Group Hug: Platform Competition with User-group

Online Appendix B: Absolute Number of Group and Individual Users

By

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1 Introduction

In our base model we measure the size of the group in proportional terms. Alternatively, one could assume a market with x group members and y individual users. In this case, however, changes in the size of the group are confounded with changes in network effects. In this appendix, we extend our model to the case where the size of the individual users is not affected by a change in the size of the group. We show that, as in our base model, a pivotal group joins the low-quality platform when it is not too large relative to the size of the individual users. The larger the number of individual users, the larger the group needs to be to choose the efficient platform. Moreover, the equilibrium utility of an individual user is independent of the size of the group as their utility is only a function of the network effects they create to each other. The utility of a group user is, in general, increasing in the group size with the exception of the case where the quality difference across the platforms is relatively large. Total consumers surplus, platforms' profits, and thus total welfare, are always increasing in the size of the group as it does not imply a decrease in the size of the individual users.

2 Model

Consider our linear example: $V_A(n_A) = \lambda n_A$ and $V_B(n_B) = Q + \lambda n_B$, where λ represents the network effects and Q the relative quality advantage platform B offers. Further, assume that there is a fixed mass of individual users of size y and a group of size x such

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that the overall size of the market is $x + y$. Under this setting, a change in the size of the group does not affect the size of the individual users. Rather, a change in the size of the group changes the overall size of the market. This setup allows us to disentangle the effect of changes in the size of the group from changes in the size of the individual users.

As in our base model, suppose that network effects are more important to individual users than platform B's quality advantage: $0 < Q < \lambda y$. This is an equivalent assumption to the one in our base model, where we assume that $0 < Q < \lambda$. This assumption implies that without the group, platform B cannot win the individual users. The timing of the game is the same as in our base model: platforms first compete on the group and then compete on individual users.

2.1 Solution to the second-stage: competition on the individual users

As in our base model, we start with solving the second stage–competition over the individual users. Suppose that the group joined platform A. An equilibrium in which platform A wins the individual users satisfies the following two conditions:

$$\lambda(x + y) - p_A \geq Q - p_B, \quad p_B = 0 \quad \Longrightarrow \quad p_A = \lambda(x + y) - Q, \tag{1}$$

$$\pi_A(x, y; A) = y p_A = y\left(\lambda(x + y) - Q\right) > 0. \tag{2}$$

That is, platform A charges the highest price that ensures that individual users prefer joining its focal platform A over joining platform B; and platform B charges the lowest price that ensures non-negative profits. The second condition guarantees that platform A earns positive profit from attracting the individual users. Since $Q < \lambda y$, the inequality in equation (2) holds for all $x > 0$.

To see that given that the group joins platform A there is no equilibrium in which platform B wins the individual users, note that if such equilibrium were to exist, $p_A = 0$ and $\lambda(x + y) - p_A = Q - p_B$, implying that $p_B = Q - \lambda(x + y)$ and platform B earns: $\pi_B(x, y; A) = y\left(Q - \lambda(x + y)\right) < 0$. That is, when platform A wins the group, it always also wins the individual users.

Suppose now that the group joins platform B. An equilibrium in which platform A

wins the individual users satisfies the following condition:

$$\lambda y - p_A \geq Q + \lambda x - p_B, \quad p_B = 0 \quad \Longrightarrow \quad p_A = \lambda(y - x) - Q, \tag{3}$$

Platform A then earns: $\pi_A(x, y; B) = y\left(\lambda(y-x) - Q\right)$. Likewise, in an equilibrium in which platform B wins the individual users, it charges and earns, respectively,

$$p_B = \lambda(x - y) + Q, \quad \pi_B(x, y; B) = y\left(\lambda(x-y) + Q\right). \tag{4}$$

Hence, platform A wins the individual users iff $y\left(\lambda(y-x) - Q\right) > 0$. As in our base model, letting $\widehat{x}$ denote the solution to $\pi_A(x, y; B) = 0$, we get that $\widehat{x} = y - \frac{Q}{\lambda}$. That is, if $x < y - \frac{Q}{\lambda}$, platform A wins the individual users regardless of the group's decision. Once the group becomes larger and $x > y - \frac{Q}{\lambda}$, the group becomes pivotal and the individual users join the platform the group joins. Notice that the threshold $\widehat{x} = y - \frac{Q}{\lambda}$ is equivalent to the threshold in example in Section V in the paper. To see why, we can impose the restriction $x + y = 1$ by substituting $y = 1 - x$ into $x = y - \frac{Q}{\lambda}$ and obtain $x = \frac{1}{2} - \frac{Q}{2\lambda}$, which is the same threshold as in the example. The quality gap Q and the degree of network effects λ affect $\widehat{x}$ in a qualitatively similar way as in our base model. Moreover, $\widehat{x}$ is increasing in y. Intuitively, when the group joins platform B, the larger the number of individual users, the larger the group needs to be in order to balance platform A's focality advantage and enable platform B to attract the individual users. The following Lemma summarizes the results:

Lemma 1. ***(The group may be pivotal)*** *Suppose that there are y individual users and a group of size x. Then, there is a threshold $\widehat{x} = y - \frac{Q}{\lambda}$ such that when $x < \widehat{x}$, platform A always wins the individual users. When $x > \widehat{x}$, the group is pivotal and the platform that wins the group wins the individual users.*

2.2 Solution to the first stage: competition on the group

Moving to the first stage, as in our base model, we start with the case where the group is pivotal, $x \geq \widehat{x}$. In this case, the platform that wins the group also wins the individual users. The group prefers joining platform A over joining platform B if:

$$x\lambda(x + y) - p_A^G > x(Q + \lambda(x + y)) - p_B^G. \tag{5}$$

The lowest price that platform B is willing to set for the group is its profit from winning the individual users; i.e., $p_B^G = -\pi_B(x, y; B) = -y(\lambda(x-y)+Q)$. Substituting p_B^G in equation (5), in an equilibrium where platform A wins the group, it charges $p_A^G = \lambda y(y-x) - Q(x+y)$.

