Online Appendix
Intermediation and steering: Competition in prices and commissions

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In Section A of this online appendix, we provide proofs of omitted results and details from the baseline model of the main paper. In Section B we provide the formal analysis and more detailed results for Section 5 in the main paper (i.e., the policy implications), and in Section C we do likewise for Section 6 (i.e., the extensions).

A Further results of the baseline model

A.1 Sub-optimality of full disclosure by M

In the main text, we restricted M's communication space to product rankings. A natural question is whether M has an incentive to reveal everything to consumers (full disclosure) if it is able to do so. Based on the same equilibrium described in Section 3, in what follows we argue that M cannot do better by deviating to full disclosure.

Obviously, full disclosure does not affect the on-equilibrium path outcome. In the equilibrium, all firms offer symmetric commissions and prices, so M ranks product j' first if and only if $j' = \arg\max_{j'=1,..,n} \{\epsilon_{j'} - p^*\}$, and consumers will buy the top-ranked product or the outside option. If M reveals all information, consumers would not change their decision.

Consider instead the off-equilibrium path in which some firms deviate by charging off-equilibrium prices and commissions. By ranking products according to expected commissions, M's expected profit is

$$\Pi = \max_{j=1,..,n} \{\tau_j (1 - G(p_j - \epsilon_j))\}$$

given that consumers always buy the top-ranked product. By fully disclosing all information, consumers will buy the highest-surplus product instead, and the expected profit to M is

$$\Pi' = \tau_{j'} (1 - G(p_{j'} - \epsilon_{j'})) \text{ where } j' = \arg \max_{j'=1,..,n} \{\epsilon_{j'} - p_{j'}\}$$

Clearly, $\Pi \geq \Pi'$ because the definition of maximization implies

$$\max_{j=1,..,n} \{\tau_j (1 - G(p_j - \epsilon_j))\} \geq \tau_{j'} (1 - G(p_{j'} - \epsilon_{j'})).$$

A.2 General message space

The derived *informative equilibrium with steering* in the paper remains an equilibrium (PBE) in a game with a general message space. Let S be the general message space and $\bar{S}$ be the set of all messages based on M providing a ranking. Given an equilibrium in which only $\bar{S}$ is used, we can construct a new equilibrium in which all the messages in S are used and the outcome remains the same. Let N denote the number of messages in $\bar{S}$. Partition $S/\bar{S}$ into N subsets, and let S_i denote the i-th subset in the partition of $S/\bar{S}$ and $\bar{s}_i$ denote the i-th message in $\bar{S}$.

Now, in the new equilibrium, whenever M would have sent $\bar{s}_i \in \bar{S}$ in the original equilibrium, it now sends any message $\{\bar{s}_i\} \cup S_i$ (or uses a mixed message with the subset as a support). As each consumer's inference after receiving a message will be the same as in the original equilibrium, M would use this strategy, and this is an equilibrium with a general message S.

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A.3 Wary beliefs

To characterize the informative equilibrium with steering in the main text, we have specified that consumers hold *passive beliefs* over unobserved commissions, as is commonly assumed in the literature on vertical contracting. An alternative approach also analyzed by that literature is called wary beliefs (McAfee and Schwartz, 1994), which has been generalized by In and Wright (2018). When observing a deviating contract offered from a common manufacturer, a retailer that holds wary beliefs will try to infer how the manufacturer should have optimally (and secretly) adjusted contracts offered to other, competing downstream retailers. In what follows, we show that our equilibrium characterization in Proposition 1 remains valid under wary beliefs, provided F and G are linear.

A consumer, after inspecting a product i and observing an off-equilibrium price $p_i \neq p^*$, tries to infer how firm i, in anticipation of this deviation, should have optimally adjusted its commission τ_i. When firms still expect the consumers to follow M's recommendation, we will show that given F and G are linear, an individual firm's optimal τ_i is independent of p_i. In other words, consumers are unable to infer anything new about τ_i from the observed p_i. Consequently, under wary beliefs, consumers continue to infer that M, whose recommendation strategy remains described by (1), is recommending the highest-surplus product. Therefore, it is indeed optimal for consumers to only search the recommended product without searching further.

Recall that a deviant firm i's first-order condition for optimal commission τ_i is

$$\frac{\partial D_i}{\partial \tau_i} - \frac{\partial D_i}{\partial p_i} = 0.$$

Suppose F and G are linear over $[\underline{\epsilon}, \bar{\epsilon}]$ and $[\underline{v}, \bar{v}]$, we have

$$\begin{aligned}
\bar{x}_i(\epsilon) &= -G^{-1}\left(1 - \frac{\tau^*}{\tau_i}\left(1 - G\left(p^* - \epsilon\right)\right)\right) \\
&= -\left[(\bar{v} - \underline{v})\left(1 - \frac{\tau^*}{\tau_i}\left(1 - \left(\frac{p^* - \epsilon - \underline{v}}{\bar{v} - \underline{v}}\right)\right)\right) + \underline{v}\right] \\
&= \frac{\tau^*}{\tau_i}\left(\bar{v} + \epsilon - p^*\right) - \bar{v}.
\end{aligned}$$

The demand function and its derivatives are given by

$$\begin{aligned}
D_i &= \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - \frac{\left(\max\left\{\bar{x}_i\left(\epsilon\right), -v\right\}\right) + p_i - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}\right] \left(\frac{n-1}{\bar{\epsilon} - \underline{\epsilon}}\right) \left(\frac{\epsilon - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}\right)^{n-2} \left(\frac{1}{\bar{v} - \underline{v}}\right) d\epsilon dv \\
\frac{\partial D_i}{\partial p_i} &= -\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1}{\bar{\epsilon} - \underline{\epsilon}}\right] \left(\frac{n-1}{\bar{\epsilon} - \underline{\epsilon}}\right) \left(\frac{\epsilon - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}\right)^{n-2} \left(\frac{1}{\bar{v} - \underline{v}}\right) d\epsilon dv < 0, \\
\frac{\partial D_i}{\partial \tau_i} &= \int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_i}{\tau^*}(\bar{v} - v) + p^* - \bar{v}}^{\bar{\epsilon}} \left[\frac{\tau_j}{\tau_i^2}\left(\bar{v} + \epsilon - p_j\right)\left(\frac{1}{\bar{\epsilon} - \underline{\epsilon}}\right)\right] \left(\frac{n-1}{\bar{\epsilon} - \underline{\epsilon}}\right) \left(\frac{\epsilon - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}\right)^{n-2} \left(\frac{1}{\bar{v} - \underline{v}}\right) d\epsilon dv > 0.
\end{aligned}$$

Let $\hat{\tau}_i$ denote, for each p_i, the optimal commission defined by the first-order condition $\frac{\partial D_i}{\partial \tau_i} = -\frac{\partial D_i}{\partial p_i}$. As per the standard supermodularity argument, using the implicit function theorem we can pin down how $\hat{\tau}_i$ changes with p_i as follows:

$$\frac{\partial \hat{\tau}_i}{\partial p_i} = -\left(\frac{\frac{\partial^2 D_i}{\partial p_i \partial \tau_i} - \frac{\partial^2 D_i}{\partial p_i^2}}{\frac{\partial^2 D_i}{\partial \tau_i^2} - \frac{\partial^2 D_i}{\partial p_i \partial \tau_i}}\right).$$

Crucially, the linearity of demand implies $\frac{\partial^2 D_i}{\partial p_i \partial \tau_i} = 0$ and $\frac{\partial^2 D_i}{\partial p_i^2} = 0$. Meanwhile, it is easily verified that $\frac{\partial^2 D_i}{\partial \tau_i^2} < 0$. Therefore $\frac{\partial \hat{\tau}_i}{\partial p_i} = 0$, meaning that firm i's optimal commission does not depend on the price it sets.

A.4 Steering with commitment

In the main text we assumed M cannot commit to its recommendation rule. While this non-commitment assumption fits our primary motivating examples of financial and insurance brokers, one may nonetheless be interested in what happens when M can announce and commit to specific recommendation rules before firms set prices and commissions. In particular, this means that M's recommendation is no longer required to be sequentially rational. All other aspects of the model follow our baseline model in Section 2 of the main text.

We first note that among all possible recommendation rules, the upper bound to the profit achievable by M is the maximized joint-industry profit, that is

$$\widehat{\Pi} \equiv \max_{p} \left\{ p \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left[(1 - G(p - \epsilon)) \right] dF^n(\epsilon) \right\}. \tag{A.1}$$

The possibility of price discrimination is ruled out because firms set their price before M observes consumer match values. With the constraint of uniform pricing by firms, the highest achievable profit is then exactly $\widehat{\Pi}$. Here, $\widehat{\Pi}$ is the same profit as obtained by a monopolist that sells a product with valuation $v + \max_{i=1,..,n} \{\epsilon_i\}$ to consumers directly assuming consumers are fully informed.

It turns out that M can exactly achieve profit $\widehat{\Pi}$ by committing to (i) recommend the highest commission product subject to the price cap $\bar{p}$; and (ii) when there are multiple products with the highest commission, M breaks ties in favor of the product with the highest surplus. To see why the price cap is necessary, suppose first that M commits to recommending the highest commission product. Each firm always has an incentive to slightly increase its level of commission provided it earns a positive margin, so as to attract the entire market. The standard Bertrand logic implies that in the resulting equilibrium, a typical firm i will set its commission at $\tau_i = p_i$ such that it earns a zero margin. Crucially, however, p_i is a choice variable so firm i can always profitably simultaneously slightly increase p_i and increase τ_i by almost the same amount to beat its rival in the competition for recommendations, and earn a positive margin, as long as the chosen p_i still leads to a positive demand (i.e., that there are realizations such that $v + \epsilon_i - p_i \geq 0$). Given the distribution support of ϵ_i and v, firm i will want to keep raising its price (and commission) until $p_i = \bar{v} + \bar{\epsilon}$. Hence, the only possible equilibrium outcome is one where all firms set $\tau_i = p_i = \bar{v} + \bar{\epsilon}$. This is clearly an undesirable outcome for M because all consumers will prefer the outside option except when the match value realization is such that $\epsilon_i = \bar{\epsilon}$ and $v = \bar{v}$, which is a zero probability event.

In contrast, with the addition of a price cap, the outcome would be firms all set commissions and prices at the level of the imposed price cap. This means that M can use its price cap to implement any desired final product price, in particular, the price that is associated with (A.1).[1] Moreover, given that all firms are offering the same commission in equilibrium, M provides an unbiased recommendation to consumers so that consumers continue to believe that M's recommendation is informative. To summarize:

Proposition 9 *Suppose M can credibly commit to always recommending the highest commission product subject to a price cap $\bar{p}$, and it breaks ties in favor of the product with the highest surplus. Then it can obtain profit $\widehat{\Pi}$, which is the highest possible profit that M can achieve among all possible recommendation rules.*

Under steering with commitment, the resulting final price is as if there is a single monopoly selling a product with valuation $v + \max_{i=1,..,n} \{\epsilon_i\}$, i.e. M's optimal price cap solves (A.1). Comparing this price level to the resulting price without steering, the following proposition, which is analogous to Propositions 2 - 4, shows that all our insights on the implications of steering in Section 4 remain valid even when M has commitment power.

[1] The outcome is similar to that arising when M sets a common per-transaction fee on all firms, as shown in Section 6.2. Here, however, because such a price-cap plus commitment eliminates the firms' margins in equilibrium, it delivers the highest possible profit to M.

