Reporting Sexual Misconduct in the #MeToo Era
Online Appendix

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I. Supplemental Detail for Main Proofs

We will need the following standard result. For any two densities g and h for a random variable z that ranges over $(-\infty, \infty)$, we say g *stochastically dominates* h $(g \succeq h)$ if $G(z) = \int_{-\infty}^{z} g(s)\ ds \leq \int_{-\infty}^{z} h(s)\ ds = H(z)\ \forall z$. If g stochastically dominates h, then for any weakly increasing function u, the expected value of u under the former weakly exceeds that of the latter.

LEMMA OA.1: *If $g \succeq h$ and $u(z)$ is a weakly increasing differentiable function of z, then:*

$$\int_{-\infty}^{\infty} u(z)\ g(z)\ dz \geq \int_{-\infty}^{\infty} u(z)\ h(z)\ dz.$$

If $u(z)$ is a weakly decreasing function of z, then the inequality is reversed.

PROOF OF LEMMA 2:

We know $\pi(r, x)$ satisfies Properties P1-P5 in Lemma 1.

[1] Stochastic dominance arguments and Property P2 implies $\pi^*(x, k)$ is strictly increasing in x. To see this, implement a change of variables with $z = -\theta$. Note that $f_z(z) = \frac{1}{\sigma} f\left(\frac{x+z}{\sigma}\right)$, and $F_z(z; x_1) = \int_{-\infty}^{z} \frac{1}{\sigma} f\left(\frac{x_1+z}{\sigma}\right) dz < \int_{-\infty}^{z} \frac{1}{\sigma} f\left(\frac{x_2+z}{\sigma}\right) dz = F_z(z; x_2)$ for any z and $x_1 < x_2$. That is, z under x_1 stochastically dominates z under x_2, because the former has more probability mass "shifted to the right." Observe that:

$$\begin{aligned}
\pi^*(x, k) &= \int_{\theta=-\infty}^{\infty} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \pi\left(1 - F\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta \\
&= \int_{z=-\infty}^{\infty} \frac{1}{\sigma} f\left(\frac{x+z}{\sigma}\right) \pi\left(1 - F\left(\frac{k+z}{\sigma}\right), x\right) dz
\end{aligned}$$

Note that $\pi\left(1 - F\left(\frac{k+z}{\sigma}\right), x_1\right)$ is a differentiable weakly decreasing function of z, and increasing function of x. Therefore, under Lemma OA.1, for $x_1 < x_2$,

$$\begin{aligned}\pi^*(x_1, k) &= \int_{-\infty}^{\infty} \frac{1}{\sigma} f\left(\frac{x_1 + z}{\sigma}\right)\left[\pi\left(1 - F\left(\frac{k+z}{\sigma}\right), x_1\right)\right] dz \\ &\leq \int_{-\infty}^{\infty} \frac{1}{\sigma} f\left(\frac{x_2 + z}{\sigma}\right)\left[\pi\left(1 - F\left(\frac{k+z}{\sigma}\right), x_1\right)\right] dz \\ &< \int_{-\infty}^{\infty} \frac{1}{\sigma} f\left(\frac{x_2 + z}{\sigma}\right)\left[\pi\left(1 - F\left(\frac{k+z}{\sigma}\right), x_2\right)\right] dz \\ &= \pi^*(x_2, k).\end{aligned}$$

Property P1 implies $\pi^*(x,k)$ is weakly decreasing in k, and Property P5 implies $\pi^*(x,k)$ is continuous in x and k. Note that for $x > 0$, $\pi^*(x,k)$ is strictly decreasing in k for $k > 0$.

[2] We show that $\{\underline{\xi}^n\}$ and $\{\bar{\xi}^n\}$ are well-defined increasing and decreasing sequences, respectively, through induction. From Property P4, we know that not reporting is dominant for $x < \underline{x}$, so $\pi^*(x,0) < 0$ for all $x < \underline{x}$. But we also know that $\pi^*(x,0) > 0$ for all $x > \bar{x}$. Define $\underline{\xi}^0 \equiv 0$ and $\bar{\xi}^0 \equiv \infty$. By continuity in x, there exists at least one solution x with $\pi^*\left(x, \underline{\xi}^0\right) = 0$, where $x \in [\underline{x}, \bar{x}]$. Call $\underline{\xi}^1$ the smallest such solution. Define $\bar{\xi}^1 \in [\underline{x}, \bar{x}]$ analogously to be the largest such solution with $\pi^*\left(x, \bar{\xi}^0\right) = 0$. Note that $\underline{\xi}^0 < \underline{\xi}^1 < \bar{\xi}^1 < \bar{\xi}^0$; if the inside inequality did not hold, then $0 = \pi^*\left(\underline{\xi}^1, \underline{\xi}^0\right) \geq \pi^*\left(\bar{\xi}^1, \underline{\xi}^0\right) > \pi^*\left(\bar{\xi}^1, \bar{\xi}^0\right) = 0$, a contradiction.

Our starting point for the induction is as follows. Given $\underline{\xi}^1$ and $\bar{\xi}^1$ with $\underline{\xi}^0 < \underline{\xi}^1 < \bar{\xi}^1 < \bar{\xi}^0$, $\pi^*\left(\underline{\xi}^1, \underline{\xi}^0\right) = 0$, and $\pi^*\left(\bar{\xi}^1, \bar{\xi}^0\right) = 0$, we claim there exists a smallest solution $\underline{\xi}^2$ of $\pi\left(\underline{\xi}^2, \underline{\xi}^1\right) = 0$ and a largest solution $\bar{\xi}^2$ of $\pi\left(\bar{\xi}^2, \bar{\xi}^1\right) = 0$, and that $\underline{\xi}^1 < \underline{\xi}^2 < \bar{\xi}^2 < \bar{\xi}^1$. We know $\pi^*\left(\underline{\xi}^1, \underline{\xi}^0\right) = 0 > \pi^*\left(\underline{\xi}^1, \underline{\xi}^1\right)$, and $\pi^*\left(\bar{\xi}^1, \bar{\xi}^0\right) = 0 < \pi^*\left(\bar{\xi}^1, \underline{\xi}^1\right)$. Note for the latter inequality that $\bar{\xi}^0 > \bar{\xi}^1 > \underline{\xi}^1$. By continuity, there exists a smallest solution $\underline{\xi}^2 \in \left(\underline{\xi}^1, \bar{\xi}^1\right)$ with $\pi^*\left(\underline{\xi}^2, \underline{\xi}^1\right) = 0$. Analogously, we know $\pi^*\left(\underline{\xi}^1, \underline{\xi}^0\right) = 0 > \pi^*\left(\underline{\xi}^1, \bar{\xi}^1\right)$, and $\pi^*\left(\bar{\xi}^1, \bar{\xi}^0\right) = 0 < \pi^*\left(\bar{\xi}^1, \bar{\xi}^1\right)$; by continuity there exists a largest solution $\bar{\xi}^2 \in \left(\underline{\xi}^1, \bar{\xi}^1\right)$ with $\pi^*\left(\bar{\xi}^2, \bar{\xi}^1\right) = 0$. Note that $\underline{\xi}^1 < \underline{\xi}^2 < \bar{\xi}^2 < \bar{\xi}^1$; if the inside inequality did not hold, then $0 = \pi^*\left(\underline{\xi}^2, \underline{\xi}^1\right) \geq \pi^*\left(\bar{\xi}^2, \underline{\xi}^1\right) > \pi^*\left(\bar{\xi}^2, \bar{\xi}^1\right) = 0$, a contradiction.

The inductive hypothesis is that, given $\underline{\xi}^n$ and $\bar{\xi}^n$ with $\underline{\xi}^{n-1} < \underline{\xi}^n < \bar{\xi}^n < \bar{\xi}^{n-1}$, $\pi^*\left(\underline{\xi}^n, \underline{\xi}^{n-1}\right) = 0$, and $\pi^*\left(\bar{\xi}^n, \bar{\xi}^{n-1}\right) = 0$, there exists a smallest solution $\underline{\xi}^{n+1}$ of $\pi\left(\underline{\xi}^{n+1}, \underline{\xi}^n\right) = 0$ and a largest solution $\bar{\xi}^{n+1}$ of $\pi\left(\bar{\xi}^{n+1}, \bar{\xi}^n\right) = 0$, and that $\underline{\xi}^n < \underline{\xi}^{n+1} < \bar{\xi}^{n+1} < \bar{\xi}^n$. We know $\pi^*\left(\underline{\xi}^n, \underline{\xi}^{n-1}\right) = 0 > \pi^*\left(\underline{\xi}^n, \underline{\xi}^n\right)$, and $\pi^*\left(\bar{\xi}^n, \bar{\xi}^{n-1}\right) = 0 < \pi^*\left(\bar{\xi}^n, \underline{\xi}^n\right)$. Note for the latter inequality that $\bar{\xi}^{n-1} > \bar{\xi}^n > \underline{\xi}^n$. By continuity, there exists a smallest solution $\underline{\xi}^{n+1} \in \left(\underline{\xi}^n, \bar{\xi}^n\right)$ with $\pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^n\right) = 0$. Similarly, we know $\pi^*\left(\underline{\xi}^n, \underline{\xi}^{n-1}\right) = 0 > \pi^*\left(\underline{\xi}^n, \bar{\xi}^n\right)$, and $\pi^*\left(\bar{\xi}^n, \bar{\xi}^{n-1}\right) = 0 < \pi^*\left(\bar{\xi}^n, \bar{\xi}^n\right)$; by continuity there exists a largest solution $\bar{\xi}^{n+1} \in \left(\underline{\xi}^n, \bar{\xi}^n\right)$ with $\pi^*\left(\bar{\xi}^{n+1}, \bar{\xi}^n\right) = 0$. Note that $\underline{\xi}^n < \underline{\xi}^{n+1} < \bar{\xi}^{n+1} < \bar{\xi}^n$; if the inside inequality did not hold, then

$0 = \pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^n\right) \geq \pi^*\left(\bar{\xi}^{n+1}, \underline{\xi}^n\right) > \pi^*\left(\bar{\xi}^{n+1}, \bar{\xi}^n\right) = 0$, a contradiction.

Note that $\{\underline{\xi}^n\}$ is bounded from above by construction. Because it is also an increasing sequence, there exists a $\underline{\xi}$ with $\lim_{n\to\infty} \underline{\xi}^n = \underline{\xi}$. Note that $\lim_{n\to\infty} \pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^n\right) = 0$ so by construction and continuity of π^*, we must have $\pi^*\left(\underline{\xi}, \underline{\xi}\right) = 0$ and that $\underline{\xi}$ is the smallest such solution to $\pi^*(\xi, \xi) = 0$. Analogously, there exists a $\bar{\xi}$ with $\lim_{n\to\infty} \underline{\xi}^n = \underline{\xi}$ and $\pi^*\left(\bar{\xi}, \bar{\xi}\right) = 0$ and that $\bar{\xi}$ is the smallest such solution to $\pi^*(\xi, \xi) = 0$. This shows, among other things, that there exists at least one threshold equilibrium $\underline{\xi}$. One can see that any such solution $\underline{\xi}$ is an equilibrium because $x_1 < \underline{\xi} < x_2$ implies $\pi^*\left(x_1, \underline{\xi}\right) < \pi^*\left(\underline{\xi}, \underline{\xi}\right) = 0 < \pi^*\left(x_2, \underline{\xi}\right)$.

[3] Note that we can write:

$$\pi^*(x, k) \quad = \quad \int_{-\infty}^{\infty} \psi(r; x, k)\, \pi(r, x)\, dr$$

Given the agent's signal x, what is her assessment of the cumulative distribution function of r, $\Psi(\tilde{r}; x, k)$? For any $\tilde{r}$, the probability that $r < \tilde{r}$ equals the probability that $\theta < k - \sigma F^{-1}(1 - \tilde{r})$. In words, the probability $\Psi(\tilde{r}; x, k) \equiv \Pr(r < \tilde{r} \mid x)$ that the true proportion of players reporting is less than $\tilde{r}$ equals the probability that the true θ satisfies $r(\theta; k) = 1 - F\left(\frac{k-\theta}{\sigma}\right) < \tilde{r}$, or equivalently that θ is such that fewer than $\tilde{r}$ players observe a signal greater than k; in turn, this equals the probability that the true θ is less than $k - \sigma F^{-1}(1 - \tilde{r})$, integrated against the conditional density $f(\theta \mid x)$. With some slight abuse of notation, we thus have:

$$\begin{aligned}
\Psi(r; x, k) &= \int_{-\infty}^{k-\sigma F^{-1}(1-r)} f(\theta \mid x)\, d\theta \\
&= \int_{-\infty}^{k-\sigma F^{-1}(1-r)} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) d\theta \\
&= \int_{z=\frac{x-k}{\sigma}+F^{-1}(1-r)}^{\infty} f(z)\, dz \text{ for } z = \frac{x-\theta}{\sigma},\ dz = -\frac{1}{\sigma} d\theta \\
&= 1 - F\left(\frac{x-k}{\sigma} + F^{-1}(1-r)\right).
\end{aligned}$$

For the marginal agent, $x = k$, so $\Psi(r; x, x) = r$. The density function of r is then $\psi(r; x, x) = 1$ over $[0, 1]$.

But then $\pi^*(x, x) = \int_0^1 \pi(r, x)\, dr$. By Property P3, there is exactly one such solution ξ. From [2], it must be that $\xi = \underline{\xi} = \bar{\xi}$ and that this is the unique threshold equilibrium. ■

PROOF OF LEMMA 3:

Let Σ be the strategy profile used by all players other than i, and denote by $\tilde{\pi}^i(\xi, \Sigma)$ the payoff gain of reporting for player i, conditional on ξ when other players play Σ. We proceed by induction.