Following the same logic, the lowest price that A is willing to set to attract the group is $p_A^G = -y(\lambda(x+y)-Q)$. Substituting this p_A^G in equation (5), in an equilibrium where B wins the group, B sets $p_B^G = (x+y)(Q-\lambda y)$. Substituting these prices into $\Pi_A(x, y; A) = \pi_A(x, y; A) + p_A^G$, and $\Pi_B(x, y; B) = \pi_B(x, y; B) + p_A^G$ and rearranging the terms we get that $\Pi_B(x, y; B) = -\Pi_A(x, y; A)$ and platform A wins iff $x < 2y(\frac{\lambda y}{Q} - 1)$ while platform B wins otherwise.[3] The following proposition summarizes the results.

Proposition 1. ***(A pivotal group may join the low-quality platform)*** *Suppose that there are y individual users and a group of size x such that the group is pivotal, $x \geq \widehat{x}$. Then, there is a threshold, $\widetilde{x} = 2y(\frac{\lambda y}{Q} - 1)$, where $\widehat{x} < \widetilde{x}$, such that if $x < \widetilde{x}$ ($x > \widetilde{x}$) platform A (B) wins the group and the individual users.*

Figure 1 illustrates the thresholds $\widehat{x}$ and $\widetilde{x}$ as a function of the quality gap between the two platforms, adjusted by the level of network effects (i.e., Q/λ) for two different individuals size: $y = 0.4$ and 0.5. The figure shows that, just like in our base model, as the quality gap between the platforms increases, the range within which the group is pivotal yet chooses the inefficient platform, $[\widehat{x}, \widetilde{x})$, becomes smaller and the range of group sizes that result in an efficient choice of platform B increases. Moreover, as Q/λ approaches y, $\widetilde{x}$ and $\widehat{x}$ approach 0, in a similar way to which $\widetilde{x}$ and $\widehat{x}$ approach 0 as Q/λ approaches 1 in our base model (recall that this appendix assumes that $Q/\lambda < y$ while our base model assumes that $Q/\lambda < 1$). Turning to changes in the absolute number of individual users, we have that both thresholds $\widehat{x}$ and $\widetilde{x}$ are increasing with the absolute number of individual users. Intuitively, a high number of individual users reduces the proportional size of the group and hence makes it more difficult for platform B to use the group for winning the individual users. As a result, the inefficient range, $[\widehat{x}, \widetilde{x})$, expands with an increase in y.

For completeness, suppose now that the group is not pivotal ($x < \widehat{x}$). In this case, platform A wins the individual users, regardless of the group's decision. In an equilibrium in which platform A wins the group, the group prefers joining A over joining

[3] It is straightforward to show that imposing $x + y = 1$ by substituting $y = 1 - x$ into $x = 2y(\frac{\lambda y}{Q} - 1)$ and solving for x, we have the same $\widetilde{x}$ as in our base model.

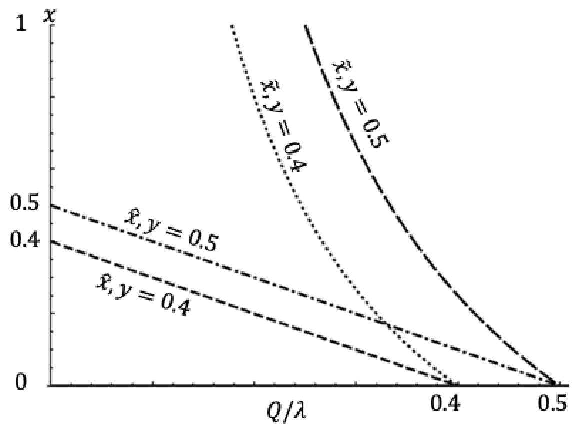

Figure 1: $\widehat{x}$ and $\widetilde{x}$ as a function of Q/λ

B if:

$$\lambda x(x+y) - p_A^G \geq x(Q+\lambda x) - p_B^G \quad, p_B^G = 0 \Longrightarrow \quad p_A^G = x\left(\lambda y - Q\right), \tag{6}$$

and platform A earns higher profit if it wins the group than if it does not attract it:

$$\pi_A(x,y;A) + p_A^G \geq \pi_A(x,y;B)$$
$$\Downarrow$$
$$x(3y\lambda - Q) \geq 0,$$

which is always positive because by assumption $\lambda y > Q$.

Next, we show that there is no equilibrium in which platform B wins the group. In this equilibrium, the lowest price that platform A is willing to charge the group is : $p_A^G = \pi_A(x,y;B) - \pi_A(x,y;A) = -2\lambda xy$. Substituting it into (6), we have that platform B can charge the group at most $p_B^G = -x(3y\lambda - Q) < 0$. As platform B cannot win the individual users when attracting the group, platform B cannot profitably win the group.

We therefore have that, just like in the base model, when the group is not pivotal, there is a unique equilibrium where platform A wins the entire market.

