Online Appendix (Not for Publication)

"A Theory of One-Size-Fits-All Recommendations"

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A Omitted Proofs

Proof of Proposition 1. By the argument preceding the statement of the proposition, when the bias is disclosed, only agent j follows recommendations of the expert and these recommendations maximize the payoff of the agent. Hence, the expected payoff of the expert is pG, and the expected payoff of agents before the bias is disclosed is $\frac{1}{2}pG$. In contrast, in the baseline model, under an informative one-size-fits-all equilibrium, the expected payoff is $p(G+\beta)$ for the expert and $\frac{1}{2}p(G+\mu)$ for each agent. Under a split recommendation equilibrium, the expected payoff is $pG+(1-p)\beta$ for the expert and $\frac{1}{2}(pG+(1-p)\mu)$ for each agent. Thus, in the informative equilibria of the baseline model, the expert's expected payoff is higher and the agents' expected payoff is lower than when the bias is disclosed. This proves the desired result. □

Proof of Proposition 2. See the argument before the statement of the proposition in the main text. □

Proof of Proposition 3. The argument for the existence of the informative equilibrium is provided in the main text. We are left to verify that the informative equilibrium is the only-neologism proof equilibrium whenever $pG+\mu>0$. First, the optimal outcome for the expert is $x_k=A$ and $x_j=A$ if and only if $\theta_j\geq 0$. Because the expert implements this outcome in the informative equilibrium, there are no self-signalling sets, and so, the informative equilibrium is neologism-proof. Second, consider the uninformative equilibrium. As we argued in Section II, in the uninformative equilibrium, the outcome is $x_i = R$ for $i \in \{1, 2\}$. Consider the set of expert types $X = \{\tau : \theta_j = G\}$. Given X, each agent prefers to approve his proposal. Further, the expert of type $\tau \in X$ gets a strictly positive utility whenever $x_i = A$ for $i \in \{1, 2\}$, which is greater than the utility that he gets in the uninformative equilibrium (which is 0). Hence, set X is self-signalling, and so, the uninformative equilibrium is not neologism-proof. □

Proof of Proposition 4. Fix a sequential equilibrium. Let K be the lowest positive number of agents who can get recommendation a on the equilibrium path. First, note that $K > M$. Indeed, if it were $K \leq M$, then it follows from $G < \beta$ that K recommendations of a would be given only to biased agents, and so, by $\mu < 0$ agents would not follow such recommendations. Next, agents are willing to follow recommendation a to K agents only if

$$\frac{M}{K}\mu + \frac{K-M}{K}\mu_K \geq 0, \tag{1}$$

where $\mu_K \equiv G\mathbb{E}[s|\mathcal{K}] - L\left(1 - \mathbb{E}[s|\mathcal{K}]\right)$, $\mathcal{K}$ is the event that the expert gives recommendation a to K agents, and s is fraction of gains among the $K - M$ proposals with which the expert has no conflict. Since $G \geq \mu_K$, we have $K \geq M\left(1 - \mu/\mu_K\right) \geq M\left(1 - \mu/G\right)$. This gives us the desired lower bound on K. □

Proof of Proposition 5. To complete the arguments in the main text regarding the conditions under which the informative one-size-fits-all equilibrium exists, it is left to show that the agents decline the proposal upon a one-size-fits-all recommendation to do so. Notice that such recommendation implies $w < w^*$, and since, $\mathbb{E}[w] = p$, it must be $\mathbb{E}[w|w < w^*] < p$. Since $\mu < 0$, the agents have incentives to follow this recommendation. □

Next, we show that for any set of parameter values, the area of the region where the unique informative equilibrium is the split recommendation is at least 23 times smaller than the area of the region with the unique informative one-size-fits-all equilibrium. This comparison follows from the following argument. Since SRE region is empty for $p \leq 1/2$, we only consider $p > 1/2$. From Figure 1, the area of the OSFA region is $\left(\frac{1-p}{p} - \frac{1-p}{1+p}\right)L^2$ and the area of the SRE region

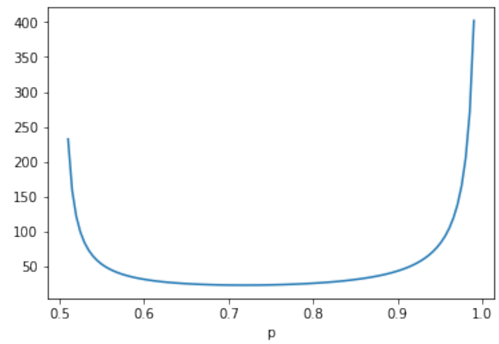

Figure 1: Ratio of the area of OSFA region to the area of SRE region as a funciton of p

is $\left(\frac{1-p}{1+p} - \frac{(1-p)^2}{p(2-p)}\right)\left(\frac{1-p}{1+p} + \frac{(1-p)^2}{p(2-p)}\right)L^2/2$. The ratio of the two equals