If everyone with $x > 0$ reports, player i's payoff is the highest, and if no one reports, player i's payoff is the lowest. Therefore:

$$\pi^*(\xi,\infty) \leq \tilde{\pi}^i(\xi,\Sigma) \leq \pi^*(\xi,0).$$

From the definition of $\underline{\xi}^1$ and monotonicity of $\pi^*(x,k)$ in x,

$$\xi < \underline{\xi}^1 \Rightarrow \text{for any } \Sigma,\ \tilde{\pi}^i_\sigma(\xi,\Sigma) \leq \pi^*(\xi,0) < \pi^*\left(\underline{\xi}^1,0\right) = 0.$$

In words, not-reporting strictly dominates reporting $(\tilde{\pi}^i_\sigma(\xi,\Sigma) < 0)$ whenever $\xi < \underline{\xi}^1$, irrespective of other players' strategies. Similarly, from the definition of $\bar{\xi}^1$ and monotonicity,

$$\xi > \bar{\xi}^1 \Rightarrow \text{for any } \Sigma,\ \tilde{\pi}^i_\sigma(\xi,\Sigma) \geq \pi^*(\xi,\infty) > \pi^*\left(\bar{\xi}^1,\infty\right) = 0.$$

In words, reporting strictly dominates not reporting $(\tilde{\pi}^i_\sigma(\xi,\Sigma) > 0)$ whenever $\xi > \bar{\xi}^1$, irrespective of other players' strategies. Thus, if $s(\xi)$ survives the first round of deletion of dominated strategies, we must have:

$$s(\xi) = \begin{cases} 0 \text{ [do not report]} & \text{if } \xi < \underline{\xi}^1 \\ 1 \text{ [report]} & \text{if } \xi > \bar{\xi}^1 \end{cases}.$$

The inductive hypothesis is that if $s(\xi)$ survives the n-th round of deletion of dominated strategies, we must have:

$$s(\xi) = \begin{cases} 0 \text{ [do not report]} & \text{if } \xi < \underline{\xi}^n \\ 1 \text{ [report]} & \text{if } \xi > \bar{\xi}^n \end{cases}$$

Let S^n denote the set of strategies that survives this n-rounds of deletion. Our claim is that if player i faces a strategy profile Σ^n consisting of those drawn from S^n, then the set of strategies that survives the next round of deletion of dominated strategies S^{n+1} satisfies:

$$s(\xi) = \begin{cases} 0 \text{ [do not report]} & \text{if } \xi < \underline{\xi}^{n+1} \\ 1 \text{ [report]} & \text{if } \xi > \bar{\xi}^{n+1} \end{cases}.$$

If everyone else is playing a $\underline{\xi}^n$-threshold strategy (reporting for $\xi > \underline{\xi}^n$), player i's payoff is maximized. Therefore:

$$\xi < \underline{\xi}^{n+1} \Rightarrow \text{for any } \Sigma,\ \tilde{\pi}^i_\sigma(\xi,\Sigma^n) \leq \pi^*(\xi,\underline{\xi}^n) < \pi^*\left(\underline{\xi}^{n+1},\underline{\xi}^n\right) = 0,$$

so that not-reporting strictly dominates reporting $(\tilde{\pi}^i_\sigma(\xi,\Sigma^n) < 0)$ whenever $\xi < \underline{\xi}^{n+1}$, irrespective of other players' strategies. Conversely, if everyone else is playing a $\bar{\xi}^n$-threshold strategy (reporting for $\xi > \bar{\xi}^n$), player i's payoff is minimized. Therefore:

$$\xi > \bar{\xi}^{n+1} \Rightarrow \text{for any } \Sigma,\ \tilde{\pi}^i_\sigma (\xi, \Sigma^n) \geq \pi^* (\xi, \underset{\sim}{\xi}^n) > \pi^* \left(\underset{\sim}{\xi}^{n+1}, \underset{\sim}{\xi}^n\right) = 0,$$

so that reporting strictly dominates not-reporting ($\tilde{\pi}^i_\sigma (\xi, \Sigma^n) > 0$) whenever $\xi > \underset{\sim}{\xi}^{n+1}$, irrespective of other players' strategies, from which the claim follows. ■

II. Distributions with Bounded Support

The key result that needs revisiting with bounded support is Lemma 2. We consider the case where ϵ has bounded support and is possibly asymmetric.

Suppose ϵ has bounded support with cumulative distribution function (CDF) F and probability density function (PDF) f. In particular, suppose ϵ has CDF representation:

$$F_\epsilon (\epsilon) = \begin{cases} 0 & \epsilon < -\underline{l} \\ \tilde{F}_\epsilon (\epsilon) & \epsilon \in \left[-\underline{l}, \bar{l}\right] \\ 1 & \epsilon > \bar{l} \end{cases},$$

where $\tilde{F}_\epsilon (\epsilon)$ is weakly increasing and has $\tilde{F}_\epsilon (-\underline{l}) = 0$ and $\tilde{F}_\epsilon (\underline{l}) = 1$, and $\underline{l}, \bar{l} > 0$. The PDF representation is:

$$f_\epsilon (\epsilon) = \begin{cases} 0 & \epsilon < -\underline{l} \\ \tilde{f}_\epsilon (\epsilon) & \epsilon \in \left[-\underline{l}, \bar{l}\right] \\ 1 & \epsilon > \bar{l} \end{cases},$$

for a density function $\tilde{f}_\epsilon$ that is continuous, positive and integrates to 1 over $\left[-\underline{l}, \bar{l}\right]$. Other than requiring ϵ to have zero mean, we place no other restrictions on $\tilde{f}$.

The PDF for $x_i = \theta + \sigma\epsilon_i$ given θ is then:

$$f_x (x \mid \theta) = \begin{cases} 0 & x < \theta - \sigma\underline{l} \\ \tilde{f}_x (x \mid \theta) & x \in \left[\theta - \sigma\underline{l}, \theta + \sigma\bar{l}\right] \\ 0 & x > \theta + \sigma\bar{l} \end{cases},$$

for:

$$\tilde{f}_x (x \mid \theta) = \frac{1}{\sigma} \tilde{f}_\epsilon \left(\frac{x - \theta}{\sigma}\right).$$

The CDF for x_i is:

$$\begin{aligned} F_x(x \mid \theta) &= \begin{cases} 0 & x < \theta - \sigma\underline{l} \\ \tilde{F}_x(x \mid \theta) & x \in \left[\theta - \sigma\underline{l}, \theta + \sigma\bar{l}\right] \\ 1 & x > \theta + \sigma\bar{l} \end{cases} \\ &= \begin{cases} 0 & x < \theta - \sigma\underline{l} \\ \tilde{F}_\epsilon\left(\frac{x-\theta}{\sigma}\right) & x \in \left[\theta - \sigma\underline{l}, \theta + \sigma\bar{l}\right] \\ 1 & x > \theta + \sigma\bar{l} \end{cases}, \end{aligned}$$

for:

$$\tilde{F}_x(x \mid \theta) = \int_{\theta - \sigma\underline{l}}^{x} \frac{1}{\sigma}\tilde{f}_\epsilon\left(\frac{s-\theta}{\sigma}\right) ds = \tilde{F}_\epsilon\left(\frac{x-\theta}{\sigma}\right).$$

The posterior in θ conditional on x equals:

$$\begin{aligned} f_\theta(\theta \mid x) &= \frac{f_x(x \mid \theta) f_\theta(\theta)}{\int_x f_x(x \mid \theta) f_\theta(\theta) d\theta} \\ &= f_x(x \mid \theta) \\ &= \begin{cases} 0 & \theta < x - \sigma\bar{l} \\ \tilde{f}_x(x \mid \theta) & \theta \in \left[x - \sigma\bar{l}, x + \sigma\underline{l}\right] \\ 0 & \theta > x + \sigma\underline{l} \end{cases} \\ &= \begin{cases} 0 & \theta < x - \sigma\bar{l} \\ \frac{1}{\sigma}\tilde{f}_\epsilon\left(\frac{x-\theta}{\sigma}\right) & \theta \in \left[x - \sigma\bar{l}, x + \sigma\underline{l}\right] \\ 0 & \theta > x + \sigma\underline{l} \end{cases}, \end{aligned}$$

under the improper prior assumption. Thus:

$$\begin{aligned} F_\theta(\theta \mid x) &= \begin{cases} 0 & \theta < x - \sigma\bar{l} \\ \int_{x-\sigma\bar{l}}^{\theta} \tilde{f}_x(x \mid \theta) d\theta & \theta \in \left[x - \sigma\bar{l}, x + \sigma\underline{l}\right] \\ 1 & \theta > x + \sigma\underline{l} \end{cases} \\ &= \begin{cases} 0 & \theta < x - \sigma\bar{l} \\ 1 - \tilde{F}_\epsilon\left(\frac{x-\theta}{\sigma}\right) & \theta \in \left[x - \sigma\bar{l}, x + \sigma\underline{l}\right] \\ 1 & \theta > x + \sigma\underline{l} \end{cases}. \end{aligned}$$

The mapping between r and θ is then as follows. Let k be the cutoff that agents play. Given k, $r(\theta;k) = \int_k^\infty f_x(x \mid \theta)\, dx = 1 - F_x(k \mid \theta)$. Then:

$$r(\theta;k) \;=\; \begin{cases} 0 & \theta < k - \sigma\bar{l} \\ 1 - \tilde{F}_\epsilon\left(\frac{k-\theta}{\sigma}\right) & \theta \in \left[k - \sigma\underline{l}, k + \sigma\bar{l}\right] \\ 1 & \theta > k + \sigma\underline{l} \end{cases} .$$

It follows that:

$$r(\theta;k) = 1 - \tilde{F}_\epsilon\left(\frac{k-\theta}{\sigma}\right) \quad \Leftrightarrow \quad \theta(r;k) = k - \sigma\tilde{F}_\epsilon^{-1}(1-r)$$

is a bijection for any $r \in (0,1)$ to $\theta \in \left(k - \sigma\underline{l}, k + \sigma\bar{l}\right)$, where the open intervals are important. The 1-1 map fails if $r = 1$, since then $\theta \geq k + \sigma\underline{l}$ and if $r = 0$, since then $\theta \leq k - \sigma\bar{l}$.

We are now prepared to revisit Lemma 2. The statement of the Lemma is unchanged with the exception of Part 1, which should now state that "π^* weakly decreases in k (strictly decreases for $x > 0$ and $k > 0$ for $k \in \left(x - \sigma(\underline{l} + \bar{l}), x + \sigma(\underline{l} + \bar{l})\right)$."

PROOF OF LEMMA 2 WITH BOUNDED SUPPORT:

The payoff gain $\pi(r,x)$ continues to satisfy Properties P1-P5 in Lemma 1.

[1]: $\pi^*(x,k)$ increases in x: As before, implement a change in variables $z = -\theta$. Then:

$$\begin{aligned} \pi^*(x,k) &= \int_{\theta = x - \sigma\bar{l},}^{\theta = x + \sigma\underline{l}} \frac{1}{\sigma} f_\epsilon\left(\frac{x-\theta}{\sigma}\right) \pi\left(1 - F_\epsilon\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta \\ &= \int_{z = -x - \sigma\underline{l},}^{z = -x + \sigma\bar{l}} \frac{1}{\sigma} f_\epsilon\left(\frac{x+z}{\sigma}\right) \pi\left(1 - F_\epsilon\left(\frac{k+z}{\sigma}\right), x\right) dz \end{aligned}$$

It is trivial to show that, for $x_1 < x_2$, $F_\epsilon(x_1 + \epsilon) \leq F_\epsilon(x_2 + \epsilon)\ \forall \epsilon$. Therefore z under x_1 stochastically dominates z under x_2, and the original proof flows.

$\pi^*(x,k)$ weakly decreases in k follows from Property P1 and that r weakly decreases in k. To obtain strictly decreasing over $x > 0$ and $k > 0$, we also need

$k \in \left(x - \sigma\left(\underline{l} + \bar{l}\right), x + \sigma\left(\underline{l} + \bar{l}\right)\right)$. Observe:

$$\begin{aligned}
\pi^*(x,k) &= \int_{\theta=x-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi\left(1 - F_\epsilon\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta \\
&= \begin{cases}
\int_{\theta=x-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi(0,x)\, d\theta = \pi(0,x) & x < k - \sigma\left(\underline{l} + \bar{l}\right) \\
\int_{\theta=x-\sigma\bar{l}}^{\theta=k-\sigma\bar{l}} f_\theta(\theta \mid x)\, \pi(0,x)\, d\theta & \\
\quad + \int_{\theta=k-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi\left(1 - F_\epsilon\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta & x \in \left(k - \sigma\left(\underline{l} + \bar{l}\right), k\right) \\
\int_{\theta=x-\sigma\bar{l}}^{\theta=k+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi\left(1 - F_\epsilon\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta & \\
\quad + \int_{\theta=k+\sigma\underline{l},}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi(1,x)\, d\theta & x \in \left(k, k + \sigma\left(\underline{l} + \bar{l}\right)\right) \\
\int_{\theta=x-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi(1,x)\, d\theta = \pi(1,x) & x > k + \sigma\left(\underline{l} + \bar{l}\right)
\end{cases}
\end{aligned}$$

Let $k_1 < k_2$ be given. Re-write as:

$$\begin{aligned}
\pi^*(x,k) &= \int_{\theta=x-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi\left(1 - F_\epsilon\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta \\
&= \begin{cases}
F_1(x,k) \equiv \int_{\theta=x-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi(0,x)\, d\theta = \pi(0,x) & k > x + \sigma\left(\underline{l} + \bar{l}\right) \\
F_2(x,k) \equiv \int_{\theta=x-\sigma\bar{l}}^{\theta=k-\sigma\bar{l}} f_\theta(\theta \mid x)\, \pi(0,x)\, d\theta & \\
\qquad + \int_{\theta=k-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi\left(1 - F_\epsilon\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta & k \in \left(x, x + \sigma\left(\underline{l} + \bar{l}\right)\right) \\
F_3(x,k) \equiv \int_{\theta=x-\sigma\bar{l}}^{\theta=k+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi\left(1 - F_\epsilon\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta & \\
\qquad + \int_{\theta=k+\sigma\underline{l},}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi(1,x)\, d\theta & k \in \left(x - \sigma\left(\underline{l} + \bar{l}\right), x\right) \\
F_4(x,k) \equiv \int_{\theta=x-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\, \pi(1,x)\, d\theta = \pi(1,x) & k < x - \sigma\left(\underline{l} + \bar{l}\right)
\end{cases}
\end{aligned}$$