Proposition 10 *When M can steer with commitment:*

1. *$\bar{p}$ is increasing in n. If $\bar{\epsilon} < \infty$ then $\lim_{n \to \infty} \bar{p} \to \tau^m$.*
2. *The level of prices and commissions as well as Π_M are higher than the equilibrium without steering.*
3. *CS, $\sum \pi_i$, and W are lower than the equilibrium without steering.*
4. *Proposition 5 still holds.*

Proof. Rewrite and expand the definition of $\bar{p}$ as

$$\bar{p} = \arg\max_p \left\{ p \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[(1 - G(p - \epsilon)) n f(\epsilon) F^{n-1}(\epsilon) \right] d\epsilon \right\}.$$

Given that log-concavity is preserved by multiplication, the log-concavity assumption on density functions ensures that the integrand is log-concave. Therefore, the demand function is log-concave because log-concavity is preserved by integration. Consequently, $\bar{p}$ can be pinned down by first-order condition:

$$\frac{1}{\bar{p}} = \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[g(\bar{p} - \epsilon) \right] dF^n(\epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - G(\bar{p} - \epsilon) \right] dF^n(\epsilon)}. \tag{A.2}$$

Log-concavity of demand ensures that the right-hand side of (A.2) is decreasing in $\bar{p}$. To establish $\frac{d\bar{p}}{dn} \geq 0$ it remains to show the right-hand side of (A.2) is decreasing in n. We have

$$\begin{aligned} \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[g(\bar{p} - \epsilon) \right] dF^n(\epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - G(\bar{p} - \epsilon) \right] dF^n(\epsilon)} &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{g(\bar{p} - \epsilon)}{1 - G(\bar{p} - \epsilon)} \right] \frac{1 - G(\bar{p} - \epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - G(\bar{p} - \epsilon) \right] dF^n(\epsilon)} dF^n(\epsilon) \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{g(\bar{p} - x)}{1 - G(\bar{p} - x)} \right] dF_{(n)}\left(x | \epsilon_{(n)} > \bar{p} - v \right), \end{aligned} \tag{A.3}$$

where $F_{(n)}\left(x | \epsilon_{(n)} > \bar{p} - v \right)$ is the CDF of the highest-order statistic $\epsilon_{(n)}$ (out of n i.i.d draws on ϵ), conditioned on the highest-order statistic being greater than $\bar{p} - v$:

$$\begin{aligned} F_{(n)}\left(x | \epsilon_{(n)} > \bar{p} - v \right) &= \Pr\left(\epsilon_{(n)} < x | \epsilon_{(n)} > \bar{p} - v \right) \\ &= \frac{\int_{\underline{\epsilon}}^{x} \left[1 - G(\bar{p} - \epsilon) \right] dF^n(\epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - G(\bar{p} - \epsilon) \right] dF^n(\epsilon)}. \end{aligned}$$

Clearly, the distribution $F_{(n)}\left(x | \epsilon_{(n)} > \bar{p} - v \right)$ is increasing in n in the sense of first-order stochastic dominance (FOSD). This fact, together with log-concavity of g (which implies that $\frac{g(\bar{p}-x)}{1-G(\bar{p}-x)}$ is decreasing in x), ensures that (A.3) is decreasing in n as required. Hence, $\frac{d\bar{p}}{dn} \geq 0$. When $n \to \infty$ and $\epsilon < \infty$, the distribution of F^n collapses to a single point $\bar{\epsilon}$, so $\bar{p} = \arg\max_p \{ p(1 - G(p - \bar{\epsilon})) \}$, the solution of which is exactly τ^m.

Recall the equilibrium price without steering is

$$p^* = \arg\max_p \left\{ p \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - F(\max\{\epsilon - p^*, -v\} + p) \right] dF(\epsilon)^{n-1} dG(v) \right\},$$

and it is the highest when $n = 1$, in which case $p^*_{n=1} = \arg\max_p \left\{ p \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[(1 - G(p - \epsilon)) f(\epsilon) \right] d\epsilon \right\} = \bar{p}_{n=1}$. We know $\bar{p}$ is increasing in n, and so $\bar{p} \geq p^*$ for all $n \geq 1$, as required. We already know the equilibrium commission equals $\bar{p} > 0$ when M steers with commitment, which is obviously higher than zero commission. Since price is higher with steering, it is immediately that CS, $\sum \pi_i$, and W are lower than the equilibrium without steering. Finally, the result of $\lim_{n \to \infty} \bar{p} \to \tau^m$ when M steers immediately implies Proposition 5 still holds. ■

In practice it would be difficult for M to credibly commit to recommending the highest commission product, given that such a recommendation is not sequentially rational. One possible way to implement this would be via an auction mechanism in which all firms could bid openly (so they could see each others' bids, thereby ensuring M sticks to its announced recommendation rule). Among other implementation issues, such a mechanism may be susceptible to collusion between the competing firms. Moreover, including a cap on prices in the mechanism may raise vertical price-fixing issues. Nonetheless, Proposition 9 remains a useful theoretical benchmark showing the highest possible profit that M can achieve when it has commitment power over recommendation rules, and that our main results continue to hold true in this case.

A.5 Equilibrium selection

In the main text, we claim that if we focus on the class of informative (i.e. non-blabbing) equilibria where the first-ranked product has a strictly higher probability to be inspected by consumers relative to other lower-ranked or unranked products, then the informative equilibrium with steering characterized in Section 3.1 is the unique symmetric equilibrium outcome — in particular, consumers inspect the top-ranked item first.

To prove this, suppose by contradiction there exists another candidate symmetric equilibrium in which the top-ranked item has a strictly higher probability to be inspected but consumers do not inspect the top-ranked item first. Therefore, consumers either inspect some lower-ranked or unranked products first, or inspect some randomly chosen products. Regardless of what consumers do, notice that in this candidate equilibrium M necessarily ranks the most suitable item in the first rank. This is because doing so yields the highest ex-ante probability of consumers purchasing something, given that by assumption the top-ranked item has a strictly higher probability to be inspected. Expecting this, consumers must choose to inspect the top-ranked item first, contradicting the initial supposition. Thus, we conclude in any symmetric equilibria where the first-ranked product has a strictly higher probability to be inspected, consumers must inspect the first-ranked product first, and the unique outcome is established by the analysis in Section 3.1.

A.6 Quasi-concavity of profit function

To prove quasi-concavity of the profit function, we first show that firm i's demand function (2) is globally log-concave in (p_i, τ_i) when f is log-concave and G is linear (i.e., g is constant). Recall that firm i's demand equals the following probability:

$$\Pr\left(\epsilon_i - p_i \geq \max\left\{-G^{-1}\left(1 - \frac{\tau^*}{\tau_i}\left(1 - G\left(p^* - \hat{\epsilon}\right)\right)\right), -v\right\}\right),$$

which can be rewritten as

$$\begin{aligned}
D_i &= \Pr\left(p^* - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}\left(1 - G\left(p_i - \epsilon_i\right)\right)\right) > \hat{\epsilon} \text{ and } \epsilon_i - p_i > -v\right) \\
&= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\Pr\left(p^* - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}\left(1 - G\left(p_i - \epsilon_i\right)\right)\right) > \hat{\epsilon}\right)\left(1 - G\left(p_i - \epsilon_i\right)\right)\right] d\epsilon_i, \\
&= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[F\left(p^* - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}\left(1 - G\left(p_i - \epsilon_i\right)\right)\right)\right)^{n-1}\left(1 - G\left(p_i - \epsilon_i\right)\right) f\left(\epsilon_i\right)\right] d\epsilon_i, \qquad \text{(A.4)}
\end{aligned}$$

where the second equality is due to the conditional independence of the two events after conditioning on ϵ_i. The key step of our proof is to show that the integrand function in (A.4):

$$I\left(p_i, \tau_i, \epsilon_i\right) \equiv F\left(p^* - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}\left(1 - G\left(p_i - \epsilon_i\right)\right)\right)\right)^{n-1}\left(1 - G\left(p_i - \epsilon_i\right)\right) f\left(\epsilon_i\right)$$

is log-concave for $(p_i, \tau_i, \epsilon_i) \in [\underline{v} + \underline{\epsilon}, v + \bar{\epsilon}]^2 \times [\underline{\epsilon}, \bar{\epsilon}]$.

We first claim that $I(p_i, \tau_i, \epsilon_i)$ is log-concave within the convex set

$$S \equiv \left\{ (p_i, \tau_i, \epsilon_i) \in [\underline{v} + \underline{\epsilon}, v + \bar{\epsilon}]^2 \times [\underline{\epsilon}, \bar{\epsilon}] \, | \epsilon_i - p_i \geq -\bar{v} \right\}.$$

By assumption $f(\epsilon_i)$ and $(1 - G(p_i - \epsilon_i)\ )$ are log-concave for $(p_i, \tau_i, \epsilon_i) \in S$. Given that log-concavity is preserved by multiplication, to establish log-concavity of $I(p_i, \tau_i, \epsilon_i)$, it remains to verify the log-concavity of $F\left(p^* - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))\right)\right)^{n-1}$ in set S. The latter, however, is simply a $n-1$ times self multiplication of $F\left(p^* - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))\right)\right)$, so it remains to check the following claim:

Claim 5 *If f is log-concave and G is linear, then $F\left(p^* - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))\right)\right)$ is log-concave for $(p_i, \tau_i, \epsilon_i) \in S$.*

Proof. Consider first the case $\epsilon_i - p_i \geq 0$. Before proceeding, we first denote $\eta \equiv \frac{d}{dx}\left(\frac{f(x)}{F(x)}\right) < 0$, which is negative because log-concavity of f implies decreasing reverse hazard rate $\frac{f}{F}$. Also, denote

$$\psi \equiv -G^{-1}\left(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))\right) = \frac{\tau_i}{\tau^*}(\bar{v} + \epsilon_i - p_i) - \bar{v},$$

where the second equality is due to the linearity assumption of G. By implicit differentiation,

$$\frac{\partial \psi}{\partial p_i} = -\frac{\partial \psi}{\partial \epsilon_i} = -\frac{\tau_i}{\tau^*}; \text{ and } \frac{\partial \psi}{\partial \tau_i} = \frac{\bar{v} + \epsilon_i - p_i}{\tau^*}.$$

It is straightforward to see that all relevant second-order derivatives and cross-derivatives are zero, except $\frac{\partial^2 \psi}{\partial \tau_i \partial p_i} = -\frac{\partial^2 \psi}{\partial \tau_i \partial \epsilon_i} = -\frac{1}{\tau^*} < 0$, and likewise for $\frac{\partial^2 \psi}{\partial p_i \partial \tau_i} = \frac{\partial^2 \psi}{\partial \epsilon_i \partial \tau_i} = -\frac{1}{\tau^*}$ due to symmetry.

Our objective is to show that $\ln(F(\psi + p^*))$ is concave $(p_i, \tau_i, \epsilon_i)$. The first-order derivatives are:

$$\begin{aligned}
\frac{\partial \ln(F(\psi + p^*))}{\partial p_i} &= \frac{f(\psi + p^*)}{F(\psi + p^*)} \frac{\partial \psi}{\partial p_i} \\
\frac{\partial \ln(F(\psi + p^*))}{\partial \epsilon_i} &= \frac{f(\psi + p^*)}{F(\psi + p^*)} \frac{\partial \psi}{\partial \epsilon_i} \\
\frac{\partial \ln(F(\psi + p^*))}{\partial \tau_i} &= \frac{f(\psi + p^*)}{F(\psi + p^*)} \frac{\partial \psi}{\partial \tau_i}.
\end{aligned}$$