$$\begin{aligned}
&\frac{\left(\frac{1-p}{p} - \frac{1-p}{1+p}\right)}{\left(\frac{1-p}{1+p} - \frac{(1-p)^2}{p(2-p)}\right)\left(\frac{1-p}{1+p} + \frac{(1-p)^2}{p(2-p)}\right)/2} \\
=&\frac{2\left(\frac{1}{p} - \frac{1}{1+p}\right)}{(1-p)\left(\frac{1}{1+p} - \frac{1-p}{p(2-p)}\right)\left(\frac{1}{1+p} + \frac{1-p}{p(2-p)}\right)} \\
=&\frac{2\frac{1}{p(1+p)}}{(1-p)\left(\frac{p(2-p)-(1-p^2)}{p(1+p)(2-p)}\right)\left(\frac{p(2-p)+1-p^2}{p(1+p)(2-p)}\right)} \\
=&\frac{2}{(1-p)\frac{2p-1}{2-p}\frac{1+2p-2p^2}{p(1+p)(2-p)}} \\
=&\frac{2p(1+p)(2-p)^2}{(1-p)(2p-1)(1+2p-2p^2)},
\end{aligned}$$

which is the function of one parameter p. Numerically, the minimum of this function on $[1/2, 1]$ is ≈ 23.47, which is attained at ≈ 0.718. The plot of this function is depicted in Figure 1.

Proof of Proposition 6. Similar to the arguments in the proof of Theorem 1, an informative one-size-fits-all equilibrium exists only if $\beta < L$ and $\frac{1}{2}(G+\mu) > -c$, where the latter holds if and only if $G > \frac{1-p}{1+p}L - \frac{2c}{1+p}$. The latter condition implies that convincing the agent to follow a recommendation to approve the proposal is easier when disobeying entails a cost of $c > 0$ on the agent. Similarly, a split equilibrium exists only if $\beta < G$ and $pG + (1-p)\mu \geq -c$, where the latter holds if and only if $G \geq \frac{(1-p)^2}{1-(1-p)^2}L - \frac{c}{1-(1-p)^2}$. A rubber stamp equilibrium exists only if $\mu \geq -c$, as the agent must have incentives to follow a recommendation to approve the proposal even if he believes the expert is biased toward his proposal for sure. Notice that no other equilibrium exists, since if both (a,r) and (r,a) are on the path, inducing (r,r) is always dominated by either (a,r) or (r,a) as the expert is always biased toward one of the proposals.

If $\mu > -c$, then the rubber stamp equilibrium is the only neologism-proof equilibrium. Indeed, since under the rubber stamp equilibrium the expert obtains his first best, a deviation of the expert from any other equilibrium to a recommendation that induces agent j to decline the proposal if and only if $\theta_j = -L$ and agent k to approve it is always in the expert's best interest. Since $\mu > -c$, agent k will follow the recommendation to approve. Agent j will follow the recommendation to reject the proposal since it fully reveals $\theta_j = -L$. If $\mu < -c$, then a deviation to (r,a) or (a,r) as described above is not feasible since the agent will disobey a recommendation to approve the proposal if the other agent receives the opposite recommendation. Therefore, similar to the arguments in the proof of Theorem 1, a split equilibrium exists if and only if $\beta < G$ and $L\frac{(1-p)^2}{1-(1-p)^2} - \frac{c}{1-(1-p)^2} < G < L\frac{1-p}{1+p} - \frac{2c}{1+p}$. A one-size-fits-all equilibrium exists if and only if $\beta < L$ and $L\frac{1-p}{1+p} - \frac{2c}{1+p} < G$. In all other cases, the equilibrium must be babbling. □

Proof of Proposition 7. We start with showing the following three claims.

Claim A.1. *On the equilibrium path, recommendation r to agent 1 at $t = 1$ is always followed by recommendation r to agent 2 at $t = 2$.*

Proof: Suppose to contradiction that recommendation (r,a) is on the equilibrium path. If (a,a) is also on the equilibrium path, then the expert prefers recommendation (r,a) to recommendation (a,a) whenever he is biased to the proposal of agent 2 and the proposal of agent 1 is a loss. Thus, recommendation (r,a) cannot be influential. If (a,a) is out of the equilibrium path, then on the equilibrium path the only recommendations are either (1) (r,a) and (a,r), or (2) (r,a) and (r,r), or (3) (r,a), (a,r) and (r,r). The first case is ruled out by the same argument as in Theorem 1 using the neologism-proofness and $p < \frac{1}{2}$. The second case is ruled out by the same argument as in Online Appendix D. Since the expert is always biased toward at least one proposal, (r,r) is dominated by either (r,a) or (a,r). Therefore, the third case can be ruled our similarly to the first case. *QED*

Claim A.2. *If recommendation a to agent 1 at $t = 1$ is on the equilibrium path, then both recommendations a and r are on the equilibrium path at $t = 2$ after recommendation a at $t = 1$.*

Proof: Indeed, let us first argue that it is not possible that the expert only recommends a at $t = 2$ after recommending a at $t = 1$. In this case, only the expert who is not biased to proposal 2 and who knows that $\theta_2 = -L$ prefers to give recommendation r at $t = 2$, hence, the neologism-proofness requires that recommendation (a, r) is credible.