Notice that $F_1\left(x, k^1\right) < F_2\left(x, k^2\right) < F_3\left(x, k^3\right) < F_4\left(x, k^4\right)$ for every x for any $k^1 > k^2 > k^3 > k^4$ satisfying the conditions of k for each function. So if $k_2 > k_1$ in any way that crosses these regions, $\pi^*(x, k_2) < \pi^*(x, k_1)$. If $k_2 > k_1$ but each both lie within a single region, evidently $\pi^*(x, k_2) = \pi^*(x, k_1)$ in regions 1 and

4. In Region 2:

$$
\begin{aligned}
F_2(x,k_2)-F_2(x,k_1) &= \int_{\theta=x-\sigma\bar{l}}^{\theta=k_2-\sigma\bar{l}} f_\theta(\theta \mid x)\,\pi(0,x)\;d\theta \\
&\quad+\int_{\theta=k_2-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_2-\theta}{\sigma}\right),x\right)\;d\theta \\
&\quad-\int_{\theta=x-\sigma\bar{l}}^{\theta=k_1-\sigma\bar{l}} f_\theta(\theta \mid x)\,\pi(0,x)\;d\theta \\
&\quad-\int_{\theta=k_1-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_1-\theta}{\sigma}\right),x\right)\;d\theta \\
&= \int_{\theta=k_1-\sigma\bar{l}}^{\theta=k_2-\sigma\bar{l}} f_\theta(\theta \mid x)\,\pi(0,x)\;d\theta \\
&\quad+\int_{\theta=k_2-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_2-\theta}{\sigma}\right),x\right)\;d\theta \\
&\quad-\int_{\theta=k_1-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_1-\theta}{\sigma}\right),x\right)\;d\theta \\
&\leq \int_{\theta=k_1-\sigma\bar{l}}^{\theta=k_2-\sigma\bar{l}} f_\theta(\theta \mid x)\,\pi(0,x)\;d\theta \\
&\quad+\int_{\theta=k_2-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_1-\theta}{\sigma}\right),x\right)\;d\theta \\
&\quad-\int_{\theta=k_1-\sigma\bar{l}}^{\theta=x+\sigma\underline{l}} f_\theta(\theta \mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_1-\theta}{\sigma}\right),x\right)\;d\theta \\
&= \int_{\theta=k_1-\sigma\bar{l}}^{\theta=k_2-\sigma\bar{l}} f_\theta(\theta \mid x)\,\pi(0,x)\;d\theta \\
&\quad-\int_{\theta=k_1-\sigma\bar{l}}^{\theta=k_2+\sigma\underline{l}} f_\theta(\theta \mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_1-\theta}{\sigma}\right),x\right)\;d\theta \\
&< 0,
\end{aligned}
$$

since π is an increasing function of r and $1-F_\epsilon\left(\frac{k_1-\theta}{\sigma}\right)>0$ for $\theta>k_1-\sigma\bar{l}$. In

Region 3:

$$\begin{aligned}
F_3(x,k_2)-F_3(x,k_1) &= \int_{\theta=x-\sigma\bar{l}}^{\theta=k_2+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_2-\theta}{\sigma}\right),x\right)d\theta \\
&\quad+\int_{\theta=k_2+\sigma\underline{l},}^{\theta=x+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi(1,x)\,d\theta \\
&\quad-\int_{\theta=x-\sigma\bar{l}}^{\theta=k_1+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_1-\theta}{\sigma}\right),x\right)d\theta \\
&\quad-\int_{\theta=k_1+\sigma\underline{l},}^{\theta=x+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi(1,x)\,d\theta \\
&= -\int_{\theta=k_1+\sigma\underline{l},}^{\theta=k_2+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi(1,x)\,d\theta \\
&\quad+\int_{\theta=x-\sigma\bar{l}}^{\theta=k_2+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_2-\theta}{\sigma}\right),x\right)d\theta \\
&\quad-\int_{\theta=x-\sigma\bar{l}}^{\theta=k_1+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_1-\theta}{\sigma}\right),x\right)d\theta \\
&\leq -\int_{\theta=k_1+\sigma\underline{l},}^{\theta=k_2+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi(1,x)\,d\theta \\
&\quad+\int_{\theta=x-\sigma\bar{l}}^{\theta=k_2+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_2-\theta}{\sigma}\right),x\right)d\theta \\
&\quad-\int_{\theta=x-\sigma\bar{l}}^{\theta=k_1+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_2-\theta}{\sigma}\right),x\right)d\theta \\
&= \int_{\theta=k_1+\sigma\underline{l}}^{\theta=k_2+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi\left(1-F_\epsilon\left(\frac{k_2-\theta}{\sigma}\right),x\right)d\theta \\
&\quad-\int_{\theta=k_1+\sigma\underline{l},}^{\theta=k_2+\sigma\underline{l}} f_\theta(\theta\mid x)\,\pi(1,x)\,d\theta \\
&< 0,
\end{aligned}$$

since π is an increasing function of r and $1-F_\epsilon\left(\frac{k_2-\theta}{\sigma}\right)<1$ for $\theta<k_2+\sigma\underline{l}$.

Continuity in x and k should follow from Property P5, which is unchanged.

[2] This portion of the proof follows very similarly from before, with a few additional arguments to account for the fact that $\pi^*(x,k)$ strictly decreases in k only locally when k is close to x.

We show that $\{\underline{\xi}^n\}$ and $\left\{\bar{\xi}^n\right\}$ are well-defined increasing and decreasing sequences, respectively, through induction. From Property P4, we know that not reporting is dominant for $x<\underline{x}$, so $\pi^*(x,0)<0$ for all $x<\underline{x}$. But we also know that $\pi^*(x,0)>0$ for all $x>\bar{x}$. Define $\underline{\xi}^0\equiv 0$ and $\bar{\xi}^0\equiv\infty$. By continuity in x, there exists at least one solution x with $\pi^*\left(x,\underline{\xi}^0\right)=0$, where $x\in[\underline{x},\bar{x}]$. Call $\underline{\xi}^1$ the smallest such solution. Define $\bar{\xi}^1$ analogously to be the largest such solution with $\pi^*\left(x,\bar{\xi}^0\right)=0$. Note that $\underline{\xi}^0<\underline{\xi}^1<\bar{\xi}^1<\bar{\xi}^0$; if the inside inequality did not hold and $\underline{\xi}^1\geq\bar{\xi}^1$, then $0=\pi^*\left(\underline{\xi}^1,\underline{\xi}^0\right)>\pi^*\left(\underline{\xi}^1,\underline{\xi}^1\right)\geq\pi^*\left(\bar{\xi}^1,\underline{\xi}^1\right)\geq\pi^*\left(\bar{\xi}^1,\bar{\xi}^0\right)=0$, a contradiction. The inequalities are because $\pi^*\left(\underline{\xi}^1,\underline{\xi}^1\right)$ strictly decreases in k locally, the contradiction assumption with π^* strictly increasing in x, and that π^* is globally weakly decreasing with $\underline{\xi}^1<\bar{\xi}^0=\infty$, respectively.

Our starting point for the induction is as follows. Given $\underline{\xi}^1$ and $\bar{\xi}^1$ with $\underline{\xi}^0<\underline{\xi}^1<\bar{\xi}^1<\bar{\xi}^0$, $\pi^*\left(\underline{\xi}^1,\underline{\xi}^0\right)=0$, and $\pi^*\left(\bar{\xi}^1,\bar{\xi}^0\right)=0$, we claim there exists a smallest solution $\underline{\xi}^2$ of $\pi\left(\underline{\xi}^2,\underline{\xi}^1\right)=0$ and a largest solution $\bar{\xi}^2$ of $\pi\left(\bar{\xi}^2,\bar{\xi}^1\right)=0$, and that $\underline{\xi}^1<\underline{\xi}^2<\bar{\xi}^2<\bar{\xi}^1$. We know $\pi^*\left(\underline{\xi}^1,\underline{\xi}^0\right)=0>\pi^*\left(\underline{\xi}^1,\underline{\xi}^1\right)$, and $\pi^*\left(\bar{\xi}^1,\bar{\xi}^0\right)=0<\pi^*\left(\bar{\xi}^1,\bar{\xi}^1\right)\leq\pi^*\left(\bar{\xi}^1,\underline{\xi}^1\right)$, where the first inequality is strict because $\pi^*\left(\bar{\xi}^1,\bar{\xi}^1\right)$ is strictly decreasing in k. By continuity, there exists a smallest solution $\underline{\xi}^2\in\left(\underline{\xi}^1,\bar{\xi}^1\right)$ with $\pi^*\left(\underline{\xi}^2,\underline{\xi}^1\right)=0$. Analogously, we know $\pi^*\left(\underline{\xi}^1,\underline{\xi}^0\right)=0>\pi^*\left(\underline{\xi}^1,\underline{\xi}^1\right)\geq\pi^*\left(\underline{\xi}^1,\bar{\xi}^1\right)$, and $\pi^*\left(\bar{\xi}^1,\bar{\xi}^0\right)=0<\pi^*\left(\bar{\xi}^1,\bar{\xi}^1\right)$; by continuity there exists a largest solution $\bar{\xi}^2\in\left(\underline{\xi}^1,\bar{\xi}^1\right)$ with $\pi^*\left(\bar{\xi}^2,\bar{\xi}^1\right)=0$. Note that $\underline{\xi}^1<\underline{\xi}^2<\bar{\xi}^2<\bar{\xi}^1$; if the inside inequality did not hold and $\underline{\xi}^2\geq\bar{\xi}^2$, then $0=\pi^*\left(\underline{\xi}^2,\underline{\xi}^1\right)>\pi^*\left(\underline{\xi}^2,\underline{\xi}^2\right)\geq\pi^*\left(\bar{\xi}^2,\underline{\xi}^2\right)\geq\pi^*\left(\bar{\xi}^2,\bar{\xi}^1\right)=0$, a contradiction. The inequalities are because $\pi^*\left(\underline{\xi}^2,\underline{\xi}^2\right)$ strictly decreases in k locally, the contradiction assumption with π^*strictly increasing in x, and that π^* is globally weakly decreasing with $\underline{\xi}^2\in\left(\underline{\xi}^1,\bar{\xi}^1\right)$, respectively.

The inductive hypothesis is that, given $\underline{\xi}^n$ and $\bar{\xi}^n$ with $\underline{\xi}^{n-1}<\underline{\xi}^n<\bar{\xi}^n<\bar{\xi}^{n-1}$, $\pi^*\left(\underline{\xi}^n,\underline{\xi}^{n-1}\right)=0$, and $\pi^*\left(\bar{\xi}^n,\bar{\xi}^{n-1}\right)=0$, there exists a smallest solution $\underline{\xi}^{n+1}$ of $\pi\left(\underline{\xi}^{n+1},\underline{\xi}^n\right)=0$ and a largest solution $\bar{\xi}^{n+1}$ of $\pi\left(\bar{\xi}^{n+1},\bar{\xi}^n\right)=0$, and that $\underline{\xi}^n<\underline{\xi}^{n+1}<\bar{\xi}^{n+1}<\bar{\xi}^n$. We know $\pi^*\left(\underline{\xi}^n,\underline{\xi}^{n-1}\right)=0>\pi^*\left(\underline{\xi}^n,\underline{\xi}^n\right)$, and $\pi^*\left(\bar{\xi}^n,\bar{\xi}^{n-1}\right)=0<\pi^*\left(\bar{\xi}^n,\bar{\xi}^n\right)\leq\pi^*\left(\bar{\xi}^n,\underline{\xi}^n\right)$. By continuity, there exists a smallest solution $\underline{\xi}^{n+1}\in\left(\underline{\xi}^n,\bar{\xi}^n\right)$ with $\pi^*\left(\underline{\xi}^{n+1},\underline{\xi}^n\right)=0$. Similarly, we know $\pi^*\left(\underline{\xi}^n,\underline{\xi}^{n-1}\right)=0>\pi^*\left(\underline{\xi}^n,\underline{\xi}^n\right)\geq\pi^*\left(\underline{\xi}^n,\bar{\xi}^n\right)$, and $\pi^*\left(\bar{\xi}^n,\bar{\xi}^{n-1}\right)=0<\pi^*\left(\bar{\xi}^n,\bar{\xi}^n\right)$; by continuity there exists a largest solution $\bar{\xi}^{n+1}\in\left(\underline{\xi}^n,\bar{\xi}^n\right)$ with $\pi^*\left(\bar{\xi}^{n+1},\bar{\xi}^n\right)=0$. Note that $\underline{\xi}^n<\underline{\xi}^{n+1}<\bar{\xi}^{n+1}<\bar{\xi}^n$; if the inside inequality did not hold and $\underline{\xi}^{n+1}\geq\bar{\xi}^{n+1}$, then $0=\pi^*\left(\underline{\xi}^{n+1},\underline{\xi}^n\right)>\pi^*\left(\underline{\xi}^{n+1},\underline{\xi}^{n+1}\right)\geq\pi^*\left(\bar{\xi}^{n+1},\underline{\xi}^{n+1}\right)\geq\pi^*\left(\bar{\xi}^{n+1},\bar{\xi}^n\right)=0$, a contradiction. The inequalities are because $\pi^*\left(\underline{\xi}^{n+1},\underline{\xi}^{n+1}\right)$ strictly decreases in k locally, the contradiction assumption with π^*strictly increasing in x, and that π^* is globally weakly decreasing with $\underline{\xi}^{n+1}\in\left(\underline{\xi}^n,\bar{\xi}^n\right)$, respectively.

Note that $\{\underline{\xi}^n\}$ is bounded from above by construction. Because it is also an increasing sequence, there exists a $\underline{\xi}$ with $\lim_{n\to\infty}\underline{\xi}^n=\underline{\xi}$. Note that $\lim_{n\to\infty}\pi^*\left(\underline{\xi}^{n+1},\underline{\xi}^n\right)=0$ so by construction and continuity of π^*, we must have $\pi^*\left(\underline{\xi},\underline{\xi}\right)=0$ and that $\underline{\xi}$ is the smallest such solution to $\pi^*\left(\xi,\xi\right)=0$. Analogously, there exists a $\bar{\xi}$ with $\lim_{n\to\infty}\underline{\xi}^n=\underline{\xi}$ and $\pi^*\left(\bar{\xi},\bar{\xi}\right)=0$ and that $\bar{\xi}$ is the smallest such solution to $\pi^*\left(\xi,\xi\right)=0$. This shows, among other things, that there exists at least one threshold equilibrium $\underline{\xi}$. One can see that any such solution $\underline{\xi}$ is an equilibrium because $x_1 < \xi < x_2$ implies $\pi^*(x_1, \xi) < \pi^*(\xi, \xi) = 0 < \pi^*(x_2, \xi)$.