The corresponding Hessian matrix is

$$H = \begin{pmatrix}
\frac{\partial^2 \ln(F(\psi+p^*))}{\partial p_i^2} & \frac{\partial^2 \ln(F(\psi+p^*))}{\partial p_i \partial \epsilon_i} & \frac{\partial^2 \ln(F(\psi+p^*))}{\partial p_i \partial \tau_i} \\
\frac{\partial^2 \ln(F(\psi+p^*))}{\partial \epsilon_i \partial p_i} & \frac{\partial^2 \ln(F(\psi+p^*))}{\partial \epsilon_i^2} & \frac{\partial^2 \ln(F(\psi+p^*))}{\partial \epsilon_i \partial \tau_i} \\
\frac{\partial^2 \ln(F(\psi+p^*))}{\partial \tau_i \partial p_i} & \frac{\partial^2 \ln(F(\psi+p^*))}{\partial \tau_i \partial \epsilon_i} & \frac{\partial^2 \ln(F(\psi+p^*))}{\partial \tau_i^2}
\end{pmatrix}$$

$$= \begin{pmatrix}
\left(\frac{\tau_i}{\tau^*}\right)^2 \eta & -\left(\frac{\tau_i}{\tau^*}\right)^2 \eta & -\left(\frac{\tau_i}{\tau^*}\right)\left(\frac{\bar{v}+\epsilon_i-p_i}{\tau^*}\right)\eta - \frac{1}{\tau^*}\frac{f(\psi+p^*)}{F(\psi+p^*)} \\
-\left(\frac{\tau_i}{\tau^*}\right)^2 \eta & \left(\frac{\tau_i}{\tau^*}\right)^2 \eta & \left(\frac{\tau_i}{\tau^*}\right)\left(\frac{\bar{v}+\epsilon_i-p_i}{\tau^*}\right)\eta + \frac{1}{\tau^*}\frac{f(\psi+p^*)}{F(\psi+p^*)} \\
-\left(\frac{\tau_i}{\tau^*}\right)\left(\frac{\bar{v}+\epsilon_i-p_i}{\tau^*}\right)\eta - \frac{1}{\tau^*}\frac{f(\psi+p^*)}{F(\psi+p^*)} & \left(\frac{\tau_i}{\tau^*}\right)\left(\frac{\bar{v}+\epsilon_i-p_i}{\tau^*}\right)\eta + \frac{1}{\tau^*}\frac{f(\psi+p^*)}{F(\psi+p^*)} & \left(\frac{\bar{v}+\epsilon_i-p_i}{\tau^*}\right)^2 \eta
\end{pmatrix},$$

where all η are evaluated at $\psi + p^*$. To show H is negative semi-definite, we check that the determinants of its leading principal minors $H1$, $H2$ and $H3$ alternate in sign as follows:

$$\det(H1) = \left(\frac{\tau_i}{\tau^*}\right)^2 \eta < 0;$$

$$\det(H2) = \det\begin{bmatrix}
\left(\frac{\tau_i}{\tau^*}\right)^2 \eta & -\left(\frac{\tau_i}{\tau^*}\right)^2 \eta \\
-\left(\frac{\tau_i}{\tau^*}\right)^2 \eta & \left(\frac{\tau_i}{\tau^*}\right)^2 \eta
\end{bmatrix} \geq 0;$$

As for $\det(H3) = \det(H)$, to simplify the notation, denote H_{ij} as the (i,j) entry of the Hessian matrix.

Then

$$
\begin{aligned}
\det(H3) &= H_{11}H_{22}H_{33} + H_{12}H_{23}H_{31} + H_{21}H_{32}H_{13} \\
&\quad - (H_{31}H_{22}H_{13} + H_{11}H_{32}H_{23} + H_{33}H_{12}H_{21}) \\
&= \eta\left(\frac{\tau_i}{\tau^*}\right)^2 \left[\eta^2\left(\frac{\bar{v}+\epsilon_i-p_i}{\tau^*}\right)^2\left(\frac{\tau_i}{\tau^*}\right)^2 - 2H_{23}H_{13}\right] \\
&\quad -\eta\left(\frac{\tau_i}{\tau^*}\right)^2 \left[\eta\left(\frac{\bar{v}+\epsilon_i-p_i}{\tau^*}\right)^2 \eta\left(\frac{\tau_i}{\tau^*}\right)^2 + H_{13}^2 + H_{23}^2\right],
\end{aligned}
$$

where we have invoked symmetry of H (i.e., $H_{23} = H_{32}$, $H_{31} = H_{13}$) in the second equality. Rearranging,

$$
\begin{aligned}
\det(H3) &= -\eta\left(\frac{\tau_i}{\tau^*}\right)^2 \left[2H_{23}H_{31} + H_{13}^2 + H_{23}^2\right] \\
&= -\eta\left(\frac{\tau_i}{\tau^*}\right)^2 \left[-2H_{13}^2 + H_{13}^2 + H_{13}^2\right] \leq 0,
\end{aligned}
$$

where the second equality is due to $H_{23} = -H_{13}$. Therefore, H is indeed negative semi-definite. ■

Next, it is easy to see that $I(p_i, \tau_i, \epsilon_i) = 0$ for $(p_i, \tau_i, \epsilon_i) \notin S$, which follows directly from the fact that $1 - G(p_i - \epsilon_i) = 0$ whenever $\epsilon_i - p_i < -\bar{v}$. Combining this observation with the analysis from the previous paragraph, we have established that $I(p_i, \tau_i, \epsilon_i)$ is a function that is log-concave in convex set S and equals to zero elsewhere. By Prékopa (1971), we thus know that $I(p_i, \tau_i, \epsilon_i)$ is log-concave in the entire space $[\underline{v} + \underline{\epsilon}, v + \bar{\epsilon}] \times [\underline{\epsilon}, \bar{\epsilon}]$, as required. We now invoke the following result by Prékopa (1973), which shows that integration preserves log-concavity:

Lemma 1 *(Prékopa, 1973) Let $f(x, y)$ be a function of $n + m$ variables where x is an n-component and y is an m-component vector. Suppose that f is log-concave in $\mathbb{R}^{n+m}$ and let A be a convex subset of $\mathbb{R}^m$. Then, the function of the variable x*

$$\int_A f(x, y)\, dy$$

is log-concave in the entire space $\mathbb{R}^n$.

Given that $I(p_i, \tau_i, \epsilon_i)$ is log-concave for $(p_i, \tau_i, \epsilon_i) \in [\underline{v} + \underline{\epsilon}, v + \bar{\epsilon}] \times [\underline{\epsilon}, \bar{\epsilon}]$ and we are integrating over ϵ_i in the fixed convex set $[\underline{\epsilon}, \bar{\epsilon}]$, the lemma above by Prékopa implies that the demand function D_i in (A.4) is log-concave in (p_i, τ_i). It follows that $\Pi_i = (p - \tau_i) D_i$ is log-concave hence quasi-concave.

When G is non-linear, an analytical proof for quasiconcavity is difficult because $\bar{x}(\epsilon)$ is a composite function of G^{-1} and G, which makes the evaluation of the Hessian matrix intractable. In order to determine the global of the profit function in this case, we rely on numerical calculations. Specifically, we plot firm i's profit function in equilibrium when F and G follows $N(\mu, \sigma)$, for all combinations of $\mu \in \{0, 1, 2\}$, $\sigma \in \{1, 2, 3\}$, and $n \in \{2, 3, 5, 10\}$. Details and codes of the numerical calculations are available from the authors upon request. For example, for $\mu = 0$ and $\sigma = 1$, we obtain the set of plots in Figure 2. In all the cases considered, we confirmed from the contour plots that the quasiconcavity assumption was satisfied, suggesting it does not require very special conditions to hold.

A.7 Lump-sum payments

In establishing the informative equilibrium with steering in Proposition 1, we have ruled out the feasibility of a firm offering M a lump-sum payment in return for M steering all consumers to that firm given that it is not sequentially rationale for M to do so. In this section, we examine what happens when we relax the requirement of sequential rationality such that a lump-sum contract becomes feasible.

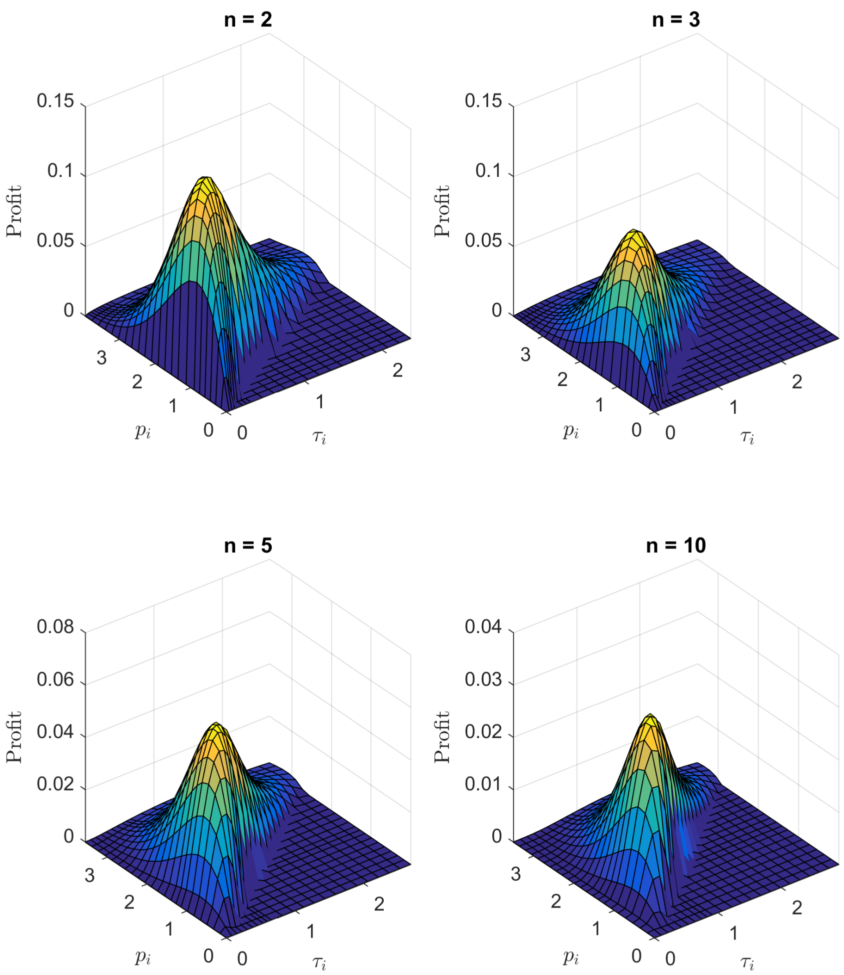

Figure 2: Profit function $\Pi\left(p_i, \tau_i\right)$, assuming all other firms set the equilibrium price and commission.

Obviously, there is no equilibrium in which any firm offers a lump-sum contract to M in equilibrium in return for being exclusively recommended. Whenever such a contract is offered in equilibrium, consumers rationally expect that M's recommendation is completely biased and uninformative, so that they will search through products as if there is no intermediation.

In what follows, we examine whether Proposition 1 remains a valid equilibrium when firms can deviate off-equilibrium by offering lump-sum contracts. Suppose that lump-sum contracts are unobservable to consumers so that their search behavior remains the same as in Section 3.1. For M to agree to an exclusive lump-sum contract, a deviating firm (say i) must offer a lump-sum payment T that compensates M for the total commission that M obtains from all other firms, i.e.,

$$T = \tau^* \int_{\underline{v}}^{\bar{v}} \left[1 - F\left(p^* - v\right)^n\right] dG\left(v\right). \tag{A.5}$$

Provided that M agrees to the contract, firm i no longer needs to pay any commission to M, and the firm becomes a monopoly with net deviation profit

$$\Pi_i' - T = \max_p \left\{ p \int_{\underline{v}}^{\bar{v}} \left[1 - F\left(p - v\right)\right] dG\left(v\right) \right\} - T.$$

Given that firm i is earning a non-negative profit in the initial equilibrium, a necessary condition for the deviation by firm i to be profitable is $\Pi_i' - T > 0$. Note from (A.5) that T is a function of n. Suppose $\bar{\epsilon}$ is finite and n is sufficiently large, Proposition 4 implies that M earns a "$\bar{\epsilon}$-product monopoly profit", that is

$$\begin{aligned} & \Pi_i' - T \\ \rightarrow \quad & \Pi_i' - \max\left\{p\left(1 - G\left(p - \bar{\epsilon}\right)\right)\right\} < 0. \end{aligned}$$

It follows that when n is sufficiently large, the deviating exclusive contract is not profitable for firm i.

In cases where n is small, in principle the deviation by lump-sum contract may be profitable so that the informative equilibrium in Proposition 1 becomes unsustainable when such contracts can be used. When that happens, the equilibrium becomes an uninformative one whereby all consumers ignore M's recommendation and search sequentially through the products as if there is no intermediary, and all firms pay no commission to M. Hence, allowing for exclusive lump-sum contracts and M's commitment to such contracts may sometimes cause the informative equilibrium to break down.[2]

A.8 Additional figures for Section 4

Figure 3 compares the price, consumer surplus, and welfare when M steers versus when it does not, whereby F and G are linear with distribution support $[-1, 1]$.

Figure 4 corresponds to Figure 1 in Section 4.3 of the main text, assuming F and G take the standard normal distribution $N(0, 1)$. The equilibrium price and commission still increase with n, even though condition (8) does not hold.