Next, let us argue that it is not possible that the expert only recommends r at $t = 2$ after recommending a at $t = 1$. If this were the case, then the expert would recommend a at $t = 1$ whenever either $k = 1$ or $k = 2$ and $\theta_1 = G$. Then, agent 1 would be willing to follow this recommendation if and only if $\frac{1}{2}\mu + \frac{1}{2}pG > 0$. Now, if the expert gives recommendation a at $t = 1$, then he benefits from recommending a to agent 2 at $t = 2$ only if either $k = 2$ or $k = 1$ and $\theta_2 = G$. Conditional on this and the fact that the expert gave recommendation a to agent 1 at $t = 1$, the expected payoff of agent 2 from following recommendation a equals $\frac{1}{2}\mu + \frac{1}{2}G > \frac{1}{2}\mu + \frac{1}{2}pG > 0$. Thus, by the neologism-proofness, recommendation (a, a) should also be on-path. *QED*

Claim A.3. *If recommendation a to agent 1 at $t = 1$ is on the equilibrium path, then recommendation (r, r) is also on the equilibrium path.*

Proof: If recommendation (r, r) is not on the equilibrium path, but recommendations (a, a) and (a, r) are on the equilibrium path, then it must be that agent 1 approves her proposal irrespective of the expert's recommendation. However, this contradicts our assumption that $\mu < 0$. *QED*

Claims A.1, A.2, and A.3 imply that the equilibrium of the sequential model either takes form (8) or is babbling. Note that the condition for this equilibrium to be informative is that $\frac{1}{2}\mu + \frac{1}{2}pG > 0$, or equivalently, $G > \frac{1-p}{2p}L$. The probability that the expert sends the same messages in two periods equals $1 - (1 - p)/2$ in the informative equilibrium (i.e., whenever $G > \frac{1-p}{2p}L$, or equivalently, $p > L/(L + 2G)$) and 1 in the uninformative equilibrium. When agents rubber-stamp recommendations, the probability that both recommendations coincide equals $1 - (1 - p)$, which is always less than in the sequential model. □

B General Distribution

In this section, we extend the baseline model to a general distribution of gains/losses from the policy. The game is as in the baseline model in Section II. Suppose that instead of the two-point distribution in (1), each θ_i is distributed according to a cumulative distribution function (CDF) F. As a regularity condition, we suppose that F is either a continuous distribution or a discrete distribution that does not assign a positive probability to 0. As before, we assume $\mu \equiv \mathbb{E}[\theta_i] < 0$. For general distribution of θ_i, the main message of our paper that when recommendations are public, only one-size-fits-all recommendations are possible still holds.

Proposition B.1. *Suppose that recommendations are public and* $F(0) \geq \frac{1}{2}$*. There is a unique equilibrium, which is the one-size-fits-all equilibrium whenever* $\mathbb{E}\left[\theta_i | \theta_i \geq -\beta\right] + \mathbb{E}[\theta_i] > 0$*, and the babbling equilibrium whenever* $\mathbb{E}\left[\theta_i | \theta_i \geq -\beta\right] + \mathbb{E}[\theta_i] < 0$*.*

Proof. We consider two cases.

Case 1: First, suppose that message (a, a) is sent on the equilibrium path. We first show that then messages (r, a) or (a, r) can not be sent on the equilibrium path. Suppose to contradiction that this is not the case in some equilibrium. Consider expert's types that send message (r, a). Then, it must be that $k = 2$. Indeed, if it were $k = 1$, then the expert would be able to increase his payoff (by β) by sending message (a, a) instead of (r, a). Since $k = 2$, agent 2 infer from message (r, a) that recommendation a comes from the biased expert, but do not infer anything about the θ_i. Hence, by $\mathbb{E}[\theta_i] < 0$, they will not follow it, which is a contradiction to (r, a) occurring on the equilibrium path. Therefore, the only two messages that can be sent in equilibrium are (r, r) and (a, a).

For there to be an equilibrium with only messages (r, r) and (a, a) on the equilibrium path, it is necessary that the agents want to follow recommendation (a, a). The expert sends message (a, a) if and only if $\beta + \theta_j \geq 0$. Thus, the probability of recommendation (a, a) is

$\mathbb{P}(\theta_j \geq -\beta) = 1 - F(-\beta)$. Agent i is willing to follow recommendation (a, a) if and only if

$$\frac{\frac{1}{2}\mathbb{P}\left(\theta_i \geq -\beta\right)\mathbb{E}[\theta_i|\theta_i \geq -\beta] + \frac{1}{2}\mathbb{P}\left(\theta_{-i} \geq -\beta\right)\mathbb{E}[\theta_i]}{\frac{1}{2}\mathbb{P}\left(\theta_i \geq -\beta\right) + \frac{1}{2}\mathbb{P}\left(\theta_{-i} \geq -\beta\right)} =$$
$$\frac{\frac{1}{2}(1 - F(-\beta))\mathbb{E}[\theta_i|\theta_i \geq -\beta] + \frac{1}{2}(1 - F(-\beta))\mathbb{E}[\theta_i]}{\frac{1}{2}(1 - F(-\beta)) + \frac{1}{2}(1 - F(-\beta))} =$$
$$\frac{1}{2}\mathbb{E}[\theta_i|\theta_i \geq -\beta] + \frac{1}{2}\mathbb{E}[\theta_i] \geq 0.$$