[3] Given the agent's signal x, what is her assessment of the cumulative distribution function of r, $\Psi(\tilde{r}; x, k)$? For $\tilde{r} \in (0,1)$, the argument follows from before, which we restate here. The probability that $r < \tilde{r}$ equals the probability that $\theta < k - \sigma F_\epsilon^{-1}(1-\tilde{r})$. In words, the probability $\Psi(\tilde{r}; x, k) \equiv \Pr(r < \tilde{r} \mid x)$ that the true proportion of players reporting is less than $\tilde{r}$ equals the probability that the true θ satisfies $r(\theta; k) = 1 - F_\epsilon\left(\frac{k-\theta}{\sigma}\right) < \tilde{r}$, or equivalently that θ is such that fewer than $\tilde{r}$ players observe a signal greater than k; in turn, this equals the probability that the true θ is less than $k - \sigma F_\epsilon^{-1}(1-\tilde{r})$, integrated against the conditional density $f_\theta(\theta \mid x)$.

Importantly, conditional on x, we must have $\theta \in \left[x - \sigma\bar{l}, x + \sigma\underline{l}\right]$.

Given x, what is the agent's probability assessment that $r = 0$? This must equal the posterior probability that $\theta < k - \sigma\bar{l}$ given x. If $k - \sigma\bar{l} \in \left[x - \sigma\bar{l}, x + \sigma\underline{l}\right]$, then $x \in \left[k - \sigma\left(\bar{l} + \underline{l}\right), k\right]$ then:

$$\begin{aligned}
\Psi(r; x, k) &= \int_{x-\sigma\bar{l}}^{k-\sigma\bar{l}} f_\theta(\theta \mid x)\, d\theta \\
&= \int_{x-\sigma\bar{l}}^{k-\sigma\bar{l}} \frac{1}{\sigma} f_\epsilon\left(\frac{x-\theta}{\sigma}\right) d\theta \\
&= \int_{z=\frac{x-k}{\sigma}+\bar{l}}^{z=\bar{l}} f_z(z)\, dz \text{ for } z = \frac{x-\theta}{\sigma},\ dz = -\frac{1}{\sigma} d\theta \\
&= 1 - F_\epsilon\left(\frac{x-k}{\sigma} + \bar{l}\right).
\end{aligned}$$

If $k - \sigma\bar{l} < x - \sigma\bar{l}$ then $x > k$ and the probability is zero by the definition of $f_\theta(\theta)$. If $k - \sigma\bar{l} > x + \sigma\underline{l}$ or equivalently if $\frac{k-x}{\sigma} > \bar{l} + \underline{l}$ then the probability is 1. So:

$$\Psi(0; x, k) = \begin{cases} 0 & x \geq k \\ 1 - F_\epsilon\left(\frac{x-k}{\sigma} + \bar{l}\right) & x \in \left(k - \sigma\left(\bar{l} + \underline{l}\right), k\right) \\ 1 & x \leq k - \sigma\left(\bar{l} + \underline{l}\right) \end{cases}.$$

Given x, what is the agent's probability assessment that $r \leq 1$? Trivially, this must be 1, since this equals the probability that $r < 1$, which is the probability that $\theta < k + \sigma\underline{l}$, plus the probability that $r = 1$, which is the probability that $\theta \geq k + \sigma\underline{l}$. Thus, $\Psi(1) = 1$.

Given $\tilde{r} \in (0,1)$, what is the agent's probability assessment that $r < \tilde{r}$? Given k, we know $\theta(r; k) = k - \sigma\tilde{F}_\epsilon^{-1}(1-r) \in \left(k - \sigma\underline{l}, k + \sigma\bar{l}\right)$. We also must have $\theta \in$

$\left[x-\sigma\bar{l},x+\sigma\underline{l}\right]$ in the posterior distribution of θ. Some useful facts to reference:

$$\begin{aligned}
k-\sigma\tilde{F}_{\epsilon}^{-1}\left(1-r\right)>x-\sigma\bar{l} \quad &\Leftrightarrow \quad r>1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}+\bar{l}\right)\\
x\geq k+\sigma\left(\bar{l}+\underline{l}\right) \quad &\Rightarrow \quad 1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}+\bar{l}\right)=1\\
x< k+\sigma\left(\bar{l}+\underline{l}\right) \quad &\Rightarrow \quad 1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}+\bar{l}\right)<1\\
x<k \quad &\Rightarrow \quad 1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}+\bar{l}\right)=0,\\
k-\sigma\tilde{F}_{\epsilon}^{-1}\left(1-r\right)<x+\sigma\underline{l} \quad &\Leftrightarrow \quad r<1-\tilde{F}_{\epsilon}^{-1}\left(\frac{k-x}{\sigma}-\underline{l}\right)\\
x\leq k-\sigma\left(\bar{l}+\underline{l}\right) \quad &\Rightarrow \quad 1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}-\underline{l}\right)=0\\
x> k-\sigma\left(\bar{l}+\underline{l}\right) \quad &\Rightarrow \quad 1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}-\underline{l}\right)>0\\
x>k \quad &\Rightarrow \quad 1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}-\underline{l}\right)=1.
\end{aligned}$$

Suppose first $x\in\left(k-\sigma\left(\bar{l}+\underline{l}\right),k+\sigma\left(\bar{l}+\underline{l}\right)\right)$. Then $1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}-\underline{l}\right)>0$ and $1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}+\bar{l}\right)<1$. Several cases can occur:

$$\begin{aligned}
k-\sigma\tilde{F}_{\epsilon}^{-1}\left(1-r\right)>x+\sigma\underline{l} \quad &\Rightarrow \quad r\in\left(1-\tilde{F}_{\epsilon}^{-1}\left(\frac{k-x}{\sigma}-\underline{l}\right),1\right)\\
k-\sigma\tilde{F}_{\epsilon}^{-1}\left(1-r\right)\in\left(x-\sigma\bar{l},x+\sigma\underline{l}\right) \quad &\Rightarrow \quad r\in\left(1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}+\bar{l}\right),1-\tilde{F}_{\epsilon}^{-1}\left(\frac{k-x}{\sigma}-\underline{l}\right)\right)\\
k-\sigma\tilde{F}_{\epsilon}^{-1}\left(1-r\right)<x-\sigma\bar{l} \quad &\Rightarrow \quad r\in\left(0,1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}+\bar{l}\right)\right)
\end{aligned}$$

If $k-\sigma\tilde{F}_{\epsilon}^{-1}\left(1-r\right)>x-\sigma\bar{l}$, then $r\in\left(1-\tilde{F}_{\epsilon}\left(\frac{k-x}{\sigma}+\bar{l}\right),1\right)$, and furthermore:

$$\begin{aligned}
\Psi\left(r;x,k\right) &= \int_{x-\sigma\bar{l}}^{k-\sigma\tilde{F}_{\epsilon}^{-1}(1-r)} f_{\theta}\left(\theta\mid x\right)\,d\theta\\
&= \int_{x-\sigma\bar{l}}^{k-\sigma\tilde{F}_{\epsilon}^{-1}(1-r)} \frac{1}{\sigma}\tilde{f}_{\epsilon}\left(\frac{x-\theta}{\sigma}\right)\,d\theta\\
&= 1-\tilde{F}_{\epsilon}\left(\frac{x-k}{\sigma}+\tilde{F}_{\epsilon}^{-1}\left(1-r\right)\right).
\end{aligned}$$

If $k-\sigma\tilde{F}_\epsilon^{-1}(1-r)<x-\sigma\bar{l}$, then $r\in\left(0,1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}+\bar{l}\right)\right)$ and furthermore by the definition of f_θ:

$$\Psi(r;x,k)=-\int_{k-\sigma F_\epsilon^{-1}(1-r)}^{x-\sigma\bar{l}} f_\theta(\theta\mid x)\,d\theta=0.$$

Suppose next $x\geq k+\sigma\left(\bar{l}+\underline{l}\right)$. Then $1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}+\bar{l}\right)=1$ and $1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}-\underline{l}\right)>0$. From before, $1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}+\bar{l}\right)=1$ implies $k-\sigma\tilde{F}_\epsilon^{-1}(1-r)<x-\sigma\bar{l}$ since $r<1$. But then, following a similar argument,

$$\Psi(r;x,k)=-\int_{k-\sigma F_\epsilon^{-1}(1-r)}^{x-\sigma\bar{l}} f_\theta(\theta\mid x)\,d\theta=0.$$

Suppose finally $x\leq k-\sigma\left(\bar{l}+\underline{l}\right)$. Then $1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}-\underline{l}\right)=0$ and $1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}+\bar{l}\right)<1$. From before, $1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}-\underline{l}\right)=0$ implies $k-\sigma\tilde{F}_\epsilon^{-1}(1-r)>x+\sigma\underline{l}$ since $r>0$. But then:

$$\Psi(r;x,k)=\int_{x-\sigma\bar{l}}^{x+\sigma\underline{l}} f_\theta(\theta\mid x)\,d\theta=1.$$

To summarize:

$$\Psi\left(r;x\geq k+\sigma\left(\bar{l}+\underline{l}\right)\right) = \begin{cases} 0 & r=0\\ 0 & r\in(0,1)\\ 1 & r=1,\end{cases}$$

$$\Psi\left(r;x\in\left(k,k+\sigma\left(\bar{l}+\underline{l}\right)\right)\right) = \begin{cases} 0 & r=0\\ 0 & r\in\left(0,1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}+\bar{l}\right)\right)\\ 1-\tilde{F}_\epsilon\left(\frac{x-k}{\sigma}+\tilde{F}_\epsilon^{-1}(1-r)\right) & r\in\left(1-\tilde{F}_\epsilon\left(\frac{k-x}{\sigma}+\bar{l}\right),1\right)\\ 1 & r=1,\end{cases}$$

$$\Psi\left(r;x\in\left(k-\sigma\left(\bar{l}+\underline{l}\right),k\right)\right) = \begin{cases} 1-F_\epsilon\left(\frac{x-k}{\sigma}+\bar{l}\right) & r=0\\ 1-\tilde{F}_\epsilon\left(\frac{x-k}{\sigma}+\tilde{F}_\epsilon^{-1}(1-r)\right) & r\in(0,1)\\ 1 & r=1,\end{cases}$$

$$\Psi\left(r;x\leq k-\sigma\left(\bar{l}+\underline{l}\right)\right) = \begin{cases} 1 & r=0\\ 1 & r\in(0,1)\\ 1 & r=1,\end{cases}$$

$$\Psi(r;x=k) = \begin{cases} 0 & r=0\\ r & r\in(0,1)\,.\\ 1 & r=1.\end{cases}$$

Therefore, the marginal agent has a uniform belief over r.

Given this belief, we can solve for the threshold x^*. Recall the expected payoff gain equals:

$$\pi^*(x,k) = \int_{\theta=x-\sigma\bar{l},}^{\theta=x+\sigma\underline{l}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \pi\left(1-F\left(\frac{k-\theta}{\sigma}\right), x\right) d\theta.$$

For the marginal agent with $x = k$, this payoff equals:

$$\pi^*(x,x) = \int_{x-\sigma\bar{l},}^{x+\sigma\underline{l}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \pi\left(1-F\left(\frac{x-\theta}{\sigma}\right), x\right) d\theta.$$

Given the marginal agent's posterior belief over θ translates into a uniform belief over r, we can write:

$$\pi^*(x,x) \quad = \quad \int_0^1 \pi(r,x)\, dr.$$

■

III. Finite Number of Agents

We show that the equilibrium solution and overall conclusions are virtually identical to that of a finite N-agent version of the game.

The manager employs N agents. This is common knowledge. Each agent who reports increments the probability of sanction by $\frac{1}{N}\gamma$ for $\gamma \leq 1$. If n agents report, probability of sanction is $\frac{n}{N}\gamma$.

If n other agents report, the payoff gain for any individual agent from reporting equals:

$$E\left(Report - NoReport \mid x_i, n \text{ other agents report}\right)$$
$$= \begin{cases} x\left(\omega - \left(1 - \frac{n+1}{N}\gamma\right)\beta\right) - c & \text{if } x \geq 0 \\ x\omega - c & \text{if } x < 0. \end{cases}$$

This payoff function implies that there are dominance regions in x_i. Agent i reports as a dominant strategy if $x_i > \overline{x} = \frac{c}{\omega - \left(1-\frac{1}{N}\gamma\right)\beta}$. Similarly, agent i does not report as a dominant strategy if $x_i < \underline{x} = \frac{c}{\omega-(1-\gamma)\beta}$.

The payoff function is also monotonic in x and n, since $\beta < \omega$ and $\gamma > 0$.

Conditional on experience x_i, agent i's posterior beliefs about other agents' experiences are determined by Bayes' Rule. Specifically, her posterior belief about any agent j's experience $x_j = \theta + \varepsilon_j$ is normal with:

1) $E[x_j \mid x_i] = E[\theta \mid x_i] = x_i$.

2) $Var\left[x_j \mid x_i\right] = Var\left[\theta \mid x_i\right] + \sigma^2 = 2\sigma^2 \Rightarrow SD\left[x_j \mid x_i\right] = \sqrt{2}\sigma$.

This derivation uses the fact that x_j and x_i are i.i.d. jointly normal. Given joint normality, $1/Var[\theta \mid x_i] = 1/Var_0[\theta] + 1/\sigma^2$; given the agent's improper prior over θ, $Var[\theta \mid x_i] = \sigma^2$.

Conjecture a symmetric strategy equilibrium where agents report if and only if $x > k$. In agent i's beliefs,

1) The probability agent j does not report equals: $Pr\left(x_j < k \mid x_i\right) = \Phi\left(\frac{k-x_i}{\sqrt{2}\sigma}\right)$.

2) The probability agent j reports equals: $Pr\left(x_j > k \mid x_i\right) = 1 - \Phi\left(\frac{k-x_i}{\sqrt{2}\sigma}\right)$. Let $p\left(x,k\right) \equiv 1 - \Phi\left(\frac{k-x}{\sqrt{2}\sigma}\right)$.

