Online Appendix

Investment and Information Acquisition

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Generalizing the Payoff Structure

In this section we confirm the robustness of our results by considering more general preferences. Suppose that the agent cares both about the principal's decision to adopt the project, and the net profit from the project, $\theta - c$. Specifically, if the project is adopted by the principal the agent's payoff is

$$(1-\alpha) + \alpha(\theta - c),$$

where $\alpha \in [0,1]$ measures the weight that the agent assigns to the net profit from the project. Thus, α parameterizes the degree of preference alignment between the players. We obtain the following result.

Proposition 6. *For each* $c \in \left[\frac{n+4}{2(n+2)}, \frac{n}{n+2}\right]$ *there exists a non-empty interval* $(0, \alpha'(c)]$*, such that:*

(i) For all $\alpha \in (0, \alpha'(c)]$ *the equilibrium allocation is strictly interior and the investment decreases in the project cost;*

(ii) The upper bound $\alpha'(c)$ *increases in the project cost, so that for a higher cost, there is a larger range of preferences where the result (i) holds;*

(iii) The equilibrium investment weakly increases in α*, as long as* $\alpha'(c) \leq \frac{3}{4}$.

Theorem 6 provides a robustness check of the results in Theorem 1. Particularly, when the alignment between the agent's and the principal's preferences is limited, in the sense that α is sufficiently small, the equilibrium has the same qualitative features as in the baseline model: the agent splits the resources between productive investment and information acquisition, and an increase in project cost leads to a lower productive investment and more information acquisition.

An increase in preference alignment embodied in higher α raises the agent's willingness to make productive investment. This is natural since the principal prefers that all resources are invested. So, a higher α is associated with smaller inefficiency in resource allocation.

Finally, the range of α on which the comparative statics of Theorem 1 holds increases in the project cost. So, when the project cost is sufficiently high, the investment decreases in the project cost even for large α's i.e., more aligned players' preferences.

Proof of the Proposition 6: Recall that in equilibrium the principal approves the project if $j \geq j^* = \lceil c(n+2) \rceil - (k+1)$, where k is the principal's belief about the agent strategy.

Suppose that the agent deviates to investment $k-d$, $d \in \{-(n-k), .., k\}$, while the principal believes that investment level is k. Then, the agent's payoff is

$$Pr(j \geq j^*|k, d)\Big(\alpha(\mathbb{E}[\theta|k, d] - c) + (1-\alpha)\Big),$$

which can be rewritten as:

$$\begin{aligned}
&D(k, d, c, \alpha) \equiv \sum_{j=j^*(k)}^{n-(k-d)} \frac{(1+(k-d))(j+(k-d))!(n-(k-d))!}{j!(n+1)!}\left(\alpha\left(\frac{(k-d)+j+1}{n+2}\right)+(1-\alpha)\right) = \\
&\frac{2-\alpha+k-d+\alpha c(2+k-d)}{2+k-d} - \\
&\frac{1}{2+k-d}\Bigg(\frac{(n-k+d)!(\lceil c(n+2)\rceil - d - 1)!(\alpha(k-d+1)\lceil c(n+2)\rceil}{(n+2)!(\lceil c(n+2)\rceil - (k+2))!} + \\
&\frac{\alpha(c(n+2)(d-k-2)-(k-d)(n+d)+d-2(k+n+2))+(n+2)(k-d+2))}{(n+2)!(\lceil c(n+2)\rceil - (k+2))!}\Bigg).
\end{aligned}$$

The equilibrium requires that at the investment level k^*, there is no deviation incentive for the agent to any feasible $k-d$, $d \neq 0$. In other words, the equilibrium condition requires that the function $D(k, d, c, \alpha)$ is maximized at $d = 0$. Formally, the following has to be satisfied in an equilibrium:

$$\frac{\partial D(k, d, c, \alpha)}{\partial d}\bigg|_{d=0} = 0 \iff D_3(k, c, \alpha) \equiv -\alpha + \frac{(n-k)!(\lceil c(n+2)\rceil - 1)!}{(n+2)!(\lceil c(n+2)\rceil - (k+2))!} \times$$

$$\Big[\Big(-\alpha\lceil c(n+2)\rceil(k+1)(k+2)+(2+k)(\alpha(1+c)-1)(2+k)(2+n)\Big)(\psi(n-k+1)-\psi(\lceil c(n+2)\rceil))+$$