The interpretation is that with probability $\frac{1}{2}$, the expert is not biased to agent i and his recommendation means that $\theta_i \geq -\beta$, hence, the agents' estimate of θ_i is $\mathbb{E}[\theta_i|\theta_i \geq -\beta]$. With probability $\frac{1}{2}\mathbb{P}\left(\theta_{-i} \geq -\beta\right)$, the expert is biased to agent i and his recommendation is uninformative, hence, the agents' estimate of θ_i is $\mathbb{E}[\theta_i]$. In this equilibrium, the ex-ante expected payoff of agents equals

$$\frac{1}{2}\mathbb{P}[\theta_i \geq -\beta]\mathbb{E}[\theta_i|\theta_i \geq -\beta] + \frac{1}{2}\mathbb{P}[\theta_{-i} \geq -\beta]\mathbb{E}[\theta_i] = \frac{1}{2}(1 - F(-\beta))\left(\mathbb{E}[\theta_i|\theta_i \geq -\beta] + \mathbb{E}[\theta_i]\right).$$

The ex-ante expected payoff of the expert equals

$$(1 - F(-\beta))\left(\beta + \mathbb{E}[\theta_i|\theta_i \geq -\beta]\right).$$

This equilibrium (whenever it exists) is neologism-proof. To prove this, suppose to contradiction that there is a credible neologism $n(X)$ for some self-signalling subset of expert's types X. Without loss of generality, suppose that conditional on $\tau \in X$, it is optimal for agent 1 to choose R and for agent 2 to choose A. Hence, $\mathbb{E}[\theta_1|\tau \in X] \leq 0 \leq \mathbb{E}[\theta_2|\tau \in X]$. Since X is self-signalling, the payoff of expert's types in X from $x_1 = R$ and $x_2 = A$ is higher than $\max\{\theta_j + \beta, 0\}$. This implies that X includes only types with $\theta_1 < 0$ and $k = 2$. In turn, this implies that conditional on $\tau \in X$, the payoff of agent 2 equals $\mathbb{E}[\theta_2] < 0$, which is a contradiction. Therefore, this equilibrium is indeed neologism-proof.

Case 2: Now, suppose that message (a, a) is not sent on the equilibrium path. Then, message (r, r) should not be sent on the equilibrium path either. Indeed, if (r, r) were sent in equilibrium by some expert types, then the expert could benefit from sending (a, r) or (r, a) (one of which should be available by the hypothesis that the equilibrium is informative) whenever he is biased to proposal 1 or 2, respectively. Thus, the only two messages on the equilibrium path are (r, a) and (a, r) (by the symmetry of equilibrium). In such an equilibrium, when message (r, a) is sent, either $k = 1$ and $\theta_2 \geq \beta$, or $k = 2$ and $\theta_1 < \beta$. Thus, agent i follows recommendation a in the message (r, a) if and only if

$$\frac{\frac{1}{2}\mathbb{P}[\theta_i \geq \beta]\mathbb{E}[\theta_i|\theta_i \geq \beta] + \frac{1}{2}\mathbb{P}[\theta_{-i} < \beta]\mathbb{E}[\theta_i]}{\frac{1}{2}\mathbb{P}[\theta_i \geq \beta] + \frac{1}{2}\mathbb{P}[\theta_{-i} < \beta]} \geq 0,$$

or equivalently,

$$(1 - F(\beta))\mathbb{E}[\theta_i|\theta_i \geq \beta] + F(\beta)\mathbb{E}[\theta_i] \geq 0.$$

However, this equilibrium is not neologism-proof. Since $\mathbb{E}[\theta_i] < 0$, function $\int_\beta^{\overline{\theta}} \theta dF(\theta) + F(\beta)\mathbb{E}[\theta_i]$ is strictly decreasing in β. Hence, function $(1 - F(\beta))\mathbb{E}[\theta_i|\theta_i \geq \beta] + F(\beta)\mathbb{E}[\theta_i]$ is also strictly decreasing in β. Thus,

$$\begin{aligned}(1 - F(\beta))\mathbb{E}[\theta_i|\theta_i \geq \beta] + F(\beta)\mathbb{E}[\theta_i] <& (1 - F(0))\mathbb{E}[\theta_i|\theta_i \geq 0] + F(0)\mathbb{E}[\theta_i] \\ \leq& \frac{1}{2}\mathbb{E}[\theta_i|\theta_i \geq 0] + \frac{1}{2}\mathbb{E}[\theta_i],\end{aligned}$$

where the second inequality is by $F(0) \geq \frac{1}{2}$. Suppose that $(1-F(\beta))\mathbb{E}[\theta_i|\theta_i \geq \beta]+F(\beta)\mathbb{E}[\theta_i] \geq 0$ so that the second type of equilibrium exists. Consider $X = \{\tau : \theta_j \geq 0\}$. Conditional on X, the expected payoff from A of each agent is

$$\begin{aligned}\frac{\frac{1}{2}\mathbb{P}[\theta_i \geq 0]\mathbb{E}[\theta_i|\theta_i \geq 0] + \frac{1}{2}\mathbb{P}[\theta_{-i} \geq 0]\mathbb{E}[\theta_i]}{\frac{1}{2}\mathbb{P}[\theta_i \geq 0] + \frac{1}{2}\mathbb{P}[\theta_{-i} \geq 0]} &= \frac{1}{2}\mathbb{E}[\theta_i|\theta_i \geq 0] + \frac{1}{2}\mathbb{E}[\theta_i] \\ &> (1 - F(\beta))\mathbb{E}[\theta_i|\theta_i \geq \beta] + F(\beta)\mathbb{E}[\theta_i] \geq 0.\end{aligned}$$