3) The probability of m other agents reporting equals:

$$Pr\left(n = m \mid x_i; k\right) = \left(\begin{array}{c} N-1 \\ m \end{array}\right) p(x,k)^m \left(1 - p(x,k)\right)^{N-1-m}.$$

The expected payoff gain for any agent conditional on experience x equals:

$$\pi^*\left(x,k\right) = \sum_{m=0}^{N-1} \left(\begin{array}{c} N-1 \\ m \end{array}\right) p\left(x,k\right)^m \left(1 - p\left(x,k\right)\right)^{N-1-m} \left[x\left(\omega - \left(1 - \frac{m+1}{N}\gamma\right)\beta\right) - c\right]$$

The indifference condition and cutoff threshold x^* satisfies $\pi^*\left(x^*, x^*\right) = 0$. Note that $p\left(x^*, x^*\right) = 1/2$. An agent who draws $x = x^*$ assigns 1/2 probability to each other agent when agents play threshold strategies around x^*. Thus, the indifference condition satisfies:

$$\begin{aligned}
0 =& \sum_{m=0}^{N-1} \left(\begin{array}{c} N-1 \\ m \end{array}\right) \left(\frac{1}{2}\right)^{N-1} \left[x\left(\omega - \left(1 - \frac{m+1}{N}\gamma\right)\beta\right) - c\right] \\
=& \left(\frac{1}{2}\right)^{N-1} x\left(2^{N-1}\left(\omega - \beta\right) + \frac{1}{N}\sum_{m=0}^{N-1} \left(\begin{array}{c} N-1 \\ m \end{array}\right)\left(m+1\right)\gamma\beta\right) - \left(\frac{1}{2}\right)^{N-1} 2^{N-1}c \\
=& x\left(\left(\omega - \beta\right) + \left(\frac{1}{2}\right)^{N-1}\frac{1}{N}\left(\left(N-1\right)2^{N-2} + 2^{N-1}\right)\gamma\beta\right) - c \\
=& x\left(\left(\omega - \beta\right) + \frac{N+1}{N}\frac{1}{2}\gamma\beta\right) - c,
\end{aligned}$$

where we use the fact that $\sum_{m=0}^{n} \left(\begin{array}{c} n \\ m \end{array}\right) = 2^n$ and $\sum_{m=0}^{n} m \left(\begin{array}{c} n \\ m \end{array}\right) = n2^{n-1}$.

The above analysis implies that a candidate equilibrium cutoff is given by:

$$x^* = \frac{c}{(\omega - \beta) + \frac{N+1}{N}\frac{1}{2}\gamma\beta}.$$

Existence and uniqueness of the equilibrium follow from the fact that π^* increases in x and decreases in k. Since π^* increases in x, $\pi^*(x, x^*) > 0$ for $x > x^*$ and $\pi^*(x, x^*) < 0$ for $x < x^*$. Thus, x^* is an equilibrium. Observing that $\pi^*(x, k)$ also decreases in k, the usual global games argument shows that this is the unique equilibrium due to the iterated deletion of strictly dominated strategies (Morris and Shin, 2003).

The claim that $\pi^*(x, k)$ increases in x follows because $p(x, k)$ increases in x. Stochastic dominance arguments then show that $\pi^*(x, k)$ increases in x, just as in the proof of Lemma A2. To see this, note that $p(x, k)$ increases in x. This implies that, for any $x_1 < x_2$, the binomial distribution of the number of reports out of $N-1$ agents under $p(x_2, k)$ stochastically dominates the distribution under $p(x_1, k)$. Since $x\left(\omega - \left(1 - \frac{i+1}{N}\gamma\right)\beta\right) - c$ is itself an increasing function of x, then stochastic dominance also implies that $\pi^*(x, k)$ increases under x.

The claim that $\pi^*(x, k)$ decreases in k follows because $x\left(\omega - \left(1 - \frac{m+1}{N}\gamma\right)\beta\right) - c$ is invariant to k. By the same stochastic dominance argument, we have $\pi^*(x, k)$ decreases in k.

We summarize with:

PROPOSITION OA.1 (Baseline Equilibrium with N agents): *Suppose agents have improper uniform priors over θ. There exists a globally unique symmetric threshold strategy equilibrium where all agents play a threshold strategy and report $(s(x_i) = 1)$ if and only if $x_i \geq x^*$, where*

$$x_N^* = \frac{c}{(\omega - \beta) + \frac{N+1}{N}\frac{1}{2}\gamma\beta}.$$

and $x^* > \underline{x} > 0$. The threshold strategy is the unique strategy that survives the iterated deletion of strictly dominated strategies. In equilibrium, the belief of the marginal agent who draws $x_i = x^*$ over the number of agents reporting r is uniformly distributed over $[0, 1]$.

Proposition OA.1 is a close analog of Proposition 1 in the text. Notice that for N large, x_N^* from Proposition OA.1 converges to $x^* = \frac{c}{(\omega-\beta)+\frac{1}{2}\gamma\beta}$ from Proposition 1. The slight difference in the equilibrium cutoff occurs because agents are atomistic in the continuum agent game but not in the finite agent game. In the finite agent game, each agent's report increments the probability of sanction by a discrete amount, $1/N$. All else equal, greater N results in a greater equilibrium reporting threshold because the effect of each agent's own report on the probability of sanction diminishes. As the number of agents approaches infinity, the effect of each agent's own report on the probability of sanction becomes vanishingly small, and the cutoff converges to the solution with a continuum of agents.

IV. Endogenous Sanction Functions

We provide a few stylized models that describe channels through which the outside party's objective function can generate a coordination problem among agents and show that the results still hold qualitatively when $\Gamma(r)$ is endogenous.

In each of these models, the agents receive their x_i's and decide whether or not to report. Then the outside party (the firm) observes r and decides whether or not to sanction the manager. For simplicity, there is no uncertainty in the sanction process.

1) Model 1: Suppose the firm values the manager's productivity but that its payoff decreases in r.

   Here, the firm doesn't mind how the manager treats his subordinates, but does find it costly to deal with reports of misconduct. This payoff function generates an endogenous $\Gamma(r)$ where the firm sanctions the manager if reports r exceed an equilibrium sanction threshold r' that depends only on the manager's productivity parameters. Because the endogenous $\Gamma(r)$ does not depend on agents' payoff parameters, all results from the exogenous $\Gamma(r)$ model described in the paper apply.

   The manager's value (productivity) to the firm is $v > 0$. If the firm does not sanction the manager, then it keeps the manager but there is a $c_I(r)$ (due to the possibility of employee turnover, violation of federal law, reputational risks, etc). Let $c_I(r)$ strictly increase in r with $c_I(0) = 0$. If the firm sanctions the manager, he is replaced by another manager with productivity v'.

   Given r, the firm's payoff is $v - c_I(r)$ if it does not sanction, and v' if it sanctions. Note that if $v - c_I(1) > v'$, then the firm would never sanction. If $v < v'$, then the firm would always want to sanction/fire the manager regardless. Thus we focus on the case of interest, $v - c_I(1) < v' < v$.

   Thus, the firm would sanction for any $r \geq r'$ where $v - c_I(r') = v'$ and not sanction for any $r < r'$. Since $c_I(r)$ is strictly increasing in r and $v - c_I(1) < v' < v$, then there exists a unique $r' \in (0, 1)$ such that $v - c_I(r') = v'$. Note that r' decreases in v' and increases in v.

   This implies that when deciding whether or not to report, agents take the sanction function $\Gamma(r)$ as

$$\Gamma(r) = \begin{cases} 0 & \text{if } r < r' \\ 1 & \text{if } r \geq r'. \end{cases}$$

Since we have shown in the manuscript that $x^* = \frac{c}{\omega-\beta+\beta\left(\int_0^1 \Gamma(r)dr\right)}$, then the solution to the equilibrium is

$$x^* = \frac{c}{\omega - \beta r'}$$
$$v - c_I(r') = v'.$$

Because the threshold r' does not depend on agents' payoff parameters, all results from the exogenous $\Gamma(r)$ model described in the manuscript follow.

2) Model 2: Suppose the firm values the manager's productivity but that its payoff decreases in θ.

Here, the firm does not want to employ a manager that treats his subordinates poorly, irrespective of the number of reports. This generates an endogenous $\Gamma(r)$ with an equilibrium sanction threshold r' that depends on agents' payoff parameters through their equilibrium reporting threshold x^*. As such, changing agents' payoff parameters affects x^* both directly and indirectly through its effect on r'. Nevertheless, we show that the comparative statics of this game for x^* qualitatively match the comparative statics of the paper where $\Gamma(r)$ is exogenous.

The manager's value to the firm (productivity) is $v > 0$. The firm dislikes bad apples (i.e., high-θ type). If the firm sanctions the manager, he is replaced by another manager with productivity v'. The firm's payoff is $v - f(\theta)$ if it does not sanction the manager, where $f(\theta)$ is non-negative and weakly increasing in θ. The firm's payoff is v' if it sanctions the manager. Assume that $v > v'$ so the firm does not want to fire the manager ex ante.

Conjecture an equilibrium in which agents use threshold strategies. Given a realized r, in equilibrium the firm learns θ. Since $f(\theta)$ is non-negative and weakly increasing in θ, then the firm will sanction for any $\theta \geq \overline{\theta}$ where $v - v' - f(\overline{\theta}) = 0$ and not sanction for any $\theta < \overline{\theta}$.

Since the use of threshold strategies generates a one-to-one mapping between r and θ where $r = 1 - \Phi\left(\frac{x^*-\theta}{\sigma}\right) = \Phi\left(\frac{\theta-x^*}{\sigma}\right)$, then in equilibrium the firm sanctions for any $r \geq r'$ where $r' = \Phi\left(\frac{\overline{\theta}-x^*}{\sigma}\right)$ and does not sanction if $r < r'$.

This implies that when deciding whether or not to report, agents take the sanction function $\Gamma(r)$ as

$$\Gamma(r) = \begin{cases} 0 & \text{if } r < r' \\ 1 & \text{if } r \geq r'. \end{cases}$$

Thus the solution to the equilibrium is $(\overline{\theta}, r', x^*)$ such that

$$x^* = \frac{c}{\omega - \beta r'} \tag{1}$$

$$r' = \Phi\left(\frac{\overline{\theta} - x^*}{\sigma}\right) \tag{2}$$

$$v - v' - f(\overline{\theta}) = 0. \tag{3}$$

Note that r' is the solution to

$$r' - \Phi\left(\frac{f^{-1}(v - v') - \frac{c}{\omega - \beta r'}}{\sigma}\right) = 0, \tag{4}$$

and

$$\overline{\theta} = f^{-1}(v - v'). \tag{5}$$

Let $G(r) = r - \Phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega - \beta r}}{\sigma}\right)$. Since $G(r = 1) > 0$ and $G(r = 0)$, then there exists an $r' \in (0, 1)$ that satisfies Equation 4. Moreover,

$$\begin{aligned}\frac{\partial G}{\partial r} &= 1 - \Phi'\left(\frac{f^{-1}(v - v') - \frac{c}{\omega - \beta r}}{\sigma}\right)(-\frac{\beta c}{\sigma})(\omega - \beta r)^{-2} \\ &= 1 + \phi\left(\frac{f^{-1}(v - v') - \frac{c}{\omega - \beta r}}{\sigma}\right)(\frac{\beta c}{\sigma})(\omega - \beta r)^{-2} > 0.\end{aligned}$$

Thus this r' is unique.

Here, note that the threshold r' depends on agents' payoff parameters. Intuitively, as agents become more reluctant to report (x^* increases), then a lower r reveals θ in equilibrium so a lower threshold r' is required to generate sanction for a given θ.

Consider the total effect on x^* when β increases $(\frac{\partial x^*}{\partial \beta})$.

Applying the implicit function theorem, we have

$$\frac{\partial r'}{\partial \beta} = -\frac{\frac{\partial G}{\partial \beta}}{\frac{\partial G}{\partial r'}}. \tag{6}$$

We have already shown that $\frac{\partial G}{\partial r'} > 0$.

$$\frac{\partial G}{\partial \beta} = -\Phi'\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(-\frac{rc}{\sigma})(\omega-\beta r)^{-2}$$
$$= \phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{rc}{\sigma})(\omega-\beta r)^{-2} > 0.$$

Thus, consistent with the above intuition $\frac{\partial r'}{\partial \beta} < 0$.

Since $x^* = \frac{c}{\omega-\beta r'}$, then the effect of $\frac{\partial x^*}{\partial \beta}$ is determined by $\frac{\partial}{\partial \beta}(\beta r')$:

$$\frac{\partial}{\partial \beta}(\beta r') = r' + \beta(\frac{\partial r'}{\partial \beta})$$
$$= r' - \beta\left(\frac{\phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{rc}{\sigma})(\omega-\beta r)^{-2}}{1+\phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{\beta c}{\sigma})(\omega-\beta r)^{-2}}\right).$$

Note that

$$\frac{\phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{rc}{\sigma})(\omega-\beta r)^{-2}}{1+\phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{\beta c}{\sigma})(\omega-\beta r)^{-2}} < \frac{\phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{rc}{\sigma})(\omega-\beta r)^{-2}}{\phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{\beta c}{\sigma})(\omega-\beta r)^{-2}}$$
$$= \frac{r}{\beta}.$$

Thus,

$$\frac{\partial}{\partial \beta}(\beta r') = r' + \beta(\frac{\partial r'}{\partial \beta})$$
$$= r' - \beta\left(\frac{\phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{rc}{\sigma})(\omega-\beta r)^{-2}}{1+\phi\left(\frac{f^{-1}(v-v') - \frac{c}{\omega-\beta r}}{\sigma}\right)(\frac{\beta c}{\sigma})(\omega-\beta r)^{-2}}\right)$$
$$> r' - \beta(\frac{r'}{\beta}) = 0.$$

Thus, $\frac{\partial}{\partial \beta}(\beta r') > 0$, so $\frac{\partial x^*}{\partial \beta} > 0$. That is, the indirect effect of increasing β through lowering r' is weaker than the direct effect of increasing β.

A similar exercise shows that $\frac{\partial x^*}{\partial c} > 0$.

