is no labor market. With a given supply of labor in each country L_j and endogenous wages that are determined through labor market clearing conditions, all countries will produce some parts no matter what production costs are.[1] I normalize the wage in country 1 to $w_1 = 1$. I assume that labor supply is perfectly inelastic and the firm has constant returns to scale production technology. The problem of every firm then looks like

[1]As long as trade costs are not too high for a given firm.

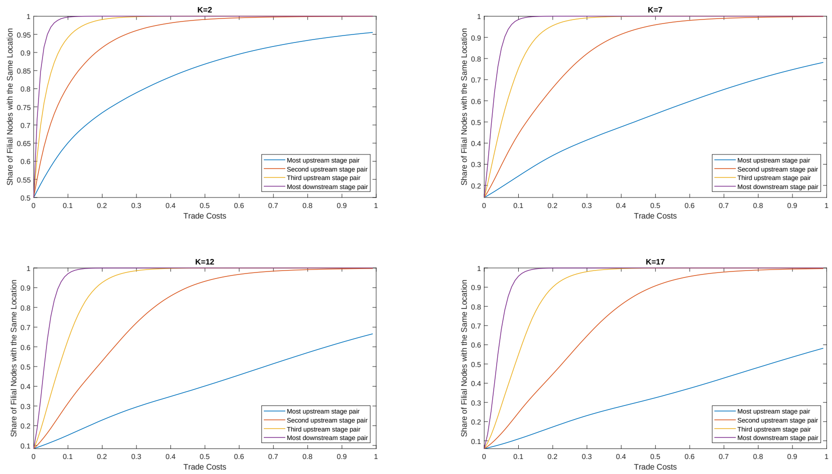

FIGURE .3. CLUSTERING AND THE NUMBER OF COUNTRIES: ICEBERG TRADE COSTS

$$MC = \min_{c_{i,b}} \sum_{i=1}^{N} \sum_{b=1}^{M^{i-1}} \left(w_j \mathbf{1}\left(c_{i,b} = k\right) a_{i,b,k} + \tau T\left(c_{i,b}, c_{i-1,\lceil \frac{b}{M} \rceil}\right)\right), \tag{.1}$$

and a firm's labor demand per unit produced is

$$L_{Dk} \equiv \sum_{i=1}^{N} \sum_{b=1}^{M^{i-1}} \mathbf{1}\left(c_{i,b} = k\right) a_{i,b,k} \text{ for } \forall k \in \{1, ..., K\}.$$

Here for simplicity I assume that transportation services are performed by independent transport companies and do not affect domestic and foreign labor markets.

LEMMA A1: *Demand of the firm from country i L_{Di} for labor in country k is a nonincreasing function of w_k.*

PROOF:

Let the wage in country k decrease, while all other wages remain constant: $w_k^A > w_k^B$ and $w_{j\neq k}^A = w_{j\neq k}^B = w_{j\neq k}$. Let A and B be optimal paths under wage schedules w^A and w^B. In case $A = B$, $L_{Dk}^A = L_{Dk}^B$. Now consider the case $A \neq B$. Then because of the optimality of A and B: (a) $MC\left(A, w^A\right) < MC\left(A, w^B\right)$

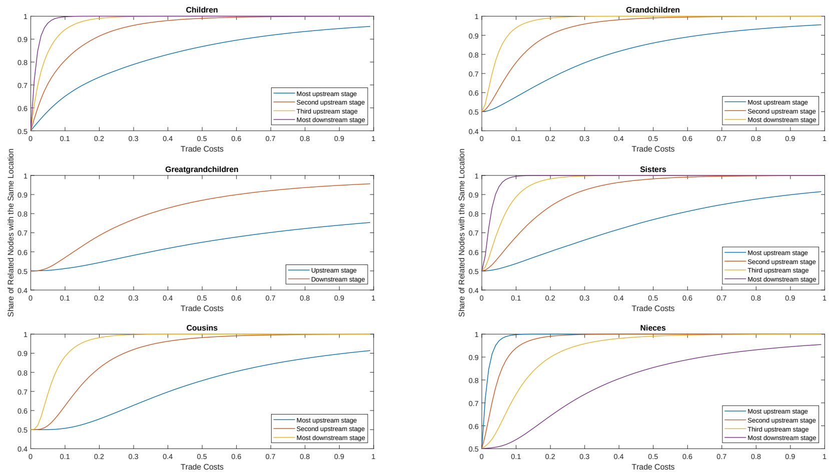

Figure .4. Direct and Indirect Clustering: Iceberg Trade Costs

and (b) $MC\left(B, w^B\right) < MC\left(A, w^B\right)$. Let $\Delta^{\tau VT} \equiv \tau VT\left(A\right) - \tau VT\left(B\right)$, and $\Delta^L \equiv \sum_{j\neq k} w_j \left(L^A_{Dj} - L^B_{Dj}\right)$. Then (a) and (b) can be rewritten as: $L^A_k w^A_k - L^B_k w^A_k + \Delta^L + \Delta^{\tau VT} < 0$, and $L^A_k w^B_k - L^B_k w^B_k + \Delta^L + \Delta^{\tau VT} > 0$, subtracting the first inequality from the second obtains: $\left(L^A_{Dk} - L^B_{Dk}\right)\left(w^B_k - w^A_k\right) > 0$, and then $L^A_{Dk} < L^B_{Dk}$.