Proposition 2. *Suppose that there are y individual users and a group of size x such that the group is not pivotal, $x < \widehat{x}$. Then, there is a a unique equilibrium in which platform A wins the group and individual users.*

3 Utility of an individual user

The utility of each individual user is $u(x, y) = \lambda(x + y) - p_A$ if A wins, and $u(x, y) = Q + \lambda(x + y) - p_B$ if B wins. Substituting p_A and p_B, given by equations (1) and (4), we get that the utility of an individual user is:

$$u(x,y) = \begin{cases} Q, & \text{if } x < \widehat{x}, \\ 2\lambda y, & \text{otherwise.} \end{cases} \tag{7}$$

As in our base model, as long as platform A wins, the utility of individual users remains fixed at Q and jumps up once the group is pivotal and joins platform B. Unlike our base model, however, when $x > \widetilde{x}$ an increase in the size of the group does not affect the utility of individual users, as their utility only depends on the network effect they create to each other. That is, in the case where the size of the individual users remains unchanged, the size of the group affects the utility of an individual users only through the groups' platform choice. As expected, as the size of individuals users (y) increases, so does the individual utility (see Figure 2).

4 Utility of a single group user

Moving to the utility of a single group user, each group user enjoys $u^G(x, y) = \lambda(x + y) - \frac{p_A^G}{x}$ if A wins, and $u^G(x, y) = Q + \lambda(x + y) - \frac{p_B^G}{x}$ if B wins, where p_A^G is given by (6) when $x \in [0, \widehat{x})$, $p_A^G = \lambda y(y - x) - Q(x + y)$ when $x \in [\widehat{x}, \widetilde{x}]$, and $p_B^G = (x + y)(Q - \lambda y)$ when $x > \widetilde{x}$. Hence,

$$u^G(x,y) = \begin{cases} Q + \lambda x, & \textit{if } x \in [0, \widehat{x}), \\ \lambda(x + 2y) + \frac{Q(x+y) - \lambda y^2}{x}, & \textit{if } x \in [\widehat{x}, \widetilde{x}), \\ \frac{\lambda(x+y)^2 - Qy}{x}, & \textit{if } x > \widetilde{x}. \end{cases} \tag{8}$$

It is easy to see from equation (8) that when $x \leq \widetilde{x}$, the utility of a group user increases with the size of the group. This is consistent with our base model and the

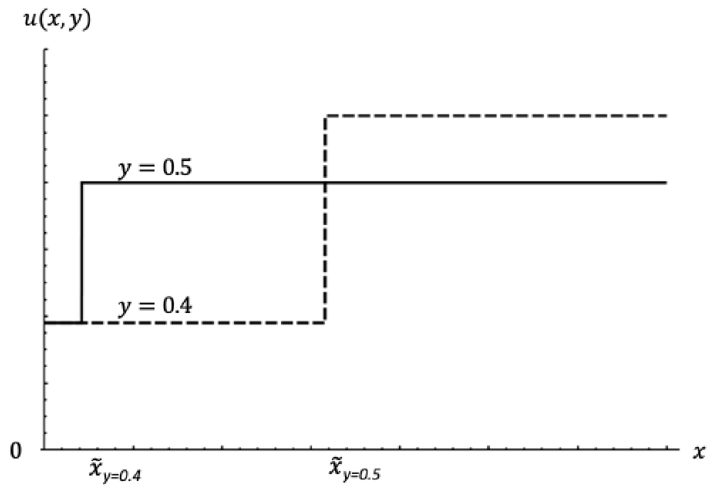

Figure 2: Individual user's utility as a function of x

Individual user's utility when $\frac{Q}{\lambda} = 0.38$ *and* $y=\{0.4, 0.5\}$.

intuition is the same: a larger group has a better outside option of joining platform B. Also, as in our base model there is a discontinuous climb at $\widehat{x}$, when the group becomes pivotal. For $x \in [\widehat{x}, \widetilde{x})$, the utility always increases with x. This differs from the case of a proportional increase in the group. As we explain in the paper, when $x \in [\widehat{x}, \widetilde{x})$ and the proportion of a pivotal group increases, on one hand its alternative option increases but at the same time the decrease in the proportion of the individual users decreases the group's market power over platform A. The second effect vanishes when we consider an absolute increase in the size of the group while keeping the size of the individual users constant. The effect of the size of the group when $x > \widetilde{x}$ is more subtle. Specifically, for group size close to $\widetilde{x}$, a group user's utility either increases with the size of the group or first decreases and then increases in it. Figure 3 presents the utility of a group user for the case where it always increases ($y = 0.5$) as well as for the case where in the area of $\widetilde{x}$ the utility first decreases and then increases ($y = 0.4$). Intuitively, as in our base model, the group needs platform B for its superior quality while platform B needs the group for attracting (and profiting from) the individual users. As x increases, the first effect becomes stronger (again, as in our base model), but now the second effect does not become weaker but instead becomes stronger because the increase in the group size does not come at the expense of the number of individual users. Hence, unlike our base model, here the utility of a group-user may decrease with x if the group is small, but