Thus, each agent prefers A conditional on $\tau \in X$. At the same time, X is self-signalling, because (A, A) is preferred to either (R, A) or (A, R) exactly when $\theta_j \geq 0$. This gives a contradiction to the definition of neologism-proofness. Therefore, this type of equilibrium is not neologism-proof. □

In the next proposition, we characterize equilibria when $F(0) < \frac{1}{2}$. We call equilibrium the *split equilibrium* if the expert sends only (r, a) or (a, r) on the equilibrium path. By the argument in Proposition $B.1$, any informative equilibrium is either the split recommendation or the one-size-fits-all equilibrium.

Proposition B.2. *Suppose $F(0) < \frac{1}{2}$. Then, there is a unique equilibrium.*

- *If $\mathbb{E}\left[\theta_i|\theta_i \geq 0\right] + \mathbb{E}[\theta_i] > 0$, then the unique equilibrium is the one-size-fits-all equilibrium whenever $\mathbb{E}\left[\theta_i|\theta_i \geq -\beta\right] + \mathbb{E}[\theta_i] > 0$, and it is the babbling equilibrium whenever $\mathbb{E}\left[\theta_i|\theta_i \geq -\beta\right] + \mathbb{E}[\theta_i] < 0$.*

- *If $\mathbb{E}[\theta_i|\theta_i \geq 0] + \mathbb{E}[\theta_i] < 0$, then the unique informative equilibrium is the split equilibrium, which exists if and only if $(1 - F(\beta))\mathbb{E}[\theta_i|\theta_i \geq \beta] + F(\beta)\mathbb{E}[\theta_i] \geq 0$.*

Proof. Observe that in the proof of Proposition $B.1$, we used the condition $F(0) > \frac{1}{2}$ only to show that $\mathbb{E}\left[\theta_i|\theta_i \geq 0\right] + \mathbb{E}[\theta_i] > 0$. Hence, when $F(0) < \frac{1}{2}$ and $\mathbb{E}\left[\theta_i|\theta_i \geq 0\right] + \mathbb{E}[\theta_i] > 0$, the characterization is identical to that in Proposition $B.1$.

Suppose $\mathbb{E}[\theta_i|\theta_i \geq 0] + \mathbb{E}[\theta_i] < 0$. Since $\mathbb{E}[\theta_i|\theta_i \geq -\beta] + \mathbb{E}[\theta_i] < \mathbb{E}[\theta_i|\theta_i \geq 0] + \mathbb{E}[\theta_i] < 0$, there is no one-size-fits-all equilibrium. The split equilibrium exists only if $(1 - F(\beta))\mathbb{E}[\theta_i|\theta_i \geq \beta] + F(\beta)\mathbb{E}[\theta_i] < 0$. We are left to show that the split equilibrium is neologism-proof. To prove the first statement, suppose to contradiction that there is a credible neologism $n(X)$ for some self-signalling subset of expert's types X. Without loss of generality, suppose that conditional on $\tau \in X$, it is optimal for both agents to choose A. Hence, $\mathbb{E}[\theta_i|\tau \in X] \geq 0$ for $i = 1, 2$. Since X is self-signalling, the payoff of expert's types in X from $x_1 = x_2 = A$ is higher than $\max\{\theta_j, \beta\}$. This implies that X includes only expert types with $\theta_j \geq 0$. In turn, this implies that conditional on $\tau \in X$, the payoff of each agent equals $\frac{1}{2}\mathbb{E}[\theta_i|\theta_i \geq 0] + \frac{1}{2}\mathbb{E}[\theta_i] < 0$, and so, it is not optimal for agents to choose A. Therefore, the split equilibrium is indeed neologism-proof. □

C Possibility of Expert Biased to Both Proposals

In this section, we relax the assumption of the baseline model that the expert is biased to only one proposal. We show that the possibility of the expert being biased to both proposals does not change qualitatively the results. Specifically, suppose that with probability $1 - \nu$ the expert is biased to both proposals, and with remaining probability ν the expert is biased to only one proposal and each agent is equally likely to have the biased proposal. The rest of the model setup is the same as the baseline model.

In this extension, we allow for mixing by the expert, which is required to ensure the existence of symmetric equilibria. As we pointed out in Section II, our results in the baseline model are not affected by this possibility. The next proposition shows that the one-size-fits-all recommendation is the unique equilibrium as is the case in the baseline model (Theorem 1).

Proposition C.1. *Define*

$$\beta^*_\nu \equiv G \cdot \mathbf{1}\left\{L\frac{(1-p)(1-\nu p)}{p\nu+(1-\nu p)p} < G < L\frac{(1-p)(1-\nu+\nu p/2)}{p(1-\nu/2+\nu p/2)}\right\}. \tag{2}$$

Then, the equilibrium is unique and is a one-size-fits-all recommendation if and only if $\beta \geq \beta^*_\nu$. *Moreover, the one-size-fits-all equilibrium is informative if and only if* $\beta < L$ and $L\frac{(1-p)(1-\nu+\nu p/2)}{p(1-\nu/2+\nu p/2)} < G$.