Finally, note that, for example, if v increases (the manager becomes more valuable to the firm), this clearly increases r' and therefore increases x^*. Increasing v is akin to increasing γ in the model described in the manuscript.

3) Model 3: Suppose the firm values the manager's productivity, that its payoff decreases θ, and faces a constraint whereby it must have $r \geq \underline{r}$ reports to sanction the manager.

Here, the firm may not want to employ a manager that treats his subordinates poorly but is required to provide "just cause" in the form of a sufficient number of reports to sanction the manager. Depending on the parameters, this case generates endogenous $\Gamma(r)$ that is akin to either Model 1 or Model 2. Thus, the results still hold qualitatively.

The manager's value to the firm (productivity) is $v > 0$. The firm dislikes high-θ types. However, it is constrained to provide "just cause" for sanctioning the manager, which means that it must also provide $r \geq \underline{r}$ to sanction (take $\underline{r}$ as an exogenous constraint, like a legal requirement). If the firm sanctions the manager, he is replaced by another manager with productivity v'. The firm's payoff is $v - f(\theta)$ if it does not sanction the manager, where $f(\theta)$ is non-negative and weakly increasing in θ. The firm's payoff is v' if it sanctions the manager. Assume that $v > v'$ so the firm does not want to fire the manager ex ante.

To derive the solution, there are two relevant cases based on whether the "just cause" constraint binds in equilibrium:

a) The "just cause" constraint does not bind in equilibrium. In this case, the solution must satisfy Equations 1, 4, and 5. Thus, the solution is the same as in Model 2.

When does this case occur? The requirement that $r \geq \underline{r}$ does not bind if the solution r' to Equations 1, 4, and 5 satisfies $r' \geq \underline{r}$. Intuitively, this must be where $\overline{\theta}$ is sufficiently high (we solve for it explicitly after analyzing the second case).

b) The "just cause" constraint binds in equilibrium. That is, the solution r' to Equations 1, 4, and 5 fails $r' \geq \underline{r}$.

In this case, agents know that $\underline{r}$ binds as the minimal reporting at which the firm sanctions. Thus, agents take $\Gamma(r)$ as

$$\Gamma(r) = \begin{cases} 0 & \text{if } r < \underline{r} \\ 1 & \text{if } r \geq \underline{r}. \end{cases}$$

Thus, the solution is analogous to that of Model 1 and the equilibrium

TABLE OA.1—AGENT PAYOFFS WHEN SHE EARNS A PAYOFF IN (NO REPORT, SANCTION)

	Sanction	No Sanction
Report	$x_i\omega - c$	$x_i(\omega - \mathbf{1}_{[x_i>0}\beta]) - c$
No Report	ax_i	0

reporting threshold is

$$x^* = \frac{c}{\omega - \beta \underline{r}}. \tag{7}$$

Let $\underline{\theta}$ be the minimal manager type such that the firm sanctions managers if and only if $\theta \geq \underline{\theta}$. Then we must have:

$$\underline{r} = \Phi\left(\frac{\underline{\theta} - x^*}{\sigma}\right)$$

$$\underline{\theta} = x^* + \sigma\Phi^{-1}(\underline{r})$$

$$\underline{\theta} = \frac{c}{\omega - \beta\underline{r}} + \sigma\Phi^{-1}(\underline{r}). \tag{8}$$

Thus, the solution must satisfy Equations 7 and 8. Note that it must be that $v - v' - f(\underline{\theta}) < 0$, i.e., $\underline{\theta} > \overline{\theta}$. That is, if the "just cause" constraint binds, then the firm would prefer to sanction managers with $\theta \in [\overline{\theta}, \underline{\theta})$, but cannot do so because there is not enough reporting for the firm to legally do so.

Thus $\overline{\theta}$ is pinned down by Equation 5 and $\underline{\theta}$ is pinned down by Equation 8. When $\overline{\theta} \geq \underline{\theta}$, the solution satisfies Equations 1, 4, and 5 (akin to Model 2) because the "just cause" constraint does not bind in equilibrium. When $\overline{\theta} < \underline{\theta}$, the solution satisfies Equations 7 and 8 (akin to Model 1) because the "just cause" constraint binds in equilibrium.

V. Alternative Payoffs From Not Reporting

We derive a variation of the model that is identical to that of Section 1 in the manuscript except that the agent's payoff from not reporting equals ax_i, where $a \geq 0$, instead of zero when the manager is sanctioned. The baseline model in Section 1 corresponds to $a = 0$. We show that all qualitative insights remain as long as $a < \beta$ (i.e., the payoff function is monotone in r).

Table OA.1 describes agent payoffs.

The payoff gain $\pi(r, x_i)$ is monotone in r and x_i if $a < \beta$:

$$\pi(r, x_i) = \begin{cases} x_i(\omega - \beta + \gamma r(\beta - a)) - c & \text{if } x_i > 0, \\ x_i(\omega - \gamma r a) - c & \text{if } x_i \leq 0. \end{cases}$$

The new dominance regions are delimited by $\underline{x}$ and $\overline{x}$:

(9) $$\underline{x} = \frac{c}{\omega - \beta + \gamma(\beta - a)}$$

(10) $$\overline{x} = \frac{c}{\omega - \beta}.$$

As expected, $\frac{\partial \underline{x}}{\partial a} > 0$ and $\frac{\partial \overline{x}}{\partial a} = 0$.

Finally, $x^* = \frac{c}{\omega - \beta(1 - \frac{1}{2}\gamma) - \frac{1}{\gamma}a}$ where $\frac{\partial x^*}{\partial a} > 0$:

(11) $$\int_0^1 \pi(r, x^*) dr = 0$$

(12) $$x^*(\omega - \frac{1}{2}\gamma a - (1 - \frac{1}{2}\gamma)\beta) - c = 0$$

(13) $$x^* = \frac{c}{\omega - \beta(1 - \frac{1}{2}\gamma) - \frac{1}{2}\gamma a}.$$

Thus, the main differences are that $\underline{x}$ and x^* are higher when $a > 0$ than when $a = 0$. That is, the reporting threshold is higher when agents can free-ride off other agents' reports. Proposition 2 (and Corollary 1.1) still applies, using in the proof the new $\underline{x}$, x^*, and $H(x) \equiv x\left(\omega - \beta + \gamma \Phi(\frac{\theta - x}{\sigma})(\beta - a)\right) - c$, which is the new expected payoff of the marginal agent given that all agents use reporting threshold x and given the true θ.

Note that if instead the agent receives ax_i only if $x_i > 0$ and zero otherwise, then we would have the same results above because $a < \beta$ implies that reporting negative x_i is a dominated strategy.

VI. General Sanction Function

The outside party observes $\mathbb{X} \equiv \{(n(x), x) | \forall x \in \mathbb{R}\}$, the frequency and value of each reported x. Let $\Xi(\mathbb{X}|\mathbb{X}) : \mathbb{R}^{\mathfrak{c}} \times \mathbb{R}^{\mathfrak{c}} \to [0, 1]$ represent the sanction probability given the set of all reports and the value of those reports. Consider a class of sanction functions $\Xi(\mathbb{X}) = B(\int_x \varphi(x) n(x) dx)$, where $\varphi(x)$ is a weight for each reported x and $n(x) \geq 0$ is the frequency of reports for each x. Let $\varphi(x)$ be weakly monotone increasing and differentiable, where $\varphi(x) = 0$ if $x < 0$ and $\varphi(x) > 0$ for some $x > 0$. Let $B(s) : [0, \infty) \to [0, 1]$ be weakly monotone increasing, and strictly increasing at some s. Note that $B(s)$ does not have to be continuous. This describes a natural class of sanction functions in which the sanction probability weakly increases in the number of reports and in the severity of misconduct.

[1] We can show that when $\varphi(x) = 1$ for all $x > 0$, then $\Xi(\mathbb{X}) = B(r) = \Gamma(r)$. Thus, the sanction probability is a weakly increasing function of r alone. PROOF:

By Lemma 4, all agents use the same strategy in equilibrium. Suppose agents use threshold strategy x^*. Then $n(x) = \mathbf{1}_{[x>x^*]}\frac{1}{\sigma}\phi(\frac{x-\theta}{\sigma})$, giving

$$\begin{aligned}\Xi(\mathbb{X}) &= B(\int_x \varphi(x)n(x)dx)\\ &= B\left(\int_{x^*}^{\infty}\frac{1}{\sigma}\varphi(x)\phi\left(\frac{x-\theta}{\sigma}\right)dx\right).\end{aligned}$$

The outside party observes $\hat{r}(\theta) = \int_{x^*}^{\infty}\frac{1}{\sigma}\phi(x \mid \theta)\,dx = \Phi\left(\frac{\theta-x^*}{\sigma}\right)$, so $\theta = x^* + \sigma\Phi^{-1}(r)$. Suppose $\varphi(x) = 1$. Then

$$\begin{aligned}\Xi(\mathbb{X}) &= B\left(\int_{x^*}^{\infty}\frac{1}{\sigma}\phi\left(\frac{x-\theta}{\sigma}\right)dx\right)\\ &= B\left(1-\Phi\left(\frac{x^*-\theta}{\sigma}\right)\right)\\ &= B\left(1-\Phi\left(\frac{x^*-\left[x^*+\sigma\Phi^{-1}(r)\right]}{\sigma}\right)\right)\\ &= B\left(1-\Phi\left(-\Phi^{-1}(r)\right)\right)\\ &= B\left(1-\Phi\left(\Phi^{-1}(1-r)\right)\right)\\ &= B(r).\end{aligned}$$

Thus, $\Xi(\mathbb{X}) = B(r) = \Gamma(r)$. Thus, the sanction probability is a weakly increasing function of r alone. ■

Notable special cases: If $B(s) = \gamma s$, then $\Xi(\mathbb{X}) = \gamma r$. If $B(s) = \begin{cases} 0 \text{ if } s \in [0,\overline{r}) \\ 1 \text{ if } s \in [\overline{r},1] \end{cases}$, then $\Xi(\mathbb{X}) = \begin{cases} 0 \text{ if } s \in [0,\overline{r}) \\ 1 \text{ if } s \in [\overline{r},1] \end{cases}$ where $\overline{r} \in [0,1]$ is a constant.

[2] For more general $\varphi(x)$, the sanction function can be reformulated as $\Xi(\mathbb{X}) = \Gamma(x^*, r)$ (though there may be multiple equilibria in which agents use threshold strategies).

By Lemma 4, all agents use the same strategy in equilibrium. Suppose agents use threshold strategy x^*. Then

$$\begin{aligned}\Xi(\mathbb{X}) &= B\left(\int_{x^*}^{\infty} \frac{\varphi(x)}{\sigma}\phi\left(\frac{x-\theta}{\sigma}\right)dx\right)\\ &= B\left(\int_{x^*}^{\infty} \frac{\varphi(x)}{\sigma}\phi\left(\frac{x-x^*}{\sigma}-\Phi^{-1}(r)\right)dx\right)\\ &= \Gamma(x^*, r),\end{aligned}$$

since $\theta = x^* + \sigma\Phi^{-1}(r)$ when agents use threshold x^*. Since each agent, including the marginal agent, takes x^* as given, then the marginal agent solves:

$$\int_0^1 \pi(r, x^*)dr = 0,$$

which has the implicit solution

$$x^* = \frac{c}{\omega - \beta + \left(\int_0^1 \Gamma(x^*, r)dr\right)(\alpha+\beta)}. \tag{14}$$

To show the existence of such an x^*, note that we can still define the dominance regions in which the agent never reports if $x \le \underline{x}$ (i.e., even if everyone reports and the manager is definitely sanctioned) and always reports if $x \ge \overline{x}$ (i.e., even if no one reports so the manager is definitely not sanctioned), where

$$\begin{aligned}\underline{x} &= \frac{c}{\omega+\alpha}\\ \overline{x} &= \frac{c}{\omega-\beta}.\end{aligned}$$

Let $G(x) = x\left(\omega - \beta + \left(\int_0^1 \Gamma(x, r)dr\right)(\alpha+\beta)\right) - c$, so x^* satisfies $G(x^*) = 0$. Note that $G(\overline{x}) > 0$ if and only if $\int_0^1 \Gamma(x^*, r)dr > 0$. Given that $f(x)$ is weakly monotone increasing where $f(x) = 0$ if $x < 0$ and $f(x) > 0$ for some $x > 0$, and $B(s) : [0, \infty) \to [0, 1]$ is weakly monotone increasing (and strictly increasing at some s), then $\int_0^1 \Gamma(x^*, r)dr > 0$. Note that $G(\underline{x}) < 0$ if and only if $\int_0^1 \Gamma(x^*, r)dr < 1$. Since $\int_0^1 \Gamma(x^*, r)dr = 1$ if and only if $B(s) = 1$ for all s, then $\int_0^1 \Gamma(x^*, r)dr < 1$ because $B(s) : [0, \infty) \to [0, 1]$ is weakly monotone increasing, and strictly increasing at some s. Thus, such an x^* exists, and $x^* \in (\underline{x}, \overline{x})$. Given such an x^*, it is straightforward to verify that agents with $x_i < x^*$ do not report and agents with $x_i > x^*$, since $x_i\left(\omega - \beta + \left(\int_0^1 \Gamma(x^*, r)dr\right)(\alpha+\beta)\right)$ monotonically increases in x_i when agents use threshold x^*. Thus, all agents using threshold x^* is an equilibrium.

Although we have established the existence of an equilibrium in which agents use threshold x^*, which must satisfy Equation 14, multiplicity of equilibria is entirely possible. Nonetheless, in any such equilibrium, the existence of an "open secret" in which there is under-reporting, as described by Corollary 1.1, still holds. This is because when, $\sigma \to 0$, for any $\theta \in (\underline{x}, x^*)$ we still have $\pi(1,x) > 0 > \pi(0,x)$. Since $x^* \in (\underline{x}, \overline{x})$, this implies there will be under-reporting for any $\theta \in (\underline{x}, x^*)$.

[3] Even when the sanction function includes a general $\varphi(x)$, we show that under-reporting occurs in any equilibrium that involves threshold strategies, under qualitatively similar conditions as Proposition 2. When θ is sufficiently high, there always exists some $\hat{x} < x^*$ such that there is a Pareto improvement in agent payoffs. When θ is intermediate, there exists some $\hat{x} < x^*$ such that there is a Pareto improvement in agent payoffs if σ is sufficiently small.