Note that if the firm changes its optimal path, then L_{Dk} is decreasing in w_k.

PROPOSITION A1: *There exists a wage schedule that clears the labor market. In a two-country case, this schedule is unique.*

PROOF:

Existence:

The world economy can be considered as an exchange economy with M agents, where labor supply in country k is the endowment of good k and wage in country k is the price of this good. Then from Lemma A1 demand of each agent for each good is nondecreasing in price of this good, so by proposition 17.C.1 in Mas-Colell et al. (1995) an equilibrium exists.

Uniqueness for the case of two countries:

A firm's relative demand for labor $\frac{L}{L^*}$ is a nonincreasing function of the relative wage w. Every firm takes the wage as given, but decisions of the firm determine the wage through market clearing condition. Here once again I apply the revealed preferences argument. Let's assume there is path A with $\sum_{i=1}^N c_i a_{Wi} = R_{WA}$ and $\sum_{i=1}^N \left(1-c\right)_i a_{Ei} = R_{EA}$ that was chosen for $w = w_0$ and there is path

B with $\sum_{i=1}^{N} c_i a_{Ni} = R_{WB}$ and $\sum_{i=1}^{N} (1-c)_i \, a_{Si} = R_{EB}$ that was chosen for $w = w_1$; $w_1 > w_0$ and $R_{WB} > R_{WA}$. Let function $NPC\,(Y)$ be a value of nonproduction costs for path Y, then given the choice that the firm made under w_0: $NPC\,(A) + w_0 R_{WA} + R_{NA} < NPC\,(B) + w_0 R_{WB} + R_{EB}$ and under w_1: $NPC\,(A) + w_1 R_{WA} + R_{EA} > NPC\,(B) + w_1 R_{WB} + R_{EB}$. Adding $R_{WA}\,(w_1 - w_0)$ to the both parts of the first inequality, I get: $NPC\,(A) + w_1 R_{WA} + R_{EA} < NPC\,(B) + w_0 R_{WB} + R_{EB} + R_{WA}\,(w_1 - w_0) < NPC\,(B) + w_1 R_{WB} + R_{EB} < NPC\,(B) + w_1 R_{WB} + R_{EB}$ or $NPC\,(A) + w_1 R_{WA} + R_{EA} < NPC\,(B) + w_1 R_{WB} + R_{EB}$, which contradicts the condition of optimality of B under w_1.

For the case of multiple countries, proof of uniqueness of the equilibrium is nontrivial: decrease in the wage in one country can increase demand for labor in another country through the bridge FDI channel, similar to Proposition 8. As a result, the gross substitute property does not hold, and the uniqueness cannot be proven using the approach of Allen, Arkolakis and Li (2015).

REFERENCES

Allen, Treb, Costas Arkolakis, and Xiangliang Li. 2015. "On the existence and uniqueness of trade equilibria." *Manuscript, Yale Univ.*

Mas-Colell, Andreu, Michael Dennis Whinston, Jerry R Green, et al. 1995. *Microeconomic theory.* Vol. 1, Oxford university press New York.