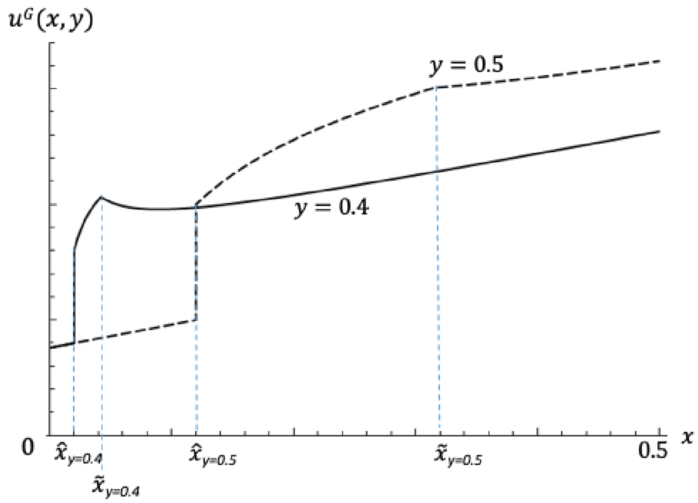

Figure 3: A group user's utility as a function of x

A group user's utility when $\frac{Q}{\lambda} = 0.38$ *and* $y=\{0.4, 0.5\}$.

increases otherwise.

5 Total consumer surplus and profits

Recall that $\Pi_i(x,y;i) \equiv \pi_i(x,y;i) + p_i^G$ denotes platform i's total profit from group and individual users for $i = \{A, B\}$ when the group chooses platform i. The platforms' profits as a function of the size of the group and individual users are then given by:

$$\Pi(x,y) = \begin{cases} (\lambda y - Q)(x+y) + \lambda xy, & \text{if } x \in [0, \widehat{x}), \\ 2y(\lambda y - Q) - Qx, & \text{if } x \in [\widehat{x}, \widetilde{x}), \\ Qx - 2y(\lambda y - Q), & \text{if } x \in [\widetilde{x}, 1]. \end{cases} \tag{9}$$

Total users' surplus is $CS(x,y) = y \times u(x,y) + x \times u^G(x,y)$. Figure 4 illustrates $CS(x,y)$ as a function of x, for two selected y values. Given the above, it is not surprising that we find that consumer surplus always increases in x. In contrast to our base model where an increase in x implies a decrease in y, when the sizes of x and y are independent, an increase in x always implies an increase in the number of users. As long as the decline in per-user utility is small, the increase in the number of overall users outweighs the decrease in per-user utility. Yet, consumer surplus may increase or decrease with

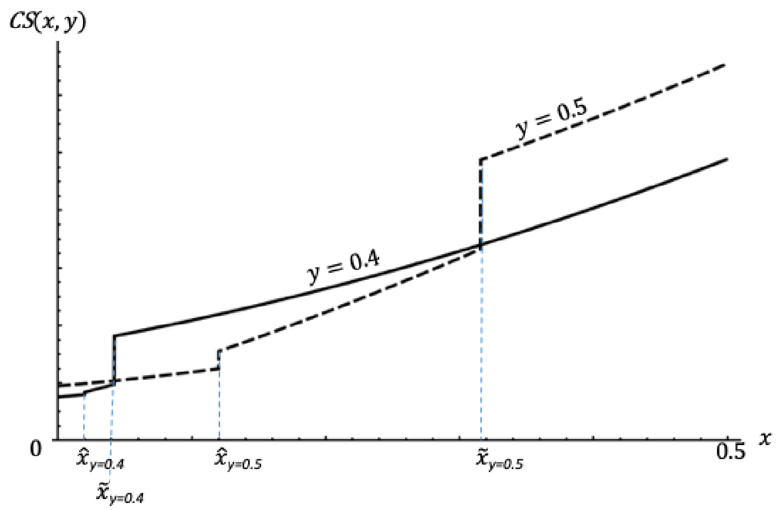

Figure 4: A group user's utility as a function of x

A group user's utility when $\frac{Q}{\lambda} = 0.38$ *and* $y=\{0.4, 0.5\}$.

y. On the one hand, an increase in y increases the threshold value $\widetilde{x}$, which reduces consumer's surplus. On the other hand, when x is sufficiently larger than $\widetilde{x}$, an increase in y increases consumer surplus because both individual and group users benefit from interacting with more individual users.

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Online Appendix C: Horizontally Differentiated Platforms

By

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1 Introduction

This appendix extends our model to horizontally differentiated platforms. We show that with product differentiation, both platforms gain a positive market share and profits. As in our base model, when the degree of horizontal differentiation is not too high, a pivotal group joins the low-quality platform A, while the high-quality platform B only serves individual users that have strong preferences for it. When the two platforms become more differentiated, individual users care about their subjective preferences towards a specific platform more than they care about the decisions of other individual users. Therefore, the focality of the low-quality platform does not affect decisions as much and the high-quality platform can win the group regardless of whether it is pivotal or not.

These results indicate that the inefficiency that our paper identifies (i.e., a pivotal group joins the "wrong" platform) becomes more prevalent as platforms become closer substitutes.

2 Model

There is a mass $y = 1$ of individual users and a mass $x > 0$ of a user-group. Users have the linear utilities: $V_A(n_A) = \lambda n_A$ and $V_B(n_B) = Q + \lambda n_B$. Following Narasimhan (1988), we assume that among the individual users, a proportion of d users are "loyal" to platform A and another proportion of d are loyal to platform B, where $0 < d < 1/2$.

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Users that are loyal to platform i would only consider buying from platform i or not buying at all. The remaining $1-2d > 0$ users are "non-loyal", who buy from the platform that provides them with the highest utility. Platforms cannot distinguish between loyal and non-loyal users and cannot price discriminate between them. Suppose for simplicity that group users are all non-loyal.[3] The parameter d measures the degree of horizontal product differentiation between the two platforms. At $d = 0$, the model is identical to our base model. As d increases, the platforms become less substitutable and as we show below, platform competition becomes less intense. At $d = 1/2$, all users are loyal, hence the two platforms are two monopolies.