A.9 Derivation of Section 4.4

In this section, we derive consumers' optimal search rule and the resulting demand function for each firm in the model of Section 4.4 in the main text. Suppose, for the moment, consumers learn the realized value of v before any search. It is then well known from Kohn and Shavell (1974) that consumers' optimal search rule

[2]Nonetheless, we have confirmed that when both F and G are linear with distribution support $[-1, 1]$, the net deviation profit is strictly smaller than the equilibrium profit for firm i for all $n \geq 2$ so that the information equilibrium still holds in this case.

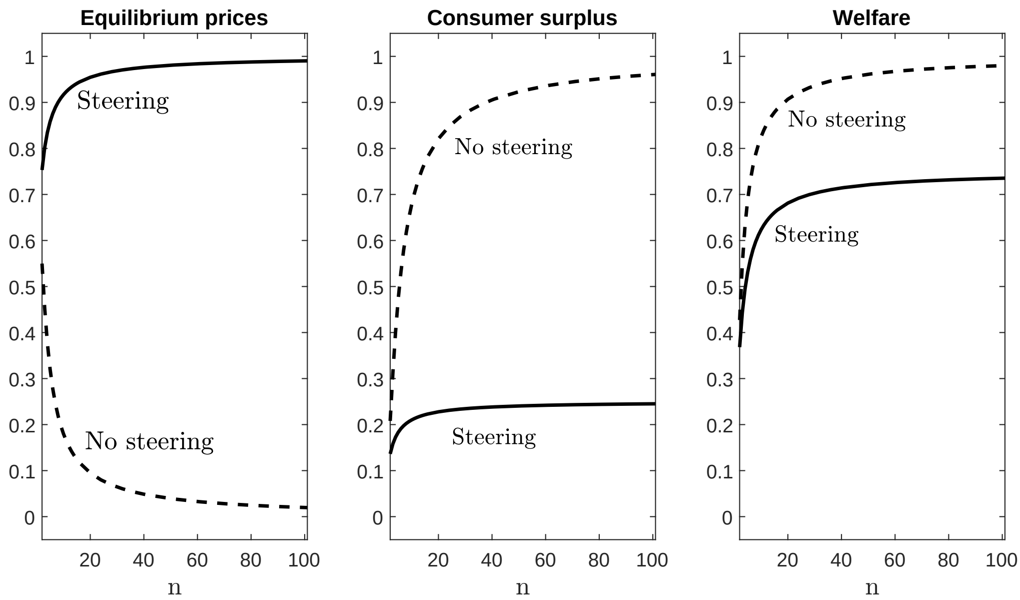

Figure 3: Price, consumer surplus and welfare when M steers versus when it does not, assuming F and G are $U\,[-1,1]$.

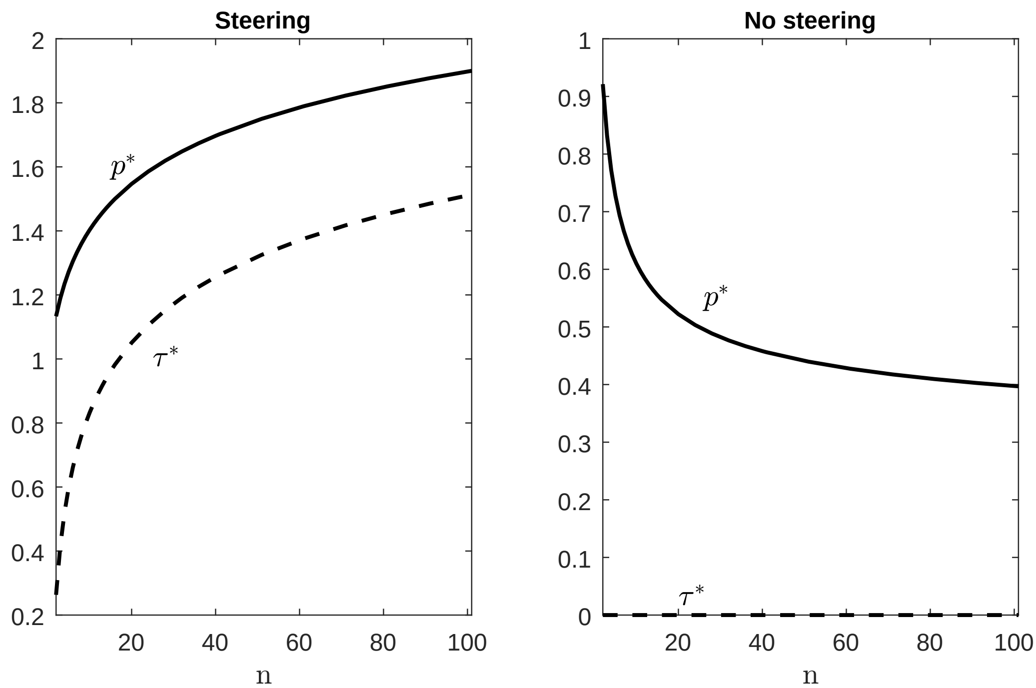

Figure 4: Prices and commissions when M steers versus when it does not, assuming F and G are $N\,(0,1)$.

in this environment is stationary and described by the standard cutoff rule. Define the reservation value $\tilde{x}$ as the solution to

$$\int_{\tilde{x}}^{\bar{\epsilon}} [\epsilon - \tilde{x}]\, dF(\epsilon) = s.$$

The left-hand side denotes the expected incremental benefit from one more search given the offer in hand $\tilde{x}$, while the right-hand side is the incremental search cost. Given that v is constant for any particular consumer, each consumer employs the following cutoff strategy when searching: (i) stops searching further if $\max\{v + \epsilon_i - p_i, 0\} \geq v + \tilde{x} - \tilde{p}$, or (ii) continues to search the next firm otherwise. Following the standard results, $v + \tilde{x} - \tilde{p}$ also represents consumers' expected surplus from initiating search once v is known.

Now suppose, consistent with our baseline model, that consumers do not actually know v before searching. Then they will carry out the first search as long as the ex-ante net surplus is positive. After the first search, consumers fully learn the realized value of v and the subsequent search problem of consumers is exactly described by the previous paragraph. Consumers with $v + \tilde{x} - \tilde{p} < 0$ expect no surplus gain from searching further relative to the outside option, so that they will stop searching and either purchase the first product or the outside option. On the other hand, consumers with $v + \tilde{x} - \tilde{p} \geq 0$ expect a positive surplus gain from costly search and they will continue searching until they find an option which gives them a surplus of at least $v + \tilde{x} - \tilde{p}$.

From the consumer search rule above, the derivation of demand facing firms is straightforward. For consumers with $v \geq \tilde{p} - \tilde{x}$, a deviating firm i's conditional demand follows the standard search model and it is given by

$$(1 - F(\tilde{x} - \tilde{p} + p_i)) \sum_{k=0}^{\infty} F(\tilde{x})^k = \frac{1 - F(\tilde{x} - \tilde{p} + p_i)}{1 - F(\tilde{x})}. \tag{A.6}$$

On the other hand, for consumers with $v < \tilde{p} - \tilde{x}$, firm i effectively becomes a local monopoly over these consumers since they do not search further. Firm i's conditional demand in this case is

$$1 - F(p_i - v). \tag{A.7}$$

Integrating both conditional demands in (A.6) and (A.7) over v gives the demand function

$$D_i(p_i) = \int_{\min\{\tilde{p}-\tilde{x},\bar{v}\}}^{\bar{v}} \left[\frac{1 - F(\tilde{x} - \tilde{p} + p_i)}{1 - F(\tilde{x})}\right] dG(v) + \int_{\underline{v}}^{\min\{\tilde{p}-\tilde{x},\bar{v}\}} [1 - F(p_i - v)]\, dG(v)$$

as stated in the proof of Proposition 5 in the appendix of the main text. The equilibrium characterization follows from Proposition 5 in the main text.

B Policy implications

In this section of the online appendix, we analyze in detail the omitted analysis described in Section 5 of the main text.

B.1 Concern for suitability

We extend our baseline model by allowing M to have a direct concern for consumer surplus. To microfound this possibility in the simplest possible fashion, we assume that, following firm i being M's recommended firm (or top ranked firm), the consumer lodges a complaint for an inappropriate recommendation with probability ρ, which results in a fixed penalty of α for M. We assume that ρ is an affine function of $1 - G(p_i - \epsilon_i)$:

$$\rho = \rho_i = \beta_1 - \beta_2 (1 - G(p_i - \epsilon_i)),$$

where $\beta_1, \beta_2 \geq 0$ are such that $0 \leq \rho \leq 1$. This functional form means that ρ is decreasing in the surplus that the consumer obtains from the product, so recommending a product that offers less surplus is more likely to lead to a consumer complaint.

In this environment, we can construct the same informative equilibrium as in Proposition 1 whereby consumers only search once and M gives unbiased recommendations in the equilibrium. The only exception is that M's recommendation off-equilibrium path is now based on the following decision rule: the top ranked product i by M satisfies

$$\tau_i \left(1 - G\left(p_i - \epsilon_i\right)\right) - \alpha\rho_i \geq \max_{j \neq i} \left\{\tau^* \left(1 - G\left(p^* - \epsilon_j\right)\right)\right\} - \alpha\rho_j,$$

or equivalently, after cancelling out common terms:

$$\left(\tau_i + \beta_2\alpha\right)\left(1 - G\left(p_i - \epsilon_i\right)\right) \geq \max_{j \neq i} \left\{\left(\tau^* + \beta_2\alpha\right)\left(1 - G\left(p^* - \epsilon_j\right)\right)\right\}. \tag{B.1}$$

Therefore, a higher penalty α implies a greater weight that M assigns to consumer surplus, meaning that its recommendation is less affected by commission differences across firms. Broadly interpreted, the parameter α captures M's concern for product suitability, as in Inderst and Ottaviani (2012a).

Given that all other aspects of the model are similar to the baseline model, we can derive the equilibrium prices and fees using the usual first order conditions. Provided that the firms' profit function is globally quasi-concave, we can state the equilibrium price and commissions with the following two equations, which parallel expressions (5) and (7) in the main text:

$$p^* = \tau^* + \frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - F\left(\max\left\{\epsilon, p^* - v\right\}\right)\right] dF\left(\epsilon\right)^{n-1} dG\left(v\right)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f\left(\max\left\{\epsilon, p^* - v\right\}\right) dF\left(\epsilon\right)^{n-1} dG\left(v\right)},$$

and

$$\tau^* = \max\left\{0, \frac{\int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} \left[\frac{1-G(p^*-\epsilon)}{g(p^*-\epsilon)} f\left(\epsilon\right)\right] dF\left(\epsilon\right)^{n-1} dG\left(v\right)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f\left(\max\left\{\epsilon, p^* - v\right\}\right) dF\left(\epsilon\right)^{n-1} dG\left(v\right)} - \beta_2\alpha\right\}.$$

We then have the following result:

Proposition 11 *If the informative equilibrium exists, then:*

1. *The equilibrium price and commission levels decrease with M's concern for product suitability (α).*
2. *Consumer surplus, firms' profit, and welfare increase with M's concern for product suitability (α).*

Proof. A total differentiation on the system of equation (p^*, τ^*) and a direct application of Cramer's rule, similar to that in the proof of Proposition 13, yields the proposition. ■

B.2 Informed consumers

We extend our baseline model by allowing for two types of consumers. With probability λ, a consumer is informed. Such a consumer knows the prices and the realizations of all match utilities before making her purchase decision. Equivalently, she has zero (search) costs of inspecting products, and so will always inspect every product. With the remaining probability $1-\lambda$, a consumer is uninformed and behaves exactly the same as the consumers in our baseline model. All purchases still go through M, meaning it receives commissions for purchases by both the informed and uninformed consumers. The realization of a consumer's type is not known to firms and M. The analysis below does not depend on whether consumers observe the decomposition of v_i after inspection.

In this setup, the model parameter λ is designed to capture that consumers are sometimes informed and do not fully rely on M's recommendation. Hence, one can interpret λ as the extent to which a representative consumer is informed. In what follows, we interpret an increase in λ as the representative consumer becoming more informed, and we explore how an increase in λ affects the equilibrium commission and price.