$$\alpha\lceil c(n+2)\rceil + (2+k)(\alpha(k+1))\Big] = 0.$$

As we know from the proof of the baseline model, $D(k^*, c, \alpha = 0) > 0$, with $k^* = n+2-\lceil c(n+2)\rceil$ (see the proof of Proposition 1).[15] Since for $\alpha = 0$, $D_3(k^* - 1, c, \alpha = 0) = 0$ which follows directly from the fact that $\psi(n-k+1) - \psi(\lceil c(n+2)\rceil) = 0$ when assuming $k = k^* - 1$, it has to be the case that $D_3(k < k^* - 1, c, \alpha = 0) < 0$ and $D_3(k > k^*, c, \alpha = 0) > 0$.

Let us consider k in the domain $[1, n-1]$. Note that as long as there exists $\alpha > 0$ and $k \in [1, .., n-1]$ that solve $D_3(k, c, \alpha) = 0$, we have $\frac{\partial D_3(k,c,\alpha)}{\partial \alpha} < 0$. This is because

$$\frac{\partial D(k, c, \alpha)}{\partial \alpha}\bigg|_{d=0} = -\frac{\frac{\Gamma(-k+n+1)((k+1)\lceil c(n+2)\rceil+(-c-1)(k'+2)(n+2))\Gamma(\lceil c(n+2)\rceil)}{\Gamma(n+3)\Gamma(-k+\lceil c(n+2)\rceil-1)} + c(k+2)+1}{k+2} < 0$$

for any feasible k.

This means that as α increases, $D_3(k, c, \alpha)$ decreases, and therefore a larger investment k

[15] Recall that due to the discrete support of $D_3(k, c, \alpha = 0)$ in the baseline model, for high enough costs the equilibrium investment is such that $D_3(k^*, c, \alpha = 0) \geq 0$.

is required to satisfy the agent's incentive constraint.

To see that there exists a non-empty interval $(0, \alpha(c)]$ that solves $D_3(k, c, \alpha)$ for any $\alpha \in (0, \alpha(c)]$, note that for $\alpha = 0$, $\frac{\partial D_3(k,c,\alpha=0)}{\partial k} > 0$ at $k = k^* - 1$, and for all k in a neighborhood $[k^* - 1 - \delta(c), k^* - 1 + \delta(c)]$, with $\delta(c) > 0$. Since increase in α decreases $D_3(\cdot)$ provided that there exists k, α solving $D_3(k', c, \alpha) = 0$, it must be that there exists a non-empty interval $(0, \alpha(c)]$ where each α in this interval solves $D_3(k', c, \alpha) = 0$.

Next, we show that $D_3(k, c, \alpha)$ increases in c for all α. This means that a lower investment solves the equality. We know that $D_3(k, c, 0)$ increases in c. To see that the derivative is positive, consider again $D_3(k, c, \alpha)$, and note the following. First, $\psi(n-k+1) - \psi(\lceil c(n+2) \rceil)$ decreases in c. The term in front of this difference is negative for $\alpha \leq \frac{3(n+2)}{6(n+1)-2n}$, where $\frac{3(n+2)}{6(n+1)-2n} \geq \frac{3}{4}$. Further, the expression

$$\frac{(n-k)!(\lceil c(n+2) \rceil - 1)!}{(n+2)!(\lceil c(n+2) \rceil - (k+2))!} \alpha \lceil c(n+2) \rceil = \frac{a(n-k)!(\lceil c(n+2) \rceil)!}{(n+2)!(\lceil c(n+2) \rceil - (k+2))!}$$

increases in c for $\alpha > 0$. Therefore, $D_3(k, c, \alpha)$ decreases in c. But then, as long as there exists α that satisfies $D_3(k, c, \alpha) = 0$, the trade-off from the baseline model holds.

Finally, since $D_3(k, c, \alpha)$ increases in c, the interval of α supporting $D_3(k, c, \alpha) = 0$ increases with higher costs. Consider $\alpha'_{max}(c)$. If the project cost increases (and so, the function $D_3(k, c, \alpha)$ increases), then there exists an additional interval for α where the model's results hold. *Q.E.D.*