The timing is as follows. First, platforms compete on the group by making simultaneous offers. The group decides which platform to join. Then, platforms compete on individual users. Here we abstract from our base model by assuming that platform A sets the price to the individual users before platform B. We make this assumption because, as Narasimhan (1988) shows, in a setting with loyal consumers there is no pure-strategy equilibrium in a simultaneous game. We show below that even though platforms set prices sequentially, the results of this model converge to the results of our online Appendix B on absolute group size when $y = 1$ and $d = 0$. Finally, individual users decide whether to join one of the platforms or not.

Following Halaburda and Yehezkel (2016), suppose that platform A is focal for the non-loyal users. Yet, each platform is focal for its loyal users: all users believe that if there is an equilibrium in which loyal users join their preferred platform, then users play this equilibrium. This implies that platform A is focal for $1 - d$ individual users. To maintain our assumption that platform A's focal position is more important than platform B's quality advantage, suppose that $Q < \lambda(1 - d)$. This assumption implies that without the group (i.e., when $x = 0$), platform A always wins the non-loyal users.

3 Main results: illustration

Before going through the technical analysis, it is useful to summarize the main results and the intuition behind them with a numerical example. We show in the next section that as in our base model, there is a cutoff, $\widehat{x}$, such that the group is pivotal if it is large enough: $x > \widehat{x}$. In the context of this extension with product differentiation, a "pivotal

[3]Our results follow trivially to the case where the group includes identical proportions of loyal users to each platform.

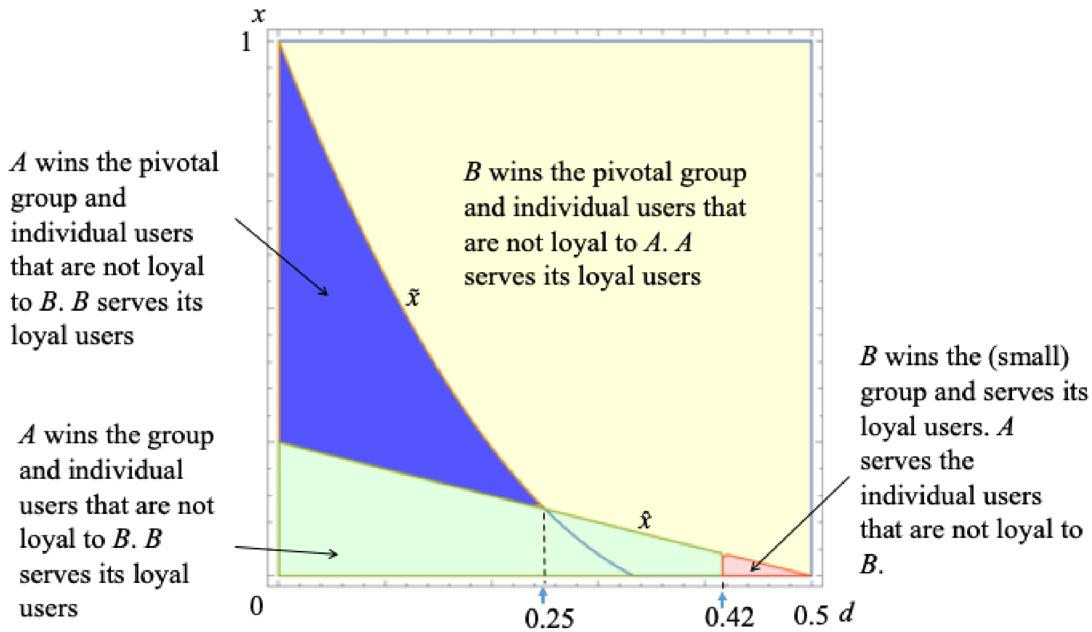

Figure 1: $\widehat{x}$ and $\widetilde{x}$ as a function of the degree of differentiation, d ($\lambda = 1$ and $Q = 1/2$)

group" means that the platform that attracts the group attracts the individual non-loyal users, while the competing platform serves the individual users that have strong preferences for it. As we show below, there is another cutoff, $\widetilde{x}$ ($\widehat{x} \leq \widetilde{x}$), such that a pivotal group joins the low-quality platform A when $\widehat{x} < x < \widetilde{x}$. Evaluated at $d = 0$ (and setting the size of individual users to $y = 1$ such that the total size of the market is $1 + x$), these two cutoffs are identical to the cutoffs of the homogeneous platform model considered in Appendix B on absolute group size. Figure 1 illustrates how the two cutoffs vary with the degree of differentiation, d. It is possible to see that there are three relevant intervals of d, which we describe below.

When the degree of differentiation is low, $d \in [0, 0.25]$, the results are qualitatively identical to our base model, with the exception that now both platforms have positive market share and profits. A pivotal group that is not too large (the dark blue region when: $\widehat{x} < x < \widetilde{x}$) joins the "wrong" platform A, driving all the individual non-loyal users to join platform A as well. Unlike our base model, platform B is now active in the market and sets prices such that it attracts the individual users that have strong preferences for it. A small group ($x < \widehat{x}$) is not pivotal and joins platform A which also attracts the individual non-loyal users and its loyal users. A large group ($x > \widetilde{x}$) makes the "right" choice of joining the high-quality platform B, in which case platform A is still active in the market and serves the individual users that are loyal to it. Hence, the dark (blue) region with $\widehat{x} < x < \widetilde{x}$ represents the inefficiency that our base model identifies. Notice that this region becomes wider as the two platforms become closer substitutes. Intuitively, more homogeneous platforms imply that the subjective preferences towards each platform are not as strong. In this case, network effects are more important, and thus individual users care more about the decisions of other users. That is, the more homogeneous the platforms, the more important the focality advantage of platform A relative to horizontal differentiation; thereby, making it easier for platform A to attract the group. Likewise, as we explain in the paper, beliefs become less important the more differentiated the platforms, and the region with the inefficient outcome shrinks as d increases.