We focus on the informative equilibrium with steering, as in the baseline model. In particular: (i) all firms adopt the same strategy; (ii) M ranks all products in order of expected commission; and (iii) consumers (if uninformed) inspect the highest ranked product without searching further given they believe that the highest-ranked product gives them the highest surplus.

Demand from uninformed consumers is given by (2), which we denote as

$$D_i^U = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - F\left(\max\left\{\bar{x}_i\left(\epsilon\right), -v\right\} + p_i\right)\right] dF\left(\epsilon\right)^{n-1} dG\left(v\right).$$

Meanwhile, an informed consumer purchases a product i if $v + \epsilon_i - p_i \geq \max_{j \neq i}\left\{v + \epsilon_j - p_j, 0\right\}$. Provided that all firms set the equilibrium price at p^*, a deviating firm i's demand from informed consumers can be derived as

$$D_i^I = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - F\left(\max\left\{\epsilon - p^*, -v\right\} + p_i\right)\right] dF\left(\epsilon\right)^{n-1} dG\left(v\right).$$

Importantly, note that D_i^I is independent of commissions, as opposed to D_i^U, which reflects that an informed consumer cannot be steered by M. Then, firm i's total demand is the weighted sum of the two demand components, i.e., $\lambda D_i^I + \left(1-\lambda\right) D_i^U$. A typical deviating firm i solves

$$\max_{p_i, \tau_i} \Pi_i = \max_{p_i, \tau_i} \left(p_i - \tau_i\right) \left[\lambda D_i^I + \left(1-\lambda\right) D_i^U\right]. \tag{B.2}$$

The demand derivatives and first-order conditions can be obtained through similar steps to those used to prove Proposition 1, with the only new step being that

$$\frac{\partial D_i^I}{\partial p_i} = -\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f\left(\max\left\{\epsilon - p^*, -v\right\} + p_i\right) dF\left(\epsilon\right)^{n-1} dG\left(v\right) < 0.$$

A useful observation is that, after imposing $p_i = p^*$ and $\tau_i = \tau^*$, the equilibrium demand and demand derivatives of informed and uninformed consumers coincide exactly:

$$\begin{aligned} \frac{\partial D_i^I}{\partial p_i} &= \frac{\partial D_i^U}{\partial p_i} = -\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f\left(\max\left\{\epsilon, p^* - v\right\}\right) dF\left(\epsilon\right)^{n-1} dG\left(v\right) \\ D_i^I &= D_i^U = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - F\left(\max\left\{\epsilon, p^* - v\right\}\right)\right] dF\left(\epsilon\right)^{n-1} dG\left(v\right). \end{aligned}$$

Consequently, we can state the equilibrium price and commissions with the following two equations, which parallel the expressions (5) and (7) in the main text:

$$p^* = \tau^* + \frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - F\left(\max\left\{\epsilon, p^* - v\right\}\right)\right] dF\left(\epsilon\right)^{n-1} dG\left(v\right)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f\left(\max\left\{\epsilon, p^* - v\right\}\right) dF\left(\epsilon\right)^{n-1} dG\left(v\right)}, \tag{B.3}$$

and

$$\tau^* = \left(1-\lambda\right) \frac{\int_{\underline{v}}^{\bar{v}} \int_{p^* - v}^{\bar{\epsilon}} \left[\frac{1 - G\left(p^* - \epsilon\right)}{g\left(p^* - \epsilon\right)} f\left(\epsilon\right)\right] dF\left(\epsilon\right)^{n-1} dG\left(v\right)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f\left(\max\left\{\epsilon, p^* - v\right\}\right) dF\left(\epsilon\right)^{n-1} dG\left(v\right)}. \tag{B.4}$$

Other aspects of the equilibrium characterization remain the same as the baseline model. Formally:

Proposition 12 *(Informative equilibrium) If profit function (B.2) is globally quasiconcave in (p_i, τ_i), then the informative equilibrium exists in which:*

1. *All firms set p^* and τ^* given by (B.3) and (B.4);*
2. *M recommends the product with highest expected commission; and*
3. *All consumers inspect the recommended product without searching further.*

Note that equilibrium existence is non-trivial because firms may profitably deviate from the informative equilibrium by offering no commission and instead focusing on selling to the informed consumers. One sufficient condition for equilibrium existence is to have λ sufficiently close to zero, so that the aforementioned deviation is unprofitable, and the sufficiency condition (linearity of G) employed in the baseline model becomes directly applicable to establish quasiconcavity of the profit function (B.2). If F and G are linear with distribution support $[-1, 1]$, we have numerically verified that the informative equilibrium is sustainable even at moderate λ provided that n is not too large. For example, it is sustainable for $\lambda \leq 0.5$ provided $n \leq 4$, and for $\lambda \leq 0.1$ provided $n \leq 10$.

We now show how the equilibrium outcome changes with λ. To proceed, we need to define consumer surplus, firms' profit, and welfare in this context. Recall that M recommends the most suitable product for uninformed consumers, so that the equilibrium consumer surplus for both informed and uninformed consumers coincides. Since only uninformed consumers incur search cost and they incur it only once, consumer surplus can be written as

$$CS \equiv \int_{\underline{v}}^{\bar{v}} \int_{v+p^*}^{\bar{\epsilon}} [v + \epsilon - p^*]\, dF(\epsilon)^n\, dG(v) - (1-\lambda)\, s,$$

which is same as the consumer surplus expression in Section 4 of the main text. Likewise,

$$\sum \pi_i = (p^* - \tau^*) \int_{\underline{v}}^{\bar{v}} [1 - F(p^* - v)^n]\, dG(v)$$

$$W = \int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} [\epsilon + v]\, dF(\epsilon)^n\, dG(v) - (1-\lambda)\, s.$$

Proposition 13 *Consider the model with informed and uninformed consumers. If the informative equilibrium exists, then:*

1. *The equilibrium price and commission levels decrease with the probability of consumers being informed (λ).*
2. *Consumer surplus, firms' profit, and welfare increase with the probability of consumers being informed (λ).*

Proof. Consider the first part of the proposition: p^* and τ^* are increasing in λ. As in the proof of Proposition 1, denote

$$\phi_1 \equiv \frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})]\, dF(\epsilon)^{n-1}\, dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\})\, dF(\epsilon)^{n-1}\, dG(v)}$$

and

$$\phi_2 \equiv \frac{\int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} \left[\frac{1-G(p^*-\epsilon)}{g(p^*-\epsilon)} f(\epsilon)\right] dF(\epsilon)^{n-1}\, dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\})\, dF(\epsilon)^{n-1}\, dG(v)}.$$

In the last part of the proof of Proposition 1, we showed that $d\phi_1/dp^* < 0$ and $d\phi_2/dp^* < 0$. Total differentiation of (B.3) and (B.4), in matrix form, gives

$$\begin{bmatrix} 1 - \frac{d\phi_1}{dp^*} & -1 \\ -(1-\lambda)\frac{d\phi_2}{dp^*} & 1 \end{bmatrix} \begin{bmatrix} \frac{dp^*}{d\lambda} \\ \frac{d\tau^*}{d\lambda} \end{bmatrix} = \begin{bmatrix} 0 \\ -\phi_2 \end{bmatrix}.$$

Denote

$$Det \equiv \det \begin{pmatrix} 1 - \frac{d\phi_1}{dp^*} & -1 \\ -(1-\lambda)\frac{d\phi_2}{dp^*} & 1 \end{pmatrix} = 1 - \frac{d\phi_1}{dp^*} - (1-\lambda)\frac{d\phi_2}{dp^*} > 0.$$

By Cramer's rule,

$$\frac{dp}{d\lambda} = \frac{1}{Det}\begin{vmatrix} 0 & -1 \\ -\phi_2 & 1 \end{vmatrix} = \frac{\phi_2}{Det} < 0 \quad \text{and} \quad \frac{d\tau}{d\lambda} = \frac{1}{Det}\begin{vmatrix} 1 - \frac{d\phi_1}{dp^*} & 0 \\ -(1-\lambda)\frac{d\phi_2}{dp^*} & -\phi_2 \end{vmatrix} = -\frac{\phi_2}{Det}\left(1 - \frac{d\phi_1}{dp^*}\right) < 0,$$

as required. It is useful to note that $|d\tau/d\lambda| > |dp^*/d\lambda|$, which signifies an incomplete pass through of the commissions into product prices.

Consider the second part of the proposition. First, CS and W are clearly decreasing in p^*, and p^* is decreasing in λ by the first part of the proposition. Moreover, the search cost incurred is decreasing in λ, so that CS and W indeed increases with λ. As for $\sum \pi_i$, due to incomplete pass through where $|d\tau/d\lambda| > |dp^*/d\lambda|$, firms' equilibrium margin must be increasing with λ while the equilibrium level of sales for each firm is decreasing in p^* (hence increasing in λ). Consequently $\sum \pi_i$ increases with λ. ■

Finally, we show how various results in the baseline model remain robust in this extended model. Formally, we have:

Proposition 14 *Consider the informative equilibrium in Proposition 12:*

1. *Compared to the informative equilibrium with steering, price and commission levels are lower in the equilibrium without steering.*
2. *Compared to the informative equilibrium with steering,* Π_M *is lower in the equilibrium without steering, while* CS*,* $\sum \pi_i$*, and* W *are higher in the equilibrium without steering.*
3. *If* F *and* G *are linear, then* τ^* *always increases with* n*. Meanwhile,* p^* *increases with* n *if* $\lambda < 1/2$*, constant in* n *if* $\lambda = 1/2$*, and decreases with in* n *if* $\lambda > 1/2$*.*

Proof. Parts (1) and (2) follow from the proof of Proposition 13 above by substituting in the special case of $\lambda = 0$. This reflects that the case of $\lambda = 0$ (where all consumers are informed) is mathematically the same as having an equilibrium without steering. It remains to prove part (3). We know from Proposition 12 that p^* and τ^* are respectively pinned down by $p^* = \phi_1 + (1-\lambda)\phi_2$ and $\tau^* = (1-\lambda)\phi_2$. By the implicit function theorem,

$$\frac{dp^*}{dn} = \frac{\partial(\phi_1 + (1-\lambda)\phi_2)/\partial n}{1 - \partial\phi_1/\partial p^* - \partial\phi_2/\partial p^*},$$

so that the sign of dp^*/dn is the same as $\partial(\phi_1 + (1-\lambda)\phi_2)/\partial n$. With the exact same steps as in the proof of Proposition 4 (imposing F is linear), we can show that $d\phi_1/dn \leq 0$, while

$$\begin{aligned} & \phi_1 + (1-\lambda)\phi_2 \\ = & \frac{1 - G(p^* - \bar{\epsilon})}{g(p^* - \bar{\epsilon})} \\ & - \left(\frac{1 - G(p^* - \bar{\epsilon})}{\bar{\epsilon} - \underline{\epsilon}}\right)^{-1} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\left((2\lambda - 1)(1 - G(p^* - \epsilon)) + \frac{(1 - G(p^* - \epsilon))^2}{g(p^* - \epsilon)^2} g'(p^* - \epsilon)\right)\left(\frac{F^{n-1}(\epsilon)}{\bar{\epsilon} - \underline{\epsilon}}\right)\right] d\epsilon, \end{aligned}$$

When G is linear, we have $g' = 0$, so that $\partial(\phi_1 + (1-\lambda)\phi_2)/\partial n$ is negative if $\lambda < 1/2$, zero if $\lambda = 1/2$, and positive if $\lambda > 1/2$.

To show the result on commission, we totally differentiate $\tau^* = \phi_2(p^*)$ to get

$$\frac{1}{1-\lambda}\frac{d\tau^*}{dn} = \frac{\partial\phi_2}{\partial n} + \frac{\partial\phi_2}{\partial p^*}\frac{dp^*}{dn}$$

and it follows from the proof of Proposition 4.2 that $\frac{\partial \phi_2}{\partial n} + \frac{\partial \phi_2}{\partial p^*}\frac{dp^*}{dn} \geq 0$. Therefore, $d\tau^*/dn \geq 0$ regardless of λ. ■

B.3 Mandatory disclosure

We first specify and verify the equilibrium considered. Similar to the benchmark case, consumers hold passive belief over any unobserved prices and commissions (where applicable).