Proof. The argument follows the steps of the proof of Theorem 1, and here, we only sketch the argument and show how the steps should be adjusted to account for the possibility that the expert is biased to both proposals. Let us first derive conditions under which the one-size-fits-all equilibrium is informative. The expert who is biased to both proposals prefers to recommend (a, a). If $\beta > L$, then the expert always prefers to recommend (a, a), and so, by $\mu < 0$ this recommendation cannot be influential. Thus, it is necessary for the one-size-fits-all equilibrium to be informative that $\beta < L$. In this case, the expert makes recommendation (a, a) if either he is biased to both proposals (with probability $1 - \nu$) or he is biased to only one proposal and the unbiased proposal is a gain (which happens with probability νp). Thus, agents are willing to follow recommendation (a, a) if and only if

$$(1-\nu)\mu + \nu p(\frac{1}{2}\mu + \frac{1}{2}G) > 0,$$

or equivalently, $G > \frac{(1-p)(1-\nu+\nu p/2)}{p(1-\nu/2+\nu p/2)}L$.

Next, we show that the are no other neologism-proof equilibria as long as $\beta \geq \beta^*_\nu$. To see this, consider first the case when (a, a) is on the equilibrium path. If recommendation (a, r) or (r, a) is also on the equilibrium path, then it is only sent by the expert who is biased to only one proposal and it is the biased proposal that receives the recommendation a. Thus, it cannot be influential (by $\mu < 0$).

If (a, a) is off path, then only recommendations (a, r) and (r, a) can be on the equilibrium path in any informative symmetric equilibrium. When $\beta > G$, recommendations (a, r) and (r, a) cannot be on the equilibrium path, because they signal that the expert is biased to the proposal that receives recommendation a, and hence, cannot be influential. Consider the case of $\beta < G$. By the same arguments as in the main text, the payoff of the agent who receives split recommendation a equals

$$\frac{\nu p \frac{1}{2} G + (1-\nu)\frac{1}{2}\mu + \nu(1-p)\frac{1}{2}\mu}{\nu p \frac{1}{2} + (1-\nu)\frac{1}{2} + \nu(1-p)\frac{1}{2}}.$$

Here, we account for the fact that with probability $1-\nu$, the expert is biased to both proposals, in which case he make a split recommendation a to a randomly chosen agent (by the symmetry of equilibria). This payoff is positive if and only if

$$\nu p G + (1-\nu p)\mu > 0,$$

or equivalently, $G > \frac{(1-\nu p)(1-p)}{\nu p + (1-\nu p)p} L$. Thus, this equilibrium does not exist whenever $G < \frac{(1-\nu p)(1-p)}{\nu p + (1-\nu p)p} L$. Further, by the argument as in the main text, this equilibrium is not neologism proof whenever the informative one-size-fits-all equilibrium exists, which in this modification of the baseline model boils down to $G > \frac{(1-p)(1-\nu+\nu p/2)}{p(1-\nu/2+\nu p/2)} L$. Thus, this split equilibrium is possible only if $L\frac{(1-p)(1-\nu p)}{p\nu+(1-\nu p)p} < G < L\frac{(1-p)(1-\nu+\nu p/2)}{p(1-\nu/2+\nu p/2)}$ and $\beta > G$. This gives the desired form of lower bound β^*_ν and completes the proof. □

D Asymmetric Equilibria

In the main text, we focused on symmetric equilibria, in which the beliefs of agent i after any message depend only on the recommendation on proposal i and the total number of recommendations of each kind. Focusing on symmetric equilibria is reasonable given the symmetry of the agents. Nevertheless, we next show that the symmetry restriction although simplifies somewhat the analysis is not necessary for Theorem 1 whenever $\beta < L$ (which is the region where the one-size-fits-all equilibrium exists). One can see that the only place where we used the symmetry assumption in the proof of Theorem 1 is when we considered the case when (a, a) is out of the equilibrium path. In this case, possible equilibria have on the equilibrium path either (a,r) and (r,a), or (a,r) and (r,r), or (r,a) and (r,r). The former case we have already analyzed in the main text.

Consider the case when only messages (a,r) and (r,r) are on the equilibrium path. We will show that this equilibrium is not neologism-proof. For this equilibrium to exist, it is necessary that whenever the recommendation is (a,r), agent 1 is willing to follow it. The expert recommends (a,r) if either $k=1$ or $k=2$ and $\theta_1 = G$. Thus, the posterior expectation of θ_1 after message (a,r) equals

$$\frac{1}{2}\mu + \frac{1}{2}pG,$$

which must be positive. If (a,a) were influential, then by $\beta < L$ it would be preferred by the expert whenever $\theta_j = G$. Conditional on this information, agents prefer to follow it whenever $\frac{1}{2}\mu + \frac{1}{2}G > 0$,which holds by $\frac{1}{2}\mu + \frac{1}{2}pG > 0$. Therefore, equilibrium with on-path messages (a,r) and (r,r) is not neologism-proof, and so, under condition $\beta < L$, only symmetric equilibria are neologism-proof.