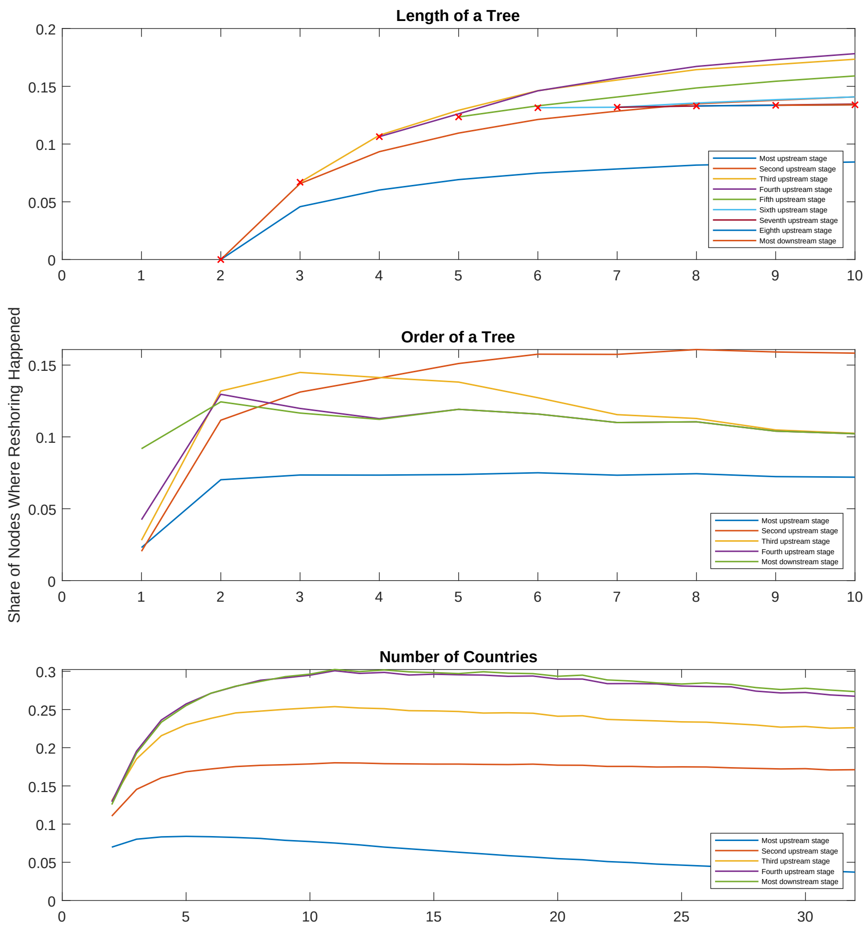

Figure .5. Reshoring: Iceberg Trade Costs

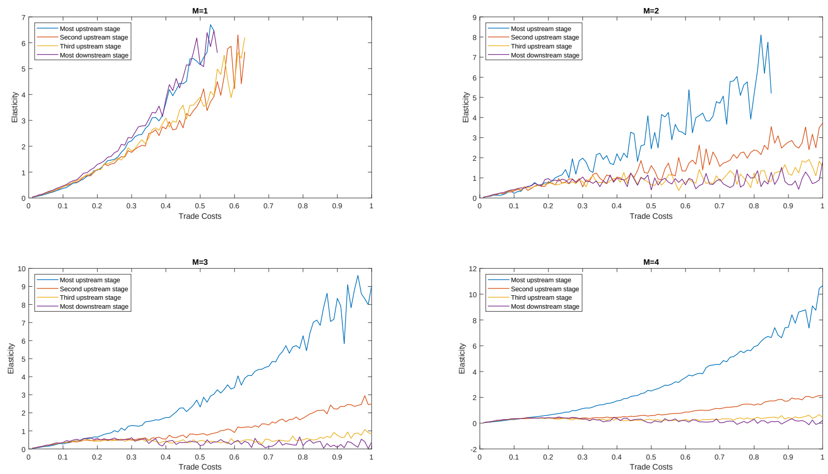

Figure .6. Trade Elasticities and Tree Order

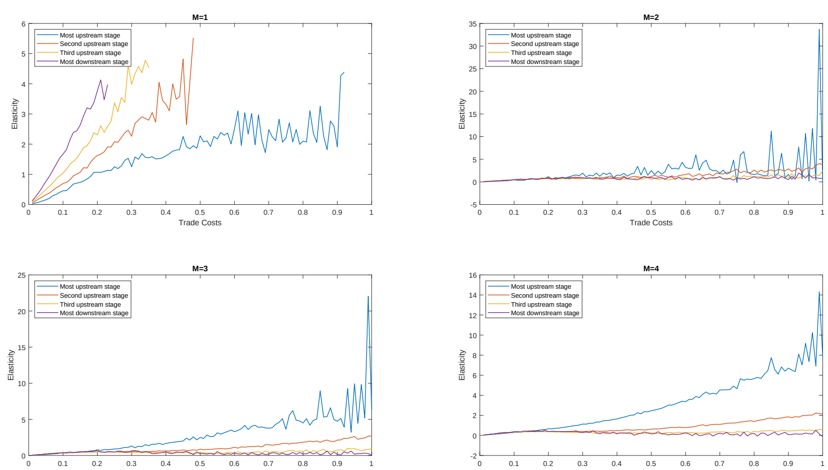

Figure .7. Trade Elasticities and Tree Order: Iceberg Trade Costs

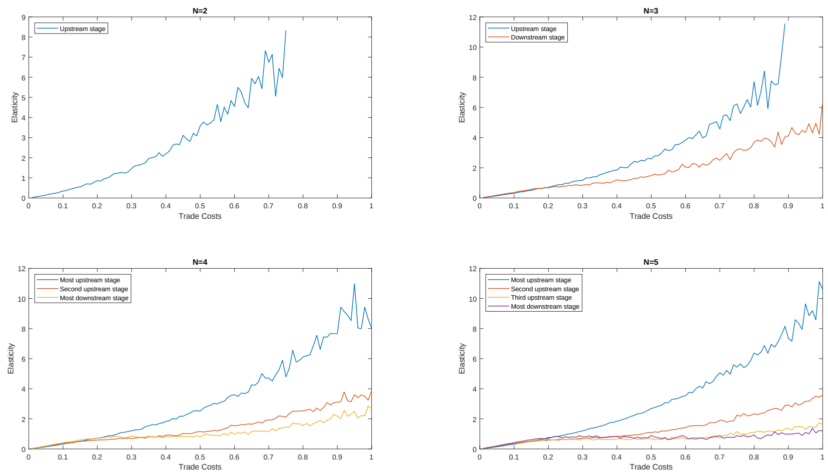

Figure .8. Trade Elasticities and Tree Length