LEMMA OA.2: *Let g and h be density functions, where $g \succeq h$ if $\int_{-\infty}^{z} g(s)ds \leq \int_{-\infty}^{z} h(s)ds$. If $g \succeq h$ and $u(z)$ is a weakly increasing and weakly positive, differentiable function of z, then $\int_a^{\infty} u(z)g(z)dz \geq \int_a^{\infty} u(z)h(z)dz$.*

PROOF:

Define $G(z) = \int_{-\infty}^{z} g(s)ds$ and $H(z) = \int_{-\infty}^{z} h(s)ds$. Note that if $g \succeq h$, then $G(z) \leq H(z)$.

Using integration by parts,

$$\begin{aligned}
\int_a^{\infty} u(z)g(z)dz &= \int_a^{\infty} u(z)G'(z)dz \\
&= u(z)G(z)|_a^{\infty} - \int_a^{\infty} G(z)u'(z)dz \\
\int_a^{\infty} u(z)h(z)dz &= \int_a^{\infty} u(z)H'(z)dz \\
&= u(z)H(z)|_a^{\infty} - \int_a^{\infty} H(z)u'(z)dz.
\end{aligned}$$

Since $G(\infty) = H(\infty) = 1$, then

$$\int_a^{\infty} u(z)g(z)dz - \int_a^{\infty} u(z)h(z)dz = u(a)[H(a) - G(z)] + \int_a^{\infty} u'(z)[H(z) - G(z)]dz \geq 0.$$

■

Let θ' be the minimum value of θ such that the marginal agent's expected probability of sanction is less than or equal to the realized probability of sanction:

$$\theta' = \min\{\theta : \int_0^1 B\left(\int_{x^*}^{\infty} \frac{\varphi(x)}{\sigma}\phi\left(\frac{x - x^*}{\sigma} - \Phi^{-1}(r)\right)dx\right)dr \leq B\left(\int_{x^*}^{\infty} \frac{\varphi(x)}{\sigma}\phi\left(\frac{x - \theta'}{\sigma}\right)\right)\}$$

Recall that $\Xi(\mathbb{X}|\theta, x^*) = B\left(\int_{x^*}^{\infty} \frac{1}{\sigma}\varphi(x)\phi\left(\frac{x-\theta}{\sigma}\right)dx\right)$. By Lemma OA.2, we know that $\Xi(\mathbb{X}|\theta, \hat{x})$ is weakly increasing in θ, so clearly such a θ' exists.

[a] Suppose $\theta > \theta'$. There exists some $\hat{x} < x^*$ such that total welfare is higher due to a Pareto improvement.

Recall that $\Xi(\mathbb{X}|\theta, x^*) = B\left(\int_{x^*}^{\infty} \frac{1}{\sigma}\varphi(x)\phi\left(\frac{x-\theta}{\sigma}\right)dx\right)$. For any $\hat{x} < x^*$, total agent payoffs equal:

$$\begin{aligned} W(\theta \mid \hat{x}) &= \int_{x^*}^{\infty} \left[x\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha+\beta)\right) - c\right] f(x \mid \theta)\, dx \\ &\quad + \int_{\hat{x}}^{x^*} \left[x\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha+\beta)\right) - c\right] f(x \mid \theta)\, dx. \end{aligned}$$

We know that equilibrium total payoffs are such that:

$$\begin{aligned} W(\theta \mid x^*) &= \int_{x^*}^{\infty} \left[x\left(\omega - \beta + \Xi(\mathbb{X}|\theta, x^*)(\alpha+\beta)\right) - c\right] f(x \mid \theta)\, dx \\ &< \int_{x^*}^{\infty} \left[x\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha+\beta)\right) - c\right] f(x \mid \theta)\, dx, \end{aligned}$$

for any $\hat{x} < x^*$.

The remaining claim is that there exists a $\hat{x} < x^*$ such that:

$$K \equiv \int_{\hat{x}}^{x^*} \left[x\left(\omega - \beta + \Xi(\mathbb{X}|\theta)(\alpha+\beta)\right) - c\right] f(x \mid \theta)\, dx \quad > \quad 0.$$

By Lemma OA.2, we know that $\Xi(\mathbb{X}|\theta, \hat{x})$ is weakly increasing in θ, which implies that when $\theta > \theta'$:

$$\begin{aligned} 0 &= x^*\left(\omega - \beta + \left(\int_0^1 \Gamma(x^*, r)dr\right)(\alpha+\beta)\right) - c \\ &< x^*\left(\omega - \beta + \Xi(\mathbb{X}|\theta, x^*)(\alpha+\beta)\right) - c. \end{aligned} \tag{15}$$

Let $H(x) \equiv x\left(\omega - \beta + \Xi(\mathbb{X}|\theta, x)(\alpha+\beta)\right) - c$. Note that $H(x^*) > 0$ from Equation 15.

Suppose H is continuous at x^*. If $H'(x^*) < 0$, then there exists a $\hat{x} < x^*$ such that:

$$\begin{aligned} 0 &< x^*\left(\omega - \beta + \Xi(\mathbb{X}|\theta, x^*)(\alpha+\beta)\right) - c \\ &< \hat{x}\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha+\beta)\right) - c. \end{aligned}$$

If $H'(x^*) > 0$, there exists some $\varepsilon > 0$ such that, for $\hat{x} = x^* - \varepsilon$,

$$\begin{aligned} 0 &< \hat{x}\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha + \beta)\right) - c \\ &< x^*\left(\omega - \beta + \Xi(\mathbb{X}|\theta, x^*)(\alpha + \beta)\right) - c. \end{aligned}$$

If $H'(x^*) = 0$, observe that because $H^{(n)}(x^*) \neq 0$ for some $n > 0$, we can apply a similar argument to find an $\hat{x} < x^*$ such that $H(\hat{x}) > 0$. Either way, there exists some $\hat{x} < x^*$ such that $H(\hat{x}) > 0$.

Suppose H is not continuous at x^* (if $B(\cdot)$ is not continuous at x^*). By direct computation, $\frac{\partial}{\partial \hat{x}} \int_{\hat{x}}^{\infty} \frac{1}{\sigma}\varphi(x)\phi\left(\frac{x-\theta}{\sigma}\right) dx < 0$. Since $B(s)$ increases in s, then H is either left-continuous at x^* or strictly decreasing in x. Either way, there exists some $\hat{x} < x^*$ such that $H(\hat{x}) > 0$.

But then:

$$\begin{aligned} K &= \int_{\hat{x}}^{x^*} \left[x\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha + \beta)\right) - c\right] f(x \mid \theta)\, dx \\ &> \int_{\hat{x}}^{x^*} \left[\hat{x}\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha + \beta)\right) - c\right] f(x \mid \theta)\, dx \\ &= \left[\hat{x}\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha + \beta)\right) - c\right] \times \left[\Phi\left(\frac{x^* - \theta}{\sigma}\right) - \Phi\left(\frac{\hat{x} - \theta}{\sigma}\right)\right] \\ &> 0. \end{aligned}$$

Note that this is a Pareto improvement because for any $x \in (\hat{x}, x^*)$,

$$x\left(\omega - \beta + \Xi(\mathbb{X}|\theta, \hat{x})(\alpha + \beta)\right) - c \quad > \quad H(\hat{x}) > 0,$$

whereas these agents all receive 0 when playing a threshold strategy around x^*.

[b] Suppose $\underline{\underline{x}} < \theta \leq \theta'$, where $\underline{\underline{x}}$ is defined below. If σ is sufficiently low, there exists some $\hat{x} \in (\underline{\underline{x}}, \theta)$ such that total welfare is higher due to a Pareto improvement.

If $\theta \leq \theta'$, then $H(x^*) \leq 0$. Note that by Lemma OA.2, for all $\hat{x} < \theta$, $B\left(\int_{\hat{x}}^{\infty} \frac{\varphi(x)}{\sigma}\phi\left(\frac{x-\theta'}{\sigma}\right)\right)$ increases as σ decreases. By direct computation, $\frac{\partial}{\partial \hat{x}} \int_{\hat{x}}^{\infty} \frac{1}{\sigma}\varphi(x)\phi\left(\frac{x-\theta}{\sigma}\right) dx < 0$. Define $\underline{\underline{x}}$ as the threshold such that

$$\underline{\underline{x}}\left(\omega - \beta + \lim_{\sigma \to 0} B\left(\int_{\underline{x}}^{\infty} \frac{\varphi(x)}{\sigma}\phi\left(\frac{x - \theta'}{\sigma}\right)\right)(\alpha + \beta)\right) - c = 0.$$

By construction, $\lim_{\sigma \to 0} H(\underline{\underline{x}}) = 0$ and $x^* > \underline{\underline{x}} \geq \underline{x}$.

Suppose $H(\cdot)$ is continuous on the interval $[\underline{\underline{x}}, \theta)$ for all σ. By direct differentiation, $\lim_{\sigma \to 0} \frac{\partial H}{\partial \hat{x}} > 0$ for all $\hat{x} < \theta$. Thus, $\lim_{\sigma \to 0} H(\hat{x}) > 0$ for all $\hat{x} \in (\underline{\underline{x}}, \theta)$. By Lemma OA.2, $H(\hat{x})$ is decreasing as σ increases. But by continuity of H, for any

$\hat{x} \in (\underline{\underline{x}}, \theta)$, there exists some $\sigma > 0$ sufficiently small that $H(\hat{x}) > 0$.

Suppose $H(x)$ is not continuous on the interval $[\underline{\underline{x}}, \theta)$ for all σ. Since $\underline{\underline{x}}$ is determined by the case of all agents reporting, then it is sufficient to consider the following to find a Pareto-improving $\hat{x} < x^*$. Let $Z = \{z : \lim_{x \uparrow z} B(x) \neq \lim_{x \downarrow z} B(x)\}$. If f is discontinuous and has a finite number of discontinuities, then Z is a finite non-empty set consisting of real numbers and $z_0 = \min(z \in Z)$ is well-defined. Furthermore, $y_0 = B(z_0)$ is also well-defined. Let $\overline{\sigma}$ satisfy:

$$B\left(\int_{\underline{\underline{x}}}^{\infty} \frac{\varphi(x)}{\overline{\sigma}} \phi\left(\frac{x-\theta}{\overline{\sigma}}\right) dx\right) = y_0.$$

Since for all $\hat{x} < \theta$, $B\left(\int_{\hat{x}}^{\infty} \frac{\varphi(x)}{\sigma} \phi\left(\frac{x-\theta'}{\sigma}\right)\right)$ increases as σ decreases, and $B(s)$ is increasing in s, then there exists some $\epsilon > 0$ such that $H(x)$ is continuous on the interval $[\underline{\underline{x}}, \underline{\underline{x}} + \epsilon)$ for all $\sigma < \overline{\sigma}$ and we can apply the preceding argument. Thus for any $\hat{x} \in (\underline{\underline{x}}, \underline{\underline{x}} + \epsilon)$, there exists some $\sigma > 0$ sufficiently small that $H(\hat{x}) > 0$.

VII. The Publicity Effect of #MeToo

The #MeToo movement was popularized by actress Alyssa Milano on Twitter, who wrote: "If all the women who have been sexually harassed or assaulted wrote 'Me too' as a status, we might give people a sense of the magnitude of the problem" (Khomami, 2017). Surveys confirm that women believe that there is value to heightened awareness of widespread problems (e.g., within the economics profession; see American Economic Association, 2019; Casselman and Tankersley, 2019).

We show that heightened public awareness of misconduct increases reporting of otherwise-hidden misconduct by coordinating beliefs over strategic uncertainty. We associate changes in public information with changes in agents' common priors (Angeletos and Lian, 2016). In Section I.D, we assumed that agents had common improper uniform priors over θ. Now suppose that agents share proper priors $p(\theta)$ that is normally distributed with mean y and standard deviation τ. Such priors could be shaped by public information about a specific manager or information about misconduct in the broader population of managers. Proposition OA.2 characterizes behavior in this environment. The proof strategy follows Morris and Shin (2004). We defer the proof of the Proposition to the end of this section.

PROPOSITION OA.2 (Equilibrium with proper priors): *Suppose agents have common proper priors that θ is distributed normally with mean y and standard deviation τ.*

1) *Existence: There exists at least one symmetric threshold strategy equilibrium $x_I^* \in (\underline{x}, \overline{x})$ where agents report if and only if $x_i \geq x_I^*$, where x_I^* is implicitly*

defined by:

$$x_I^* = \frac{c}{\omega - \beta\left(1 - \gamma\Phi\left(\frac{y - x_I^*}{\kappa}\right)\right)},$$

for $\kappa \equiv \frac{\sigma^2+\tau^2}{\sigma}\sqrt{\frac{\sigma^2+2\tau^2}{\sigma^2+\tau^2}}$ *and where* $x_I^* > \underline{x} > 0$.

2) *Uniqueness (up to either reporting or not reporting when* $x_i = x_I^*$*):*

   a) *The equilibrium* x_I^* *is a unique threshold equilibrium if* $\kappa > \frac{1}{\sqrt{2\pi}}\frac{c\gamma\beta}{(\omega-\beta)^2}$.

   b) *Whenever* $y = \frac{c}{\omega-\beta\left(1-\frac{\gamma}{2}\right)}$*, a sufficient condition for non-unique equilibria is* $\kappa < \frac{1}{\sqrt{2\pi}}\frac{\gamma\beta c}{\left(\omega-\beta\left(1-\frac{\gamma}{2}\right)\right)^2}$.

   c) *For* $y \to \infty$*, there is a unique threshold equilibrium* x_I^* *with* $x_I^* \to \underline{x}$*. For* $y \to -\infty$*, there is a unique threshold equilibrium* x_I^* *with* $x_I^* \to \overline{x}$.

   d) *If there is a unique equilibrium in threshold strategies, then the equilibrium strategy is the only strategy that satisfies the iterated deletion of strictly dominated strategies. In particular, the unique threshold strategy equilibrium is the globally unique equilibrium.*

3) *Beliefs over* r*: The marginal agent, who has experience* $x_i = x_I^*$*, has a belief over the incidence of reporting* r *characterized by the cumulative distribution function* $\Psi_I(.)$*:*

$$\Psi_I(r) \equiv \Phi\left(\frac{\sigma}{\tau}\frac{1}{\sqrt{\sigma^2+\tau^2}}(x_I^* - y) + \frac{1}{\tau}\sqrt{\sigma^2+\tau^2}\,\Phi^{-1}(r)\right),$$

and has expectation $E_I^*[r] = \Phi\left(\frac{y-x_I^*}{\kappa}\right)$.