1. **Firms.** All firms set prices and commissions equal to p^* and τ^* respectively;
2. **Intermediary.** For each consumer and at any stage in their search process, M ranks all products in order of expected commission $\tau_i (1 - G(p_i - \epsilon_i))$, with the order of any ties being chosen in favor of the product with the higher surplus $\epsilon_i - p_i$ (and randomly from among any remaining ties);
3. **Unobservant consumers.** Regardless of how many products are ranked, unobservant consumers inspect the highest ranked product (say product i) without searching further, purchasing i if $v + \epsilon_i - p_i \geq 0$, and otherwise purchasing the outside option. They believe that the first-ranked product gives them the highest surplus, and that the surplus of any lower ranked or non-ranked product is (weakly) lower than this. In case M makes no recommendation, these consumers' purchase and search behavior is optimized as if M is absent.
4. **Observant consumers.** If M's ranking includes at least one of the lowest commission product(s), observant consumers inspect the products sequentially from the highest ranked product (say product i) to the lowest ranked product. If $\tau_i \leq \tau^*$, they stop searching, purchasing i if $v_i - p_i \geq 0$, and otherwise purchasing the outside option. Otherwise, they continue searching until encountering one of the firm(s) that offers the lowest commission, at which point they select a product to buy among the products inspected and the outside option. They believe that M ranks all products in order of expected commission. In case M's ranking excludes all of the lowest commission product(s), these consumers' purchase and search behavior is optimized as if M is absent.

Proof. In the symmetric equilibrium, M offers an unbiased ranking. Therefore M and consumers' strategy are clearly optimal on the equilibrium path. In what follows, we consider an off-equilibrium path scenario in which a firm i deviates by setting $p_i \neq p^*$ and $\tau_i \neq \tau^*$.

We first check M's incentives regarding its ranking. Denote

$$j^* \equiv \arg\max_{j \neq i} \{\epsilon_j - p^*\},$$

that is, the highest surplus product excluding i. In the proposed equilibrium, the only decision that matters to M is whether to rank j^* first or to rank i first, because other rankings are either outcome-equivalent or strictly worse than one of these two. Suppose $\tau_i < \tau^*$, so that M's profit from ranking product i first is $\Pi^M(i) = \tau_i (1 - G(p_i - \epsilon_i))$, while the profit from ranking j^* first is

$$\Pi^M(j^*) = \begin{cases} \lambda\tau^* (1 - G(p^* - \epsilon_{j^*})) + (1-\lambda)\tau_i (1 - G(p_i - \epsilon_i)) & \text{if } \epsilon_i - p_i > \epsilon_{j^*} - p^* \\ \tau^* (1 - G(p^* - \epsilon_{j^*})) & \text{if } \epsilon_i - p_i \leq \epsilon_{j^*} - p^* \end{cases}.$$

A simple comparison shows $\Pi^M(i) \geq \Pi^M(j^*)$ if and only if $\tau_i (1 - G(p_i - \epsilon_i)) \geq \tau^* (1 - G(p^* - \epsilon_{j^*}))$. Suppose instead $\tau_i > \tau^*$, then M's profit from ranking product j^* first is $\Pi^M(j^*) = \tau^* (1 - G(p^* - \epsilon_{j^*}))$, while the profit from ranking i first is

$$\Pi^M(i) = \begin{cases} \tau_i (1 - G(p_i - \epsilon_i)) & \text{if } \epsilon_i - p_i > \epsilon_{j^*} - p^* \\ \lambda\tau^* (1 - G(p^* - \epsilon_{j^*})) + (1-\lambda)\tau_i (1 - G(p_i - \epsilon_i)) & \text{if } \epsilon_i - p_i \leq \epsilon_{j^*} - p^* \end{cases}.$$

Again, $\Pi^M(i) \geq \Pi^M(j^*)$ if and only if $\tau_i(1 - G(p_i - \epsilon_i)) \geq \tau^*(1 - G(p^* - \epsilon_{j^*}))$.

For unobservant consumers, given that they behave exactly the same as the consumers in our baseline model, it follows immediately that the proposed strategy is optimal under the beliefs specified.

For observant consumers, whenever M's ranking includes at least one of the lowest commission product(s), these consumers have no reason not to inspect the top-ranked product first given their beliefs, because there is no instance in which they can infer that lower-ranked or unranked products are better. Consider an observant consumer who has inspected the top-ranked product, say product i. There are two cases:

- Suppose $\tau_i \leq \tau^*$, i.e., i is one of the lowest commission products. Then the consumer can infer from M's ranking strategy that $\epsilon_i - p_i \geq \max_{j \neq i}\{\epsilon_j - p_j\}$, and so she has no incentive to keep searching on given the positive search cost. Once the consumer stops searching, she buys either product i or the outside option.
- Suppose instead $\tau_i > \tau^*$. The consumer can infer from M's ranking strategy that the next product is j^*, and $1 - G(p^* - \epsilon_{j^*}) \leq \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))$. After observing v_i and p_i from inspecting product i, the corresponding expected net incremental benefit (or option value) from inspecting the next firm is thus

$$E\left(\max\{v + \epsilon_{j^*} - p^* - \max\{v_i - p_i, 0\}, 0\} | \epsilon_{j^*} - p^* \leq -G^{-1}\left(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))\right) | v_i\right) - s, \tag{B.5}$$

where the expectation is taken with respect to ϵ_i, ϵ_{j^*} and v. Substituting for $\epsilon_i = v_i + v$ and applying iterated expectations, (B.5) becomes

$$E_v\left[E_{\epsilon_{j^*}}\left[\max\{v + \epsilon_{j^*} - p^* - \max\{v_i - p_i, 0\}, 0\} | v + \epsilon_{j^*} - p^* \leq v - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))\right)\right] | v_i\right] - s.$$

The first component is positive as long as there is a positive mass of v satisfying

$$v_i - p_i < v - G^{-1}\left(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))\right),$$

which indeed holds given that $\frac{\tau_i}{\tau^*} > 1$ and $v_i \equiv v + \epsilon_i$. Therefore, (B.5) is always positive so that the consumer will inspect the second-ranked product given that search cost is arbitrarily small. After inspecting the second ranked product (that is, product j^* by M's ranking strategy), the consumer has no incentive to search further. Once she stops, she selects among product i, product j^*, and the outside option to make the purchase.

■

Given the equilibrium characterization above, a deviating firm i's demand is the same as in the baseline model when $\tau_i \leq \tau^*$:

$$D_i(p_i, \tau_i) = \Pr\left(\epsilon_i - p_i \geq \max_{j \neq i}\{\bar{x}_i(\epsilon_j), v\}\right) \text{ if } \tau_i \leq \tau^*,$$

where $\bar{x}_i(\epsilon) \equiv -G^{-1}\left(1 - \frac{\tau^*}{\tau_i}(1 - G(p^* - \epsilon))\right)$. If $\tau_i > \tau^*$, unobservant consumers behave the same as in the baseline model. Meanwhile observant consumers buy product i if the following three conditions hold: $\epsilon_i - p_i \geq \max_{j \neq i}\{\bar{x}_i(\epsilon_j)\}$ (i is recommended) and $\epsilon_i - p_i \geq \max_{j \neq i}\{\epsilon_j - p_j, -v\}$ (consumers search beyond the recommended product and still find i to be the best). Given $\frac{\tau^*}{\tau_i} < 1$ implies $\bar{x}_i(\epsilon_j) < \epsilon_j - p_j$, the first condition is non-binding whenever the last condition hold, so that firm i's demand is simply:

$$D_i(p_i, \tau_i) = \begin{cases} (1 - \lambda)\Pr(\epsilon_i - p_i \geq \max_{j \neq i}\{\bar{x}_i(\epsilon_j), -v\}) \\ \quad +\lambda \Pr(\epsilon_i - p_i \geq \max_{j \neq i}\{\epsilon_j - p_j, -v\}) \end{cases} \text{ if } \tau_i > \tau^*.$$

Clearly, $D_i(p_i, \tau_i)$ is increasing and continuous in τ_i, but not differentiable at $\tau_i = \tau^*$ because $\frac{dD_i}{d\tau_i}|_{\tau_i \to \tau^{*-}} = \frac{1}{(1-\lambda)}\frac{dD_i}{d\tau_i}|_{\tau_i \to \tau^{*+}}$, That is, the demand derivative $\frac{dD_i}{d\tau_i}$, and by extension the profit derivative $\frac{d\Pi_i}{d\tau_i}$, "jumps" downward at $\tau_i = \tau^*$. This kinked demand form gives rise to the possibility of multiple equilibrium. For example, provided that the relevant second-order conditions hold, then some possible equilibria are

$$\frac{d\Pi_i}{dp_i}|_{p_i=p^*} = \frac{d\Pi_i}{d\tau_i}|_{\tau_i \to \tau^{*}-} = 0, \tag{B.6}$$

and

$$\frac{d\Pi_i}{dp_i}|_{p_i=p^*} = \frac{d\Pi_i}{d\tau_i}|_{\tau_i \to \tau^{*}+} = 0, \tag{B.7}$$

as well as other equilibrium that cannot be characterized through first-order conditions (which we do not consider due to tractability issues).

One sufficient condition for the second-order conditions corresponding to (B.6) and (B.7) to hold is to have λ sufficiently close to zero, so that the sufficiency condition (Assumption 1) employed in the baseline model becomes directly applicable to establish quasiconcavity of the profit function in this extension. In the case of F and G are linear with distribution support $[-1, 1]$, we have numerically verified that the informative equilibrium is sustainable even at moderate λ provided that n is not too large. For example, it is sustainable for $\lambda \leq 0.7$ provided $n \leq 5$, and for $\lambda \leq 0.5$ provided $n \leq 20$.

It is easy to see that the equilibrium characterized by (B.6) is the same as the baseline model, in which case commission disclosure has no effect on the equilibrium. The more interesting equilibrium is the one characterized by (B.7), which turns out to be the same equilibrium as the one described by Proposition 12 in the extended model with informed and uninformed consumers. Consequently, the result of Proposition 13 implies that, when $\lambda > 0$, equilibrium price and commission levels are lower, while consumer surplus, firms' profit, and welfare are higher (when compared to the case with $\lambda = 0$). This proves the mandatory disclosure result stated in Proposition 6.[3]

C Extensions

In this section of the online appendix, we analyze in detail the omitted analysis described in Section 6 of the main text.

C.1 Asymmetric firms

C.1.1 The cost of the second search is high

The derivation of demand in this case is stated in the main text. It remains to prove Proposition 7.