E Asymmetric Bias

In this section we consider the case of asymmetric bias. Specifically, with probability q the expert is biased to the proposal of agent 1 and with probability $1-q$ the expert is biased to the proposal of agent 2. The rest of the model setup is as in the baseline model. Since the probability of bias is asymmetric, we do not restrict attention to symmetric equilibria. To simplify the analysis and better illustrate the new features that arise because of the asymmetry of the bias, we additionally suppose that $\beta < L$ and $p < \frac{1}{2}$. Further, without loss of generality, we suppose that $q \in [0, \frac{1}{2}]$, that is, agent 1 is less likely to have a biased proposal. Note that when $q = \frac{1}{2}$, we get our baseline model. When $q = 0$, we get the case of disclosed bias.

Proposition E.1. *Consider the version of our model with asymmetric bias, and suppose that $\beta < L$ and $p < \frac{1}{2}$. Then, a unique equilibrium exists.*

1. *The equilibrium is a one-size-fits-all equilibrium if and only if $G > \frac{(1-q)(1-p)}{1-(1-q)(1-p)}L$.*

2. *The equilibrium has only messages (a,r) and (r,r) on the equilibrium path if and only if $G < \frac{(1-q)(1-p)}{1-(1-q)(1-p)}L$ and $G > \frac{q(1-p)}{p}L$.*

3. Otherwise, equilibrium is babbling.

Proof. We start our analysis from the case when (a, a) is on the equilibrium path. Since it is common knowledge that the expert is biased to one of the proposals, for the same reasons as in our baseline model, recommendations (a, r) and (r, a) cannot be influential if recommendation (a, a) is influential. Since $\beta < L$, the expert prefers recommendation (a, a) over recommendation (r, r) unless the approval of proposal j destroys value. At the same time, recommendation (a, a) is influential if and only if agent 2, who is more likely to have a biased proposal, is willing to follow it:

$$(1 - q)\mu + qG > 0,$$

or equivalently, $G > \frac{(1-q)(1-p)}{1-(1-q)(1-p)}L$. To show that when a one-size-fits-all equilibrium, when it exists, is also neologism-proof, one can follow the same arguments as in the proof of Theorem 1

Now, consider an equilibrium in which (a, a) is out of the equilibrium path. Then, possible equilibria have on the equilibrium path either (a, r) and (r, a), or (a, r) and (r, r), or (r, a) and (r, r).

We can proceed as in the main text to show that the equilibrium with only (a, r) and (r, a) on-path is not neologism-proof given our parameter restrictions whenever $\beta > G$. Suppose $\beta < G$. As in the baseline model, the proposal that gets a recommendation to approve either has no conflict of interest and it also increases value to the agent, or the proposal has the conflict of interest, but the other agent loses from approving her proposal. Agent 2, who is more likely to have a biased proposal, is willing to follow a recommendation to approve her proposal only if

$$\frac{pqG + (1 - q)(1 - p)\mu}{pq + (1 - q)(1 - p)} > 0$$

Notice that if the proposal without the conflict of interest increases value then the expert has incentives to deviate to (a, a) if both agents will follow the recommendation to approve their proposals. Upon such deviation, the agents will approve the proposal if $(1 - q)\mu + qG > 0$, which always holds if $\frac{pqG+(1-q)(1-p)\mu}{pq+(1-q)(1-p)} > 0$ and $p < \frac{1}{2}$. Therefore, this split equilibrium is not neologism-proof even when $\beta < G$.

Next, consider an equilibrium where only (r, a) and (r, r) are on the path. Then, the expert recommends (r, a) if either $k = 2$, or $k = 1$ and $\theta_2 = G$. Thus, the posterior expectation of θ_2

after message (r,a) equals

$$\frac{pqG+(1-q)\mu}{pq+(1-q)},$$

which must be positive for this to be an equilibrium. If (a,a) were influential, then by $\beta < L$ it would be preferred by the expert whenever $\theta_j = G$. Conditional on this information, both agents prefer to follow it whenever $(1-q)\mu+qG > 0$. which holds by $\frac{pqG+(1-q)\mu}{pq+(1-q)} > 0$. Therefore, equilibrium with on-path messages (r,a) and (r,r) is not neologism-proof.

Finally, consider an equilibrium where only (a,r) and (r,r) are on the path. Then, the expert recommends (a,r) if either $k=1$ or $k=2$ and $\theta_1 = G$. Thus, the posterior expectation of θ_1 after message (a,r) equals

$$\frac{q\mu+(1-q)pG}{q+(1-q)p},$$

which must be positive for this to be an equilibrium. If (a,a) were influential, then by $\beta < L$ it would be preferred by the expert whenever $\theta_j = G$. Conditional on this information, agents prefer to follow it whenever $(1-q)\mu + qG > 0$. Thus, this equilibrium is neologism-proof only if $G < \frac{(1-q)(1-p)}{1-(1-q)(1-p)}L$ and $G > \frac{q(1-p)}{p}L$. Notice a deviation to (r,a) is never credible, for the same reason that an equilibrium where only (r,a) and (r,r) are on the path cannot exist. This completes the characterization of equilibrium with asymmetric probability of bias. $\square$

Proposition E.1 is illustrated on Figure 2. As the probability that agent 1 does not have biased proposal increases (i.e., q decreases), it is more likely that only agent 1 gets an informative recommendation. Figure 2also demonstrates that for a range of q close to $\frac{1}{2}$, the unique equilibrium is always a one-size-fits-all equilibrium, which shows the robustness of the result in Theorem 1.