For $d \in [0.25, 0.42]$, most individual users have strong preferences towards a specific platform and therefore platform A's focality advantage is not strong enough to enable platform A to win a pivotal group. Moreover, since there is a large enough portion of individual users with strong preferences to platform B, a pivotal group always joins the high-quality platform (yellow region) which also attracts the non-loyal users. Platform A is still active, and serves its individual loyal users.

For $d \in [0.42, 0.5]$, the platforms become almost monopolies. In this region, for most group sizes, the group is large enough to be pivotal; in which case, it always chooses platform B. Interestingly, we find that when d is very large, the group joins platform B even when it is not pivotal (see light red region). The intuition is simple. When d is large, the proportion of non-loyals is too small to affect the decision of the group. The value for the group from joining the superior platform B is higher than the value from joining platform A. As d increases, the results converge to the case where the two platforms are monopolies, each operating in a different market, and the group joins platform B.

4 Detailed Solution

Solution to the second stage: competition on individual users

Suppose that platform A won the group in the first stage. We show that platform A always wins the individual non-loyal users, while platform B focuses on serving its loyal individual users. In an equilibrium in which platform A wins the non-loyal users, platform B focuses on its mass d of loyal users, sets $p_B = Q + \lambda d$ and earns $\pi_B^{loyal}(x; A) \equiv p_B d = (Q + \lambda d)d$. If platform B attempts to win the individual non-loyal users, platform B sets p_B such that:

$$-p_A + \lambda(1 - d + x) \leq -p_B + Q + \lambda d \quad \Leftrightarrow \quad p_B = p_A + Q - \lambda(1 - 2d + x).$$

Notice that as platform A is focal for its loyal users (of mass d) as well as the non-loyal users (of mass $1 - 2d$) and also has the group (of mass x), a non-loyal user joins platform B only if the utility from doing so, in the worst case scenario, where all $1 - d + x$ users join platform A and only platform B's loyal users, d, join B is higher than the utility from joining A.[4] Platform B earns from winning the loyal and non-loyal users $\pi_B^{all}(x; A) \equiv (1 - d)p_B = (1 - d)(p_A + Q - \lambda(1 - 2d + x))$. Hence, to motivate platform B to price such that it only attracts its loyal users, and not compete on the non-loyal users, platform A needs to set p_A such that:

$$\pi_B^{all}(x; A) \leq \pi_B^{loyal}(x; A) \quad \Leftrightarrow \quad p_A \leq \lambda x - 3\lambda d + \frac{\lambda - Q + 2Qd}{1 - d}. \tag{1}$$

Turning to platform A, if platform A attracts the non-loyals, it sets the p_A in (1) and earns $\pi_A^{all}(x; A) \equiv (1 - d)p_A = (1 - d)(\lambda x - 3\lambda d) + \lambda - Q + 2Qd$. If platform A prices to focus on its loyal users, it sets $p_A = \lambda(d + x)$ and earns $\pi_A^{loyal}(x; A) \equiv dp_A = d\lambda(d + x)$. The gap between the two terms is: $\pi_A^{all}(x, A) - \pi_A^{loyal}(x, A) = (1 - 2d)(\lambda(1 - d + x) - Q) > 0$, where the inequality holds for all $x > 0$ because $d < 1/2$ and $\lambda(1 - d) > Q$. Hence when platform A wins the group, it always wins the individual non-loyal users.

Next, suppose that the group joined platform B. Consider first an equilibrium in which platform A wins the individual loyal and non-loyal users. As before, platform A sets p_A such that platform B prefers focusing on its loyal users and not compete on the non-loyal. When platform B focuses on its loyal users, it now sets $p_B = Q + \lambda(d + x)$ because platform B has the group, and earns $\pi_B^{loyal}(x, B) \equiv p_B d = (Q + \lambda(d + x))d$. If platform B attempts to win the individual users, platform B sets p_B such that:

$$-p_A + \lambda(1 - d) \leq -p_B + Q + \lambda(d + x) \quad \Leftrightarrow \quad p_B = p_A + Q - \lambda(1 - 2d - x),$$

and earns $\pi_B^{all}(x, B) \equiv (1 - d)p_B = (1 - d)(p_A + Q - \lambda(1 - 2d - x))$. Hence, platform

[4] Notice that, in equilibrium, users' expectations are realized: if this condition fails, there is an equilibrium in which all non-loyal users join platform A. If this condition holds, users correctly expect that all non-loyal users join platform B and this is an equilibrium outcome.

A needs to set p_A such that:

$$\pi_B^{all}(x;B) \leq \pi_B^{loyal}(x;B) \quad \Leftrightarrow \quad p_A \leq -3\lambda d + \frac{\lambda - Q + 2Qd - \lambda(1-2d)x}{1-d}. \qquad (2)$$

Platform A earns from attracting the loyal users: $\pi_A^{all}(x;B) \equiv (1-d)p_A =-3\lambda d(1-d) + \lambda - Q + 2Qd - \lambda(1-2d)x$. If platform A focuses on its loyal users, it sets $p_A = \lambda d$ and earns $\pi_A^{loyal}(x;B) \equiv dp_A = \lambda d^2$. The gap between the two terms is: $\pi_A^{all}(x;B) - \pi_A^{loyal}(x;B) = (1-2d)(\lambda(1-d-x)-Q)$, which is positive iff $x < 1-d-Q/\lambda$.