Proof. (Proposition 7). Given that each firm's profit function is the same as in the baseline mode, and that F and G are linear over $[\underline{\epsilon}, \bar{\epsilon}]$ and $[\underline{v}, \bar{v}]$ respectively, we have:

$$\Pi_i = (p_i - c_i - \tau_i) \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - \frac{(\max\{\bar{x}_i(\epsilon), -v\}) + p_i - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}\right] \left(\frac{1}{\bar{\epsilon} - \underline{\epsilon}}\right) \left(\frac{1}{\bar{v} - \underline{v}}\right) d\epsilon dv,$$

[3] We also considered an alternative model where commission payments are observable to all consumers, while consumers face heterogenous search costs randomly distributed over interval $[0, 1]$. In this case, consumers with high search cost do not react to commission changes (as if they are uninformed), while consumers with low search cost search more than once whenever they expect M's ranking is biased (as if they are observant). Our result that mandatory disclosure reduces price and commission levels remain robust under this alternative model. Details are available from the authors upon request.

where $\bar{x}_i(\epsilon) = \frac{\tau_j}{\tau_i}(\bar{v} + \epsilon - p_j) - \bar{v}$. Then, the first-order conditions can be derived as:

$$\begin{aligned} p_i &= c_i + \tau_i + \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\bar{\epsilon} - \left(\max\left\{\frac{\tau_j}{\tau_i}(\bar{v} + \epsilon - p_j) - \bar{v}, -v\right\}\right) - p_i\right] d\epsilon dv \\ \tau_i &= \int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_i}{\tau_j}(\bar{v}-v)+p_j-\bar{v}}^{\bar{\epsilon}} \left[\frac{\tau_j}{\tau_i}(\bar{v} + \epsilon - p_j)\right] d\epsilon dv. \end{aligned}$$

The second-order conditions follow from the baseline model. Simplifying, the equilibrium $(p_1^*, p_2^*, \tau_1^*, \tau_2^*)$ is pinned down by the following system of four equations for $i, j \in \{1, 2\}$, $i \neq j$.

$$p_i^* = \frac{\bar{\epsilon} + c_i}{2} - \frac{1}{2}\int_{\underline{v}}^{\bar{v}} \left[(\bar{v} - v)\left(\frac{\tau_i^*}{\tau_j^*}(\bar{v} - v) + p_j^*\right)\right] dv + Z \tag{C.1}$$

$$\tau_i^* = \int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_i^*}{\tau_j^*}(\bar{v}-v)+p_j^*-\bar{v}}^{\bar{\epsilon}} \left[\frac{\tau_j^*}{\tau_i^*}(\bar{v} + \epsilon - p_j^*)\right] d\epsilon dv, \tag{C.2}$$

where $Z \equiv \frac{1}{2}\int_{\underline{v}}^{\bar{v}} [\bar{v}(\bar{v} + \bar{\epsilon}) - v(\bar{v} + \underline{\epsilon})] dv$ is a constant. Brouwer's fixed point argument guarantees the existence of a solution to the system. From (C.1), dividing τ_1^* by τ_2^* and rearranging, we get:

$$\left(\frac{\tau_1^*}{\tau_2^*}\right)^3 = \frac{\int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_1^*}{\tau_2^*}(\bar{v}-v)+p_2^*-\bar{v}}^{\bar{\epsilon}} [\bar{v} + \epsilon - p_2^*] d\epsilon dv}{\int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_2^*}{\tau_1^*}(\bar{v}-v)+p_1^*-\bar{v}}^{\bar{\epsilon}} [\bar{v} + \epsilon - p_1^*] d\epsilon dv}. \tag{C.3}$$

From (C.3), it is easy to verify that $\tau_1^* < \tau_2^*$ if and only if $p_1^* < p_2^*$. To prove $\tau_1^* < \tau_2^*$, suppose by contradiction $\tau_1^* \geq \tau_2^*$, which implies $p_1^* \geq p_2^*$. From (C.1), computing the difference $p_1^* - p_2^*$:

$$(p_1^* - p_2^*)\left(1 - \frac{(\bar{v} - \underline{v})^2}{4}\right) = \frac{c_1 - c_2}{2} - \frac{(\bar{v} - \underline{v})^3}{6}\left(\frac{\tau_1^*}{\tau_2^*} - \frac{\tau_2^*}{\tau_1^*}\right) < 0$$

because $c_2 > c_1$ and $\tau_1^* \geq \tau_2^*$. This contradicts (C.3), so we can conclude $\tau_1^* < \tau_2^*$ and $p_1^* < p_2^*$ must hold in equilibrium.

To prove uniqueness, from (C.1) we substitute the expression of p_j^* and rearrange to obtain:

$$\begin{aligned} p_i^*\left(1 - \frac{(\bar{v} - \underline{v})^4}{16}\right) &= \frac{\bar{\epsilon} + c_i}{2} - \frac{1}{2}\int_{\underline{v}}^{\bar{v}} \left[\frac{\tau_i^*}{\tau_j^*}(\bar{v} - v)^2\right] dv + Z \\ &\quad - \frac{(\bar{v} - \underline{v})^2}{4}\left(\frac{\bar{\epsilon} + c_j}{2} - \frac{1}{2}\int_{\underline{v}}^{\bar{v}} \left[\frac{\tau_j^*}{\tau_i^*}(\bar{v} - v)^2\right] dv + Z\right), \end{aligned} \tag{C.4}$$

which pins down p_i^* as a decreasing function of $\frac{\tau_i^*}{\tau_j^*}$, implying that p_1^* decreases with $\frac{\tau_1^*}{\tau_2^*}$ while the reverse is true for p_2^*. It follows that the right-hand side of (C.3) is decreasing in $\frac{\tau_1^*}{\tau_2^*}$, hence the solution must be unique. Substituting the unique $\frac{\tau_1^*}{\tau_2^*}$ into (C.4) leads to unique (p_1^*, p_2^*), which can then be substituted into (C.2) to obtain unique (τ_1^*, τ_2^*). Finally, we verify (9). The condition clearly holds for $i = 1$ given we have proven $\frac{\tau_2^*}{\tau_1^*} > 1$, and it also holds for $i = 2$ if $\frac{\tau_1^*}{\tau_2^*} < 1$ is close enough to 1, which is true when c_2 is sufficiently small given $\frac{\tau_1^*}{\tau_2^*}$ monotonically increases when c_2 decreases and also $\lim_{c_2 \to 0} \frac{\tau_1^*}{\tau_2^*} = 1$. ■

In the case of non-uniform distributions we solve numerically for the equilibrium prices, commissions, and condition (9) for each given c_2. Figure 5 below illustrates the numerical result for F and $G \sim N(\mu, \sigma)$, where $\mu = 0$ and $\sigma = 1$. Consistent with the case of uniform distribution, we observe that $p_1^* < p_2^*$ and $\tau_1^* < \tau_2^*$. In equilibrium (9) holds for all $c_2 \leq 3$, so that consumers indeed find it optimal to follow M's recommendation, regardless of which firm is recommended. We obtained similar observations for the other values of (μ, σ) that we tried, as well as the exponential distribution.

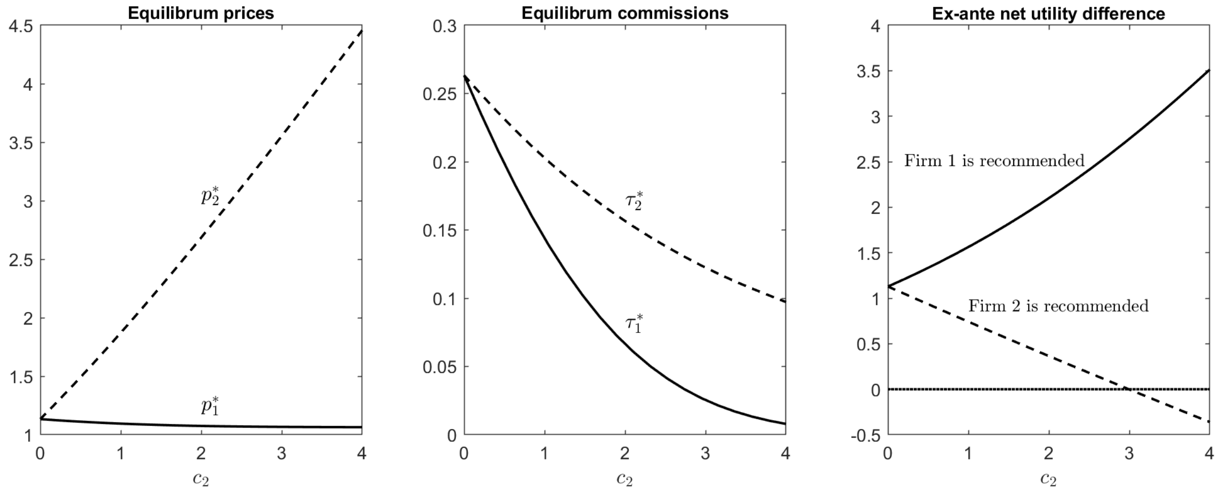

Figure 5: Asymmetric equilibrium, F and $G \sim N(0,1)$.

C.1.2 The cost of the second search is low

When the cost of the second search is arbitrarily small, we want to prove that there is no pure-statregy equilibrium with asymmetric positive commissions. Consider some arbitrarily given profile of expected and actual prices and commissions, and without loss of generality suppose $0 < \tau_1^* < \tau_2^*$. Similar to the benchmark case, consumers hold passive belief over any unobserved prices and commissions (where applicable). We can construct the following informative equilibrium in the recommendation stage:

- Consumers follow M's ranking, believing that M ranks all products in order of expected commissions. If firm 1 is ranked first, consumers inspect it without searching further. If firm 2 is ranked first, consumers inspect both products.
- M ranks all products in order of expected commission $\tau_i\,(1 - G\,(p_i - \epsilon_i))$.

Clearly, consumers' search strategy is optimal given their beliefs and M's equilibrium strategy. To verify M's strategy, note if $\epsilon_1 - p_1 \leq \epsilon_2 - p_2$ then the profit from ranking firm 1 first is $\tau_1\,(1 - G\,(p_1 - \epsilon_1))$ and from ranking firm 2 first is $\tau_2\,(1 - G\,(p_2 - \epsilon_2))$. If instead $\epsilon_1 - p_1 > \epsilon_2 - p_2$ then M's profit is always $\tau_1\,(1 - G\,(p_1 - \epsilon_1))$ regardless of the recommendation because consumers always inspect both products and buy from firm 1 whenever firm 2 is ranked first. In this case, the equilibrium strategy specifies that M breaks a tie in favor of the firm with higher $\tau_i\,(1 - G\,(p_i - \epsilon_i))$, which is required for the equilibrium to exist. From the informative equilibrium in the recommendation stage, firm 2's demand is

$$\Pr\left(\epsilon_2 - p_2 > \max\left\{\epsilon_1 - p_1, -G^{-1}\left(1 - \frac{\tau_1}{\tau_2}\,(1 - G\,(p_1 - \epsilon_1))\right), -v\right\}\right).$$

Notice that for all $\tau_2 > \tau_1$, any increase in τ_2 has no effect on the demand because $-G^{-1}\left(1 - \frac{\tau_1}{\tau_2}\,(1 - G\,(p_1 - \epsilon_1))\right) < \epsilon_1 - p_1$. Therefore, firm 2 never sets $\tau_2 > \tau_1$ as opposed to the initial supposition, i.e. any equilibrium with $0 < \tau_1^* < \tau_2^*$ is not sustainable. A mirror argument shows that equilibrium with $\tau_1^* > \tau_2^* > 0$ is also not sustainable, as claimed in the text.

C.1.3 Two groups of consumers

Suppose there are two groups of consumers: a fraction λ of which have arbitrarily small cost for the second search (*low-cost consumers*) while the remaining fraction $1 - \lambda$ have sufficiently high cost for the second search and they only search once (*high-cost consumers*). We now construct the asymmetric (pure-strategy) informative equilibrium with steering.