F Free-Riding on Subscription Fees

In this section, we show that if common factors were the underlying force behind one-size-fits-all recommendations, then the expert would not be able to charge subscription fees from agents due to free-riding. This aspect is particularly relevant in the context of proxy advisors, thus, we formulate this extension in terms of this application.

To show this argument formally, consider a variant of the baseline model in which share-

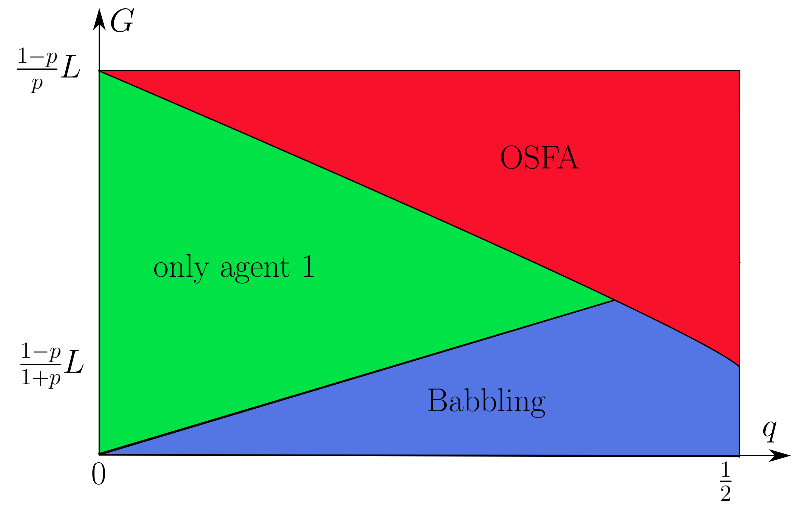

Figure 2: Asymmetric Bias

Note: OSFA denotes the region of parameters where the unique equilibrium is the informative one-size-fits-all equilibrium. Only agent 1 denotes the region of parameters where the unique equilibrium has only recommendations to agent 1, i.e., only (a, r) and (r, r). Babbling denotes the region of parameters where the unique equilibrium is babbling equilibrium.

holders of both firms simultaneously decide whether to subscribe to the recommendations of the proxy advisor for a subscription fee $f > 0$ before the proxy advisor obtains his private information. The recommendations are public as in the baseline model, however, if shareholder i does not subscribe, then the proxy advisor does not collect information specific to firm i or issue a recommendation to firm i.

Suppose first θ_is are correlated: with probability ρ, $\theta_1 = \theta_2 = \theta_c$ where θ_c is drawn from (1), and with probability $1 - \rho$, θ_is are independent draws from (1). The correlation parameter ρ captures to what extent one size fits all firms. The proxy advisor privately observes θ_c, but not θ_is. Therefore, if the recommendations are informative in equilibrium, they must provide the same information to all firms, and shareholders in different firms will make the same voting decision. The next result immediately follows.

Proposition F.1. *Suppose θ_is are correlated and proxy advisor is privately informed about the common factor θ_c. For any $f > 0$, in any equilibrium shareholders of at least one firm choose not to subscribe to the proxy advisor's recommendations.*

Intuitively, when one-size-fits-all recommendations are driven by the common factor, there is a free-rider's problem among shareholders of different firms in that they can simply rely on public recommendations given to the other firms without paying the subscription fee. Indeed, shareholder i subscribes only if the proxy advisor's recommendation is informative, but in this case the other shareholder can learn about θ_c from that recommendation, and he has no incentives to subscribe by himself. Therefore, if proxy advisors provided information only about common factors, they would likely choose a different business model from what is currently adapted in practice. As the next proposition shows, this contrasts with our baseline model where one-size-fits-all recommendations arise from the proxy advisor's attempt to conceal his conflict of interest.

Proposition F.2. *If the equilibrium in the baseline model is a one-size-fits-all equilibrium, then shareholders of both firms subscribe to the advisor's recommendations with sufficiently small subscription fee.*

Proof. If shareholders of both firms subscribe and the equilibrium in the baseline model is a one-size-fits-all equilibrium, then the proxy advisor becomes informed on both firms and the expected shareholder value is $0.5p(G+\mu) > 0$. If only shareholder 1 subscribes, then the proxy advisor's recommendation is informative only about firm 1. In this case, shareholder 2 will ignore the recommendation of firm 1 and reject the proposal of firm 2, which brings her the payoff of zero. Therefore, he has strict incentives to pay the fee as long as $f < 0.5p(G+\mu)$. □

Intuitively, since in the baseline model valuations are uncorrelated and the proxy advisor's recommendation is informative about θ_i only if shareholder i subscribes, shareholder i cannot free-ride on the subscription of the other shareholder, and therefore, has stronger incentives to pay the subscription fee.