When $x > 1-d-Q/\lambda$, the analysis above indicates that platform A focuses on its loyal users, sets $p_A = \lambda d$ and earns $\pi_A^{loyal}(x;B) \equiv dp_A = \lambda d^2$. Given $p_A = \lambda d$, platform B attracts the non-loyals by charging $p_B = p_A + Q - \lambda(1-2d-x)= Q + \lambda(3d+x-1)$ and earns $\pi_B^{all}(x;B) = (1-d)p_B = (1-d)(Q+\lambda(3d+x-1))$. Platform B prefers this option over only focusing on its loyal users because $\pi_B^{all}(x;B) - \pi_B^{loyal}(x;B) = (1-2d)(Q-\lambda(1-2d-x)) > 0$, where the inequality holds when $x > 1-d-Q/\lambda$.

The following lemma summarizes these results:

Lemma 1. ***(The group may be pivotal)*** *When the group joins platform A, there is a unique equilibrium in which platform A serves its individual loyal users as well as the non-loyal users and earns* $\pi_A^{all}(x;A) \equiv (1-d)(\lambda x - 3\lambda d) + \lambda - Q + 2Qd$. *Platform B serves its individual loyal users and earns* $\pi_B^{loyal}(x;A) = (Q+\lambda d)d$.

When platform B has the group, there is a threshold, $\widehat{x} = 1-d-Q/\lambda$, *such that:*

- (i) *When* $x < \widehat{x}$, *platform A serves its loyal individual users plus non-loyal users and earns from the individual users:* $\pi_A^{all}(x;B) = -3\lambda d(1-d)+\lambda-Q+2Qd-\lambda(1-2d)x$. *Platform B serves its loyal users and earns* $\pi_B^{loyal}(x;B) = (Q+\lambda(d+x))d$.

- (ii) *When* $x > \widehat{x}$, *platform A serves its individual loyal users and earns:* $\pi_A^{loyal}(x;B) = \lambda d^2$. *Platform B serves its individual loyal users plus non-loyal users and earns:* $\pi_B^{all}(x;B) = (1-d)(Q+\lambda(3d+x-1))$.

Lemma 1 implies that a group of size $x > \widehat{x}$ is *pivotal*: the platform that wins the group wins the non-loyal individual users. Notice that when $d = 0$, we obtain the same profits as in the homogeneous case considered in Appendix B on absolute group size when setting $y = 1$. Likewise the cutoff $\widehat{x}$ is identical to the one in Appendix B.

Solution to the first stage: competition on the group

We now move to the first stage where platforms compete on attracting the group. As in our base model, we distinguish between a pivotal group ($x > \widehat{x}$) and a small group ($x < \widehat{x}$):

Pivotal group ($x > \widehat{x}$)

Suppose that the group is pivotal: the platform that wins the group attracts the individual non-loyal users. Consider first an equilibrium in which platform A wins the group. The lowest price that platform B is willing to charge is $p_B^G = \pi_B^{loyal}(x; A) - \pi_B^{all}(x; B)$. To attract the group, platform A sets p_A^G such that:

$$\lambda(1 - d + x)x - p_A^G \geq (Q + \lambda(1 - d + x))x - p_B^G. \tag{3}$$

Notice that now, the group knows that non-loyal users join the platform that it joins. Hence, the group gains access to $1 - d$ individual users in both platforms, while on platform B the group also gains a higher base quality, Q. Substituting $p_B^G = \pi_B^{loyal}(x; A) - \pi_B^{all}(x; B)$ into (3) and solving for p_A^G, we have that platform A earns from attracting the group $\pi_A^{all}(x; A) + p_A^G$. Platform A prefers attracting the group over giving up on the group and focusing on its individual loyal users when

$$\begin{gathered} \pi_A^{all}(x; A) + p_A^G > \pi_A^{loyal}(x; B) \\ \Downarrow \\ \lambda(1 - 2d)(2 - 3d) - 2(1 - 2d)Q - Qx > 0. \end{gathered} \tag{4}$$

Next, consider an equilibrium in which platform B wins the group. In this equilibrium, the lowest price that platform A is willing to charge the group is $p_A^G = \pi_A^{loyal}(x; B) - \pi_A^{all}(x; A)$. To win the group, platform B charges p_B^G that solves (3) in equality. Then, platform B earns from attracting the group $\pi_B^{all}(x; B) + p_B^G$, where p_B^G is the solution to (3) when substituting $p_A^G = \pi_A^{loyal}(x; B) - \pi_A^{all}(x; A)$. Platform B prefers attracting the group over giving up on the group and focusing on its individual loyal users when $\pi_B^{all}(x; B) + p_B^G > \pi_B^{loyal}(x; A)$. This condition yields the opposite condition to (4).