Proposition 15 *Suppose F and G are linear. For each given $c_2 - c_1 > 0$ such that Proposition 7 holds, if λ is sufficiently small then there exists an asymmetric informative equilibrium with steering in which:*

1. *Firms set prices $p_1^* < p_2^*$ and commissions $\tau_1^* < \tau_2^*$.*
2. *M ranks all products in order of expected commission $\tau_i (1 - G(p_i - \epsilon_i))$.*
3. *All high-cost consumers inspect the recommended product without searching further.*
4. *Low-cost consumers inspect the highest ranked product first. If firm 1 is ranked first, they inspect it without searching further. If firm 2 is ranked first, they inspect both products. They believe that M ranks all products in order of expected commission.*
5. *All consumers believe that M ranks all products in order of expected commission. In case M makes no recommendation, consumers' purchase and search behavior is optimized as if M is absent.*

Proof. We first check M's incentives regarding its ranking. If $\epsilon_1 - p_1 \le \epsilon_2 - p_2$ then the profit from ranking firm 1 first is $\tau_1 (1 - G(p_1 - \epsilon_1))$ and from ranking firm 2 first is $\tau_2 (1 - G(p_2 - \epsilon_2))$. If instead $\epsilon_1 - p_1 > \epsilon_2 - p_2$, then M's profit from ranking firm 1 first is $\tau_1 (1 - G(p_1 - \epsilon_1))$ and from ranking firm 2 first is $\lambda\tau_1 (1 - G(p_1 - \epsilon_1)) + (1 - \lambda)\tau_2 (1 - G(p_2 - \epsilon_2))$. In both cases, M does best by ranking all products in order of expected commission $\tau_i (1 - G(p_i - \epsilon_i))$. Then, it remains to check consumers indeed find it optimal to follow M's ranking, i.e. check whether (9) indeed holds in equilibrium. From the informative equilibrium in the recommendation stage, we can derive firm 1's demand, which is the same as in the baseline model because all consumers only inspect once whenever firm 1 is ranked first. Firm 2's demand becomes

$$D_2(p_2, \tau_2) = \left(\begin{array}{c} (1-\lambda)\Pr\left(\epsilon_2 - p_2 > \max\left\{-G^{-1}\left(1 - \frac{\tau_1}{\tau_2}(1 - G(p_1 - \epsilon_1))\right), -v\right\}\right) \\ +\lambda \Pr\left(\epsilon_2 - p_2 > \max\left\{\epsilon_1 - p_1, -G^{-1}\left(1 - \frac{\tau_1}{\tau_2}(1 - G(p_1 - \epsilon_1))\right), -v\right\}\right) \end{array} \right).$$

Specifically, there is an extra term $\epsilon_1 - p_1$ in the second demand component because low-cost consumers inspect both products. In general, $D_2(p_2, \tau_2)$ is continuous in τ_2 but not differentiable at $\tau_2 = \tau_1$ because $\frac{dD_2}{d\tau_2}|_{\tau_2 \to \tau_1^-} = \frac{1}{(1-\lambda)} \frac{dD_2}{d\tau_2}|_{\tau_2 \to \tau_1^+}$, i.e. the slope of the demand function has a downward kink at $\tau_2 = \tau_1$. Nonetheless, given that we are focusing on an equilibrium with $\tau_1^* < \tau_2^*$, firm 2's commission is necessarily an interior one and pinned down by $-\frac{dD_2}{dp_2} = \frac{dD_2}{d\tau_2}|_{\tau_2 > \tau_1}$, as otherwise the equilibrium is violated. Therefore, we write firm 2's profit function as

$$\Pi_2 = (p_2 - c_2 - \tau_2) \left[\begin{array}{c} (1-\lambda)\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - \frac{(\max\{\bar{x}_2(\epsilon), -v\}) + p_2 - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}\right] \left(\frac{1}{\bar{\epsilon} - \underline{\epsilon}}\right)\left(\frac{1}{\bar{v} - \underline{v}}\right) d\epsilon dv \\ +\lambda \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - \frac{(\max\{\epsilon_1 - p_1, -v\}) + p_2 - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}\right] \left(\frac{1}{\bar{\epsilon} - \underline{\epsilon}}\right)\left(\frac{1}{\bar{v} - \underline{v}}\right) d\epsilon dv \end{array} \right].$$

Then, the first-order conditions for firm 2 can be derived as:

$$\begin{aligned} p_2^* &= c_2 + \tau_2^* + (1-\lambda)\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\bar{\epsilon} - \left(\max\left\{\frac{\tau_1^*}{\tau_2^*}(\bar{v} + \epsilon - p_1^*) - \bar{v}, -v\right\}\right) - p_2^*\right] d\epsilon dv \\ &\quad + \lambda \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\bar{\epsilon} - (\max\{\epsilon - p_1^*, -v\}) - p_2^*\right] d\epsilon dv \\ \tau_2^* &= (1-\lambda)\int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_1^*}{\tau_2^*}(\bar{v} - v) + p_1^* - \bar{v}}^{\bar{\epsilon}} \left[\frac{\tau_1^*}{\tau_2^*}(\bar{v} + \epsilon - p_1^*)\right] d\epsilon dv, \end{aligned}$$

while the first-order conditions for firm 1 are (C.1) and (C.2) (setting $i = 1$). Given firm 2's best responses are continuous in λ and converges to (C.1) and (C.2) (setting $i = 2$), it follows from continuity that for sufficiently small λ the equilibrium in Proposition 7 holds. In particular, the equilibrium $(p_1^*, p_2^*, \tau_1^*, \tau_2^*)$ is such that condition (9) holds for $i = 1, 2$ and also $\tau_1^* < \tau_2^*$, as required. ■

For an illustration, the figure below plots the equilibrium outcome as a function of λ, assuming F and $G \sim U[-1,1]$, and $c_2 = 0.5$. We observe $\tau_1^* < \tau_2^*$ for all $\lambda \leq 0.35$ such that the asymmetric informative equilibrium exists. For $\lambda > 0.35$, $\tau_1^* = \tau_2^*$ implies that the asymmetric informative equilibrium no longer exists. A similar observation can be obtained assuming F and $G \sim N(0,1)$, and $c_2 = 1$, in which case $\tau_1^* < \tau_2^*$ for all $\lambda \leq 0.5$ such that the asymmetric informative equilibrium exists.

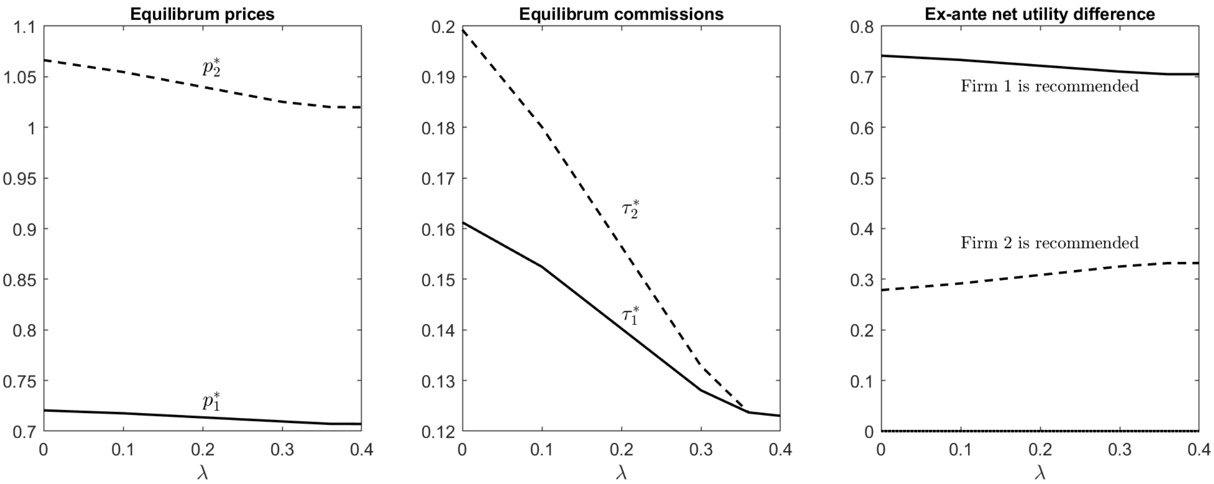

Figure 6: Asymmetric equilibrium, F and $G \sim U[-1,1]$ and $c_2 = 0.5$.

C.2 Fee-setting intermediary

For any given τ set by M, the pricing stage among n firms is simply the Perloff-Salop model where a typical firm i solves

$$\max_{p_i} (p_i - \tau) \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, -v\} + p_i)] \, dF(\epsilon)^{n-1} \, dG(v).$$

From the first-order conditions, the equilibrium price $p^* = p^*(\tau)$ satisfies

$$p^* = \tau + \underbrace{\frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})] \, dF(\epsilon)^{n-1} \, dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) \, dF(\epsilon)^{n-1} \, dG(v)}}_{\equiv \phi_1(p^*)}. \tag{C.5}$$

We note that firms' second-order conditions hold when f and g are log-concave.

Next, consider M's fee-setting problem. Since M collects a fee for all transactions, the total demand it faces is the sum of demands of all n firms, or simply the total market coverage of all n firms. Hence, it sets a commission that solves

$$\max_{\tau} \left\{ \tau \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - G(p^* - \epsilon)] \, dF^n(\epsilon) \right\} \tag{C.6}$$
$$\text{subject to } p^* = \tau + \phi_1(p^*).$$

We are now ready to prove Proposition 8 in the main text.

Proof. (Proposition 8). We first consider the limiting result. Based on the pricing constraint, we can recast M's maximization problem as choosing final product prices directly. This is possible because $\phi_1(.)$ is a strictly decreasing function (by the last part of the proof of Proposition 1), hence there is a one-to-one relationship between p^* and τ with $dp^*/d\tau \in [-1, 0]$. Hence, M solves

$$\max_p \left(p - \phi_1 (p)\right) \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - G (p - \epsilon)\right] dF^n (\epsilon) . \tag{C.7}$$

Equivalently, M is a monopolist who sells a product with valuation $\max_{j=1,\dots,n} \{\epsilon_j\}$ and faces a marginal cost at $\phi_1 (p)$. When n approaches infinity, $\phi_1 (p) \to 0$ while at the same time the distribution F collapses to a single point at $\bar{\epsilon}$ so that (C.7) becomes

$$\max_p p \left(1 - G (p - \bar{\epsilon})\right) .$$

We have $\tau^p \equiv \arg\max_p p \left(1 - G (p - \bar{\epsilon})\right) = \tau^m$ by the definition of τ^m.

In what follows, define $p^* (\tau)$ as the solution to $p^* = \tau + \phi_1 (p^*)$ for each given τ. When both F and G are linear, the associated first-order condition for (C.6) that pins down τ^p is

$$\begin{aligned}
\tau &= \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - G (p^* (\tau) - \epsilon)\right] dF^n (\epsilon)}{-\frac{dp^*}{d\tau} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[g (p^* (\tau) - \epsilon)\right] dF^n (\epsilon)} \\
&= \frac{-1}{dp^*/d\tau} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G (p^* (\tau) - \epsilon)}{g (p^* (\tau) - \epsilon)}\right] \left[\frac{g (p^* (\tau) - \epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} g (p^* (\tau) - \epsilon) \, dF^n (\epsilon)}\right] dF^n (\epsilon) \\
&= \frac{-1}{dp^*/d\tau} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G (p^* (\tau) - \epsilon)}{g (p^* (\tau) - \epsilon)}\right] \left[\frac{1}{1 - F (p^* (\tau))^n}\right] dF^n (\epsilon) ,
\end{aligned}$$

where the last equality utilizes that g is constant when G is linear. Meanwhile, recall that from (7) that after substituting for constant $f (\epsilon) = g (\epsilon)$ and changing the order of integration, the equilibrium commission τ^* (when firms set commission) is pinned down by

$$\tau = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G (p^* (\tau) - \epsilon)}{g (p^* (\tau) - \epsilon)}\right] \left[\frac{1 - G (p^* (\tau) - \epsilon)}{1 - G (p^* (\tau) - \bar{\epsilon})}\right] dF^{n-1} (\epsilon) . \tag{C.8}$$

To show $\tau^p \geq \tau^*$, we note that for any $\tau \geq 0$,

$$\begin{aligned}
& \frac{-1}{dp^*/d\tau} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G (p^* (\tau) - \epsilon)}{g (p^* (\tau) - \epsilon)}\right] \left[\frac{1}{1 - F (p^* (\tau))^n}\right] dF^n (\epsilon) \\
\geq & \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G (p^* (\tau) - \epsilon)}{g (p^* (\tau) - \epsilon)}\right] dF^n (\epsilon) \\
\geq & \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G (p^* (\tau) - \epsilon)}{g (p^* (\tau) - \epsilon)}\right] dF^{n-1} (\epsilon) \\
\geq & \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G (p^* (\tau) - \epsilon)}{g (p^* (\tau) - \epsilon)}\right] \left[\frac{1 - G (p^* (\tau) - \epsilon)}{1 - G (p^* (\tau) - \bar{\epsilon})}\right] dF^{n-1} (\epsilon) ,
\end{aligned}$$

where the first inequality is due to $\frac{-1}{dp^*/d\tau} \left(\frac{1}{1-F(p^*(\tau))^n}\right) \geq 1$ (because $dp^*/d\tau \in [-1, 0]$), the second inequality is due to first-order stochastic dominance, while the third inequality is due to $\left[\frac{1-G(p^*(\tau)-\epsilon)}{1-G(p^*(\tau)-\bar{\epsilon})}\right] \leq 1$. The final line of the expression is exactly the RHS (C.8), and it is decreasing in τ. We thus conclude that $\tau^p \geq \tau^*$. ■