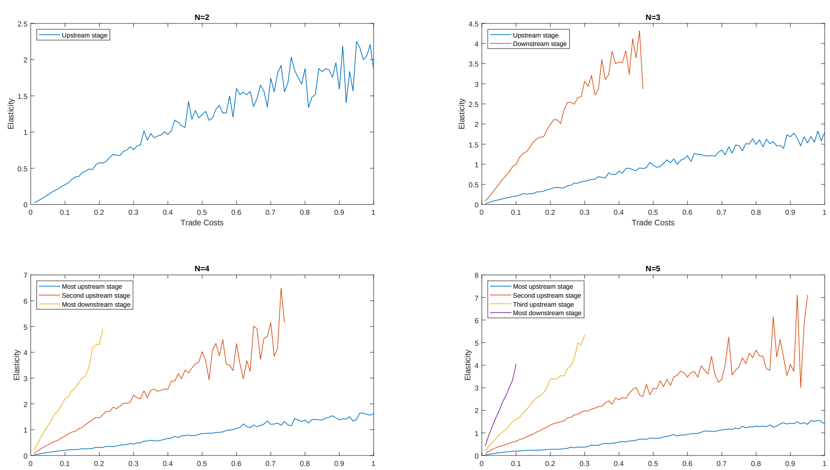

Figure .9. Trade Elasticities and Tree Length: Iceberg Trade Costs

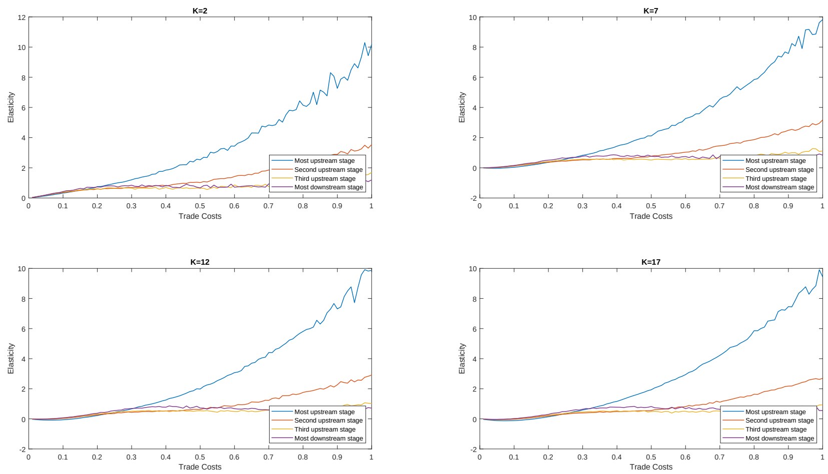

Figure .10. Trade Elasticities and the Number of Countries

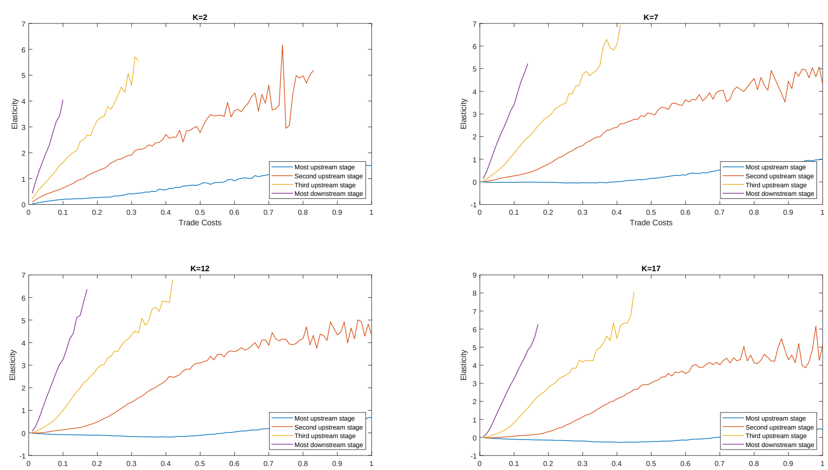

Figure .11. Trade Elasticities and the Number of Countries: Iceberg Trade Costs