Define $\widetilde{x}$ as the solution to (4) in equality. Hence, an equilibrium in which platform

A wins the group exists when $x < \widetilde{x}$, where

$$\widetilde{x} \equiv \frac{\lambda(1-2d)(2-3d)}{Q} - 2(1-2d). \tag{5}$$

It is straightforward to show that $\widehat{x} < \widetilde{x}$ if and only if $d < (\lambda - Q)/2\lambda$, where $0 < (\lambda - Q)/2\lambda < 1/2$. The following proposition summarizes our results:

Proposition 1. ***(A pivotal group may join the low-quality platform)*** *Suppose that the group is pivotal: $x > \widehat{x}$. Then:*

(*i*) *If $d \leq (\lambda - Q)/2\lambda$, there is a threshold, $\widetilde{x}$, such that for $\widehat{x} < x < \widetilde{x}$, platform A wins the group and individual users that are not loyal to platform B, while platform B serves its individual loyal users. For $\widetilde{x} < x$, platform B wins the group and individual users that are not loyal to platform A, while platform A serves its individual loyal users.*

(*ii*) *If $d > (\lambda - Q)/2\lambda$, for all values of $x > \widehat{x}$, platform B wins the group and individual users that are not loyal to platform A, while platform A serves its individual loyal users.*

Part (i) of Proposition 1 identifies the same inefficiency as in our base model: a pivotal group joins the "wrong" platform that also attracts all non-loyal users. The second part of the proposition shows that when products are highly differentiated, this inefficiency vanishes. In our numerical example in Section V, $Q = 1/2$, $\lambda = 1$ and the condition in Proposition 1 becomes $d > (\lambda - Q)/2\lambda = 0.25$, which is when $\widetilde{x}$ intersects with $\widehat{x}$ in Figure 1. The intuition for this result is that when products are highly differentiated, users care more about their own subjective preferences in comparison to network effects from interacting with other users. This reduces platform A's ability to collect rents from individual users and transfer them to the group.

Small group ($x < \widehat{x}$)

Suppose now that the group is too small to affect the decisions of non-loyal users: they will join platform A regardless of the decision of the group. Consider an equilibrium in which platform A wins the group. Unlike in the case with no differentiation, the lowest amount that platform B is willing to charge to attract a small group may not be 0 because while platform B cannot use the non-pivotal group to attract the individual non-loyal users, the group provides network effects to the individual users that are loyal to platform B. Platform B is willing to charge the group as low as $p_B^G = \pi_B^{loyal}(x; A) - \pi_B^{loyal}(x; B) = -\lambda xd$. Platform A attracts the group by charging p_A^G such that the group joins A given that non-loyal individual users join platform A regardless of the decision of the group:

$$\lambda(1 - d + x)x - p_A^G \geq (Q + \lambda(d + x))x - p_B^G. \tag{6}$$

Substituting $p_B^G = -\lambda xd$ into (6), we have $p_A^G = (\lambda(1 - 3d) - Q)x$ and platform A earns from attracting the group the total $\pi_A^{all}(x; A) + p_A^G$. In an equilibrium in which platform A wins the group, the platform earns higher profit from winning the group than from not winning the group and only serving the individual loyal and non-loyal users:

$$\begin{gathered} \pi_A^{all}(x; A) + p_A^G \geq \pi_A^{all}(x; B), \\ \Downarrow \\ Q \leq 3\lambda(1 - 2d). \end{gathered} \tag{7}$$

Next, consider an equilibrium in which platform B wins the group. In this equilibrium, the lowest price that platform A is willing to charge the group is $p_A^G = \pi_A^{all}(x; B) - \pi_A^{all}(x; A) = -\lambda x(2 - 3d)$. Substituting this p_A^G into (6) and solving for p_B^G, we have that $p_B^G = (Q - \lambda(3 - 5d))x$. Platform B finds it optimal to attract the group when $\pi_B^{loyal}(x; B) + p_B^G > \pi_B^{loyal}(x; A)$, which yields the opposite condition of (7).

We therefore have that platform A wins the group if and only if $Q \leq \lambda(3 - 6d)$. Recalling that by assumption, $Q < \lambda(1 - d)$, and because $\lambda(3 - 6d) < \lambda(1 - d)$ if and only if $d > 2/5$, we have the following result:

Proposition 2. *Suppose that the group is not pivotal: $x < 1 - d - Q/\lambda$. Then,*

(*i*) *If $d < 2/5$, platform A wins the group and individual users that are not loyal to platform B, while platform B serves its individual loyal users.*

(*ii*) *If $2/5 < d < 1/2$, there is a threshold of Q, $\overline{Q} = \lambda(3-6d)$ $(0 < \overline{Q} < \lambda(1-d))$, such that when $Q < \overline{Q}$, the same equilibrium as in (i) holds. When $Q > \overline{Q}$, platform B wins the group and then serves its individual loyal users, while platform A serves its loyal individual users and the non-loyal users.*

Proposition 2 shows that when the degree of differentiation between the two platforms is sufficiently small, platform A wins a small (non-pivotal) group. Platform B is still active in the market and serves the users that have strong preferences towards it. When the degree of differentiation is high, the same result holds, unless platform B offers a substantially higher quality than platform A. In this case, the group joins the high-quality platform B even though platform A gains the individual non-loyal users. In our numerical example in Section 3, $Q = 1/2$, $\lambda = 1$ and the condition $Q > \overline{Q} = \lambda(3 - 6d)$ becomes $d > 1/2 - \frac{Q}{6\lambda} = 0.42$, which yields the red region in which the small group joins platform B. Intuitively, when only about 0.16 of the individual users are non-loyal (as 0.42 are loyal to A and another 0.42 are loyal to B), focality over these users does not provide platform A with a sufficiently strong competitive advantage to overcome platform B's superior quality.

References

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