There is a unique equilibrium when priors are sufficiently diffuse or when private experiences are informative for θ relative to priors. Specifically, Part 2(a) of Proposition OA.2 is satisfied for high τ or when σ is low relative to τ. The reason is that κ increases in τ; furthermore, κ decreases in σ when $\sigma^2/\tau^2 < \sqrt{2}$. Indeed, as $\tau \to \infty$ or $\sigma \to 0$, we converge to the unique equilibrium in Proposition 1, because $\kappa \to \infty$ and $\Psi_I(r) \to r$.

This observation comports with well-known results in the literature about global games: precise public information induces coordination and multiple equilibria, while private information hinders coordination because signals are not common knowledge (Angeletos and Lian, 2016; Morris and Shin, 2003, 2004). Part 2(b) illustrates equilibrium multiplicity. Suppose $y = \frac{c}{\omega-\beta\left(1-\frac{\gamma}{2}\right)}$, so that $y = x_I^*$ and

Table OA.2—Comparative statics with proper priors.

Parameter	(1) Reporting Threshold x^*	(2) Aggregate Reporting $\hat{r}(\theta)$
y: Public belief of average type	-	$+$
τ: Public belief of type dispersion (for $y > x_I^*$; flip signs if $>$; 0 if "$=$")	$+$	-

Note: Comparative statics for ω, c, γ, and β are identical to those in the main text. Note that $y > x_I^*$ if and only if $y > \frac{c}{\omega - \beta\left(1 - \frac{\gamma}{2}\right)}$.

$E_I^*[r] = 1/2$. Multiplicity occurs if $\tau \approx 0$ and σ is not too large.[1] We discuss the relevance of Part 2(c) below.

With proper priors, under-reporting still occurs if and only if misconduct is widespread. The "open-secret" equilibrium of Corollary 1.1 is unchanged even when priors are proper. Proposition 2 holds in the unique equilibrium or in any threshold strategy equilibrium of Proposition OA.2 (as discussed in the main Appendix). Condition (1) holds so long as $\theta > x_I^* + \frac{\sigma}{\kappa}\left(y - x_I^*\right)$. Condition (2) holds for $\theta \in \left(\underline{x} + \frac{\sigma}{\kappa}\left(y - x_I^*\right), x_I^* + \frac{\sigma}{\kappa}\left(y - x_I^*\right)\right]$ and σ sufficiently small.

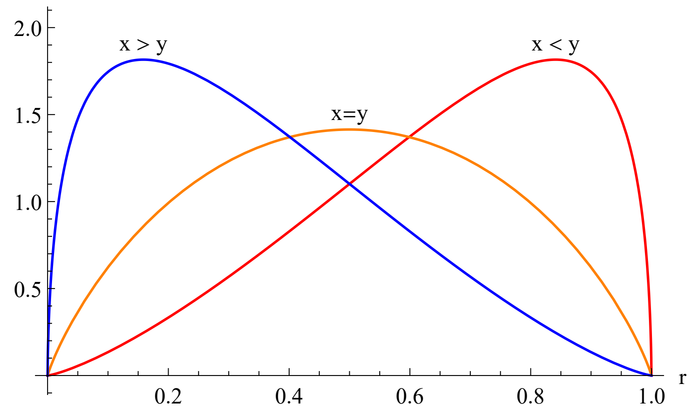
From here on, we assume the environment satisfies the condition for a unique threshold equilibrium in Part 2(a); the equilibrium is then globally unique by Part 2(d). This assumption seems plausible because public information about any prior misconduct by a specific individual is often diffuse. Such information does not accumulate smoothly in public through time, often because the accused and accusers settle claims using non-disclosure agreements (NDAs). Information about whether such NDAs exist is disperse, and the threat of enforcement is effective at keeping information about prior allegations hidden (Lobel, 2018). For example, NDAs kept information about Harvey Weinstein's misconduct hidden for many years (for other anecdotes, see Benner, 2017, or the story of "LaDonna" in Episode #647 of "This American Life"). Moreover, information about misconduct in the broader population of managers is likely less informative about a specific manager's type than experiences from that manager, suggesting Part 2(a) is satisfied.

Table OA.2 summarizes how changes in the mean and standard deviation of the prior belief, y and τ, affect equilibrium reporting. Corollary OA.2.1 highlights the key implication relevant for the #MeToo movement.[2]

[1] Small τ is insufficient to guarantee small κ and equilibrium multiplicity because $\lim_{\tau \to 0} \kappa = \sigma$. If σ is large, κ can be large, potentially satisfying the sufficient condition for uniqueness in Part 2(a). Part 2(b) guarantees this does not happen.

[2] The comparative statics for ω, c, γ, and β are identical to those in the main text. For σ, an increase in σ can either increase or decrease the reporting threshold x_I^*, because the change in σ has two effects on beliefs over r. First, it affects the marginal agent's assessment of how likely her experience was relative to what other agents might be experiencing based on her priors, but it also affects how much she revises her belief about what experience she expects others to have when forming her posteriors. Which effect dominates depends on σ^2/τ^2 and whether y is low or high. For brevity, we omit details about σ as it

FIGURE OA.1. PROBABILITY DENSITY FUNCTION $\psi_I(r)$



Note: This figure plots the probability density function of the marginal agent's belief over the number of agents reporting for three different cases denoted in the figure. The cumulative distribution function for this belief is $\Psi_I(r)$ given in Proposition OA.2.

COROLLARY OA.2.1 (Publicity effect of #MeToo): *Agents become more willing to report (x_I^* falls) when y increases. When y is high, agents also become more willing to report when τ falls.*

If y increases so that the public believes that managers as a whole are engaging in worse misconduct on average than previously believed, the reporting threshold x_I^* falls and aggregate reporting $\hat{r}(\theta)$ rises for every θ. The reason is that an increase in y makes agents believe the average experience is worse and hence more agents are likely to report.

The intuition is easiest to see when we are starting from an equilibrium where $y = \frac{c}{\omega-\beta\left(1-\frac{\gamma}{2}\right)}$ so that $x_I^* = y$. The marginal agent believes $E_I^*[r] = \frac{1}{2}$, and Figure OA.1 plots the density of her belief over r. Note that, from Equation 3, the CDF of her belief shifts "to the right" when y increases, as $\Psi_I(r)$ decreases in y for all $r \in (0,1)$ and any fixed value of x_i. This implies that, in response to an increase in y, the marginal agent will think that more agents will report because they are having worse experiences, making her willing to report and no longer indifferent.

At the higher value of y, the marginal agent in the new equilibrium must be indifferent at a less-bad experience; at the new indifference point, $x_I^* < y$. Figure OA.1 plots the marginal agent's belief over r in this new equilibrium. When $x_I^* < y$, the marginal agent believes many other agents are reporting, giving her the confidence to come forward despite a less-bad experience than the marginal agent in the previous equilibrium. Analogous logic applies if y falls: at the new indifference point where $x_I^* > y$, the marginal agent believes fewer other agents are reporting, so she must have a worse experience to come forward.

seems less directly relevant to the #MeToo movement.

How reporting responds to the reliability of the public signal τ depends on whether x_I^* is greater or less than y. Suppose $x_I^* < y$ and the marginal agent believes many other agents are reporting. In response to a lower τ, the marginal agent believes that she had an even lower-than-expected draw of x, and therefore that even more agents are reporting. She thus becomes more willing to report and is no longer indifferent, lowering the new equilibrium threshold. Analogous logic applies if $x_I^* > y$: when τ decreases, the marginal agent believes she had an even higher-than-expected x and that even fewer agents are reporting, making her less willing to report and raising the equilibrium threshold.

Corollary OA.2.1 is a robust first-order prediction irrespective of underlying parameters: Part 2(c) from Proposition OA.2 guarantees that the equilibrium is unique when $y \to \infty$ and that $x_I^* \to \underline{x}$, irrespective of other parameters. Part 2(c) also shows that Corollary OA.2.1 has first-order effects on the magnitudes of x_I^*, as changes in y can move the equilibrium threshold x_I^* across the entire range of $(\underline{x}, \overline{x})$.

Overall, Corollary OA.2.1 is consistent with more agents coming forward with accusations in the wake of #MeToo, which publicized several major incidents of misconduct and arguably raised y (and perhaps decreased τ) by raising public awareness of sexual misconduct. Our model suggests that heightened awareness led directly to more reporting even though: 1) experiences may have remained unchanged, and 2) there was no direct impact on agent payoffs $\pi(r, x)$. In particular, agents with "hidden x_i" who were previously not reporting due to a low-y environment may come forward in a higher-y (possibly lower-τ) environment purely from a change in beliefs about whether other agents are reporting.

To formally prove Proposition OA.2, proceed as follows. Suppose the prior belief of θ with density $p(\theta)$ is normally distributed with mean y and standard deviation τ, and experiences are $x = \theta + \sigma\epsilon$ where $\epsilon \sim N(0,1)$. Let $t = 1/\tau^2$ and $u = 1/\sigma^2$ denote the precisions of the prior and x. The posterior density $f(\theta \mid x)$ is a normal density with:

$$\begin{aligned} \text{mean } \lambda &= \frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2} = \frac{ty + ux}{t + u}, \\ \text{standard deviation } v &= \sqrt{\frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}} = \frac{1}{\sqrt{t+u}}, \\ \text{precision } h &= t + u = \frac{\sigma^2 + \tau^2}{\sigma^2 \tau^2}. \end{aligned}$$

Define $r(\theta, k)$ as in the proof of Proposition 1. Recall that we can consider $k \geq 0$ and that for such k there is a one-to-one map of r into θ: $r(\theta; k) = 1 - \Phi\left(\frac{k-\theta}{\sigma}\right)$ so $\theta(r; k) = k - \sigma\Phi^{-1}(1-r)$, where $\Phi(.)$ denotes the normal CDF; let $\phi(.)$ denote the normal PDF. The expected payoff gain of reporting for agent drawing x when other agents are playing threshold strategies around $k \geq 0$ when $x > 0$ equals:

$$\pi^*(x,k) = \int_{-\infty}^{\infty} f(\theta \mid x)\, \pi(r(\theta;k), x)\, d\theta$$

$$(16) \qquad = x\left(\omega - \beta + \gamma(\alpha+\beta)\int_{-\infty}^{\infty} f(\theta \mid x)\left(1 - \Phi\left(\frac{k-\theta}{\sigma}\right)\right) d\theta\right) - c.$$

For $x \leq 0$, the payoff gain equals $\pi^*(x,k) = x\omega - c < 0$.

LEMMA OA.3: *Any solution x^* to $\pi^*(x,x) = 0$ satisfies the implicit equation:*

$$x^* = \frac{c}{\omega - \beta + \gamma(\alpha+\beta)\Phi\left(\frac{y-x^*}{\kappa}\right)},$$

where we drop the I subscript on x_I^* for notational brevity.

PROOF:

We have $f(\theta \mid x) = \frac{1}{v}\phi\left(\frac{\theta-\lambda}{v}\right)$. Then Equation 16 becomes:

$$\pi^*(x,k) = x\left(\omega - \beta + \gamma(\alpha+\beta)\int_{-\infty}^{\infty} \frac{1}{v}\phi\left(\frac{\theta-\lambda}{v}\right)\left(1 - \Phi\left(\frac{k-\theta}{\sigma}\right)\right) d\theta\right) - c.$$

We know the following general relationship (Patel and Read, 1996, p.36):

$$\int_{-\infty}^{\infty} \Phi(a+bz)\,\phi(z)\,dx = \Phi\left(\frac{a}{\sqrt{1+b^2}}\right).$$

Letting $z = \frac{\theta-\lambda}{v}$ and $a + bz = \frac{k-\lambda}{\sigma} - \frac{v}{\sigma}z$, we have:

$$\int_{-\infty}^{\infty} \frac{1}{v}\phi\left(\frac{\theta-\lambda}{v}\right)\left(1-\Phi\left(\frac{k-\theta}{\sigma}\right)\right) d\theta = 1 - \int_{-\infty}^{\infty} \phi(z)\,\Phi\left(\frac{k-\lambda}{\sigma} - \frac{v}{\sigma}z\right) dz$$

$$= 1 - \Phi\left(\frac{\frac{k-\lambda}{\sigma}}{\sqrt{1+\frac{v^2}{\sigma^2}}}\right)$$

$$= \Phi\left(\frac{\lambda-k}{\sqrt{\sigma^2+v^2}}\right).$$

For any agent who draws x, the payoff gain is then:

$$(17) \qquad \pi^*(x,k) = x\left(\omega - \beta + \gamma(\alpha+\beta)\Phi\left(\frac{\lambda-k}{\sqrt{\sigma^2+v^2}}\right)\right) - c \text{ if } x > 0,$$

$$= x\omega - c \text{ if } x \leq 0.$$

For the marginal agent with $x = k$, we have $\lambda - k = \lambda - x = \frac{v^2}{\tau^2}(y - x)$. Then:

$$\Phi\left(\frac{\lambda - x}{\sqrt{\sigma^2 + v^2}}\right) = \Phi\left(\frac{v^2}{\tau^2}\frac{y - x}{\sqrt{\sigma^2 + v^2}}\right)$$
$$= \Phi\left(\frac{y - x}{\kappa}\right) \text{ for } \kappa \equiv \frac{\tau^2}{v^2}\sqrt{\sigma^2 + v^2}.$$

The payoff gain for the marginal agent (assuming $x > 0$, which we verify) then equals:

$$\pi^*(x, x) = 0 = x\left(\omega - \beta + \gamma(\alpha + \beta)\Phi\left(\frac{y - x}{\kappa}\right)\right) - c,$$

which gives us the implicit equation for x^*. Note that any solution must be positive, as required. ■

LEMMA OA.4: *There exists at least one symmetric threshold strategy equilibrium* $x_I^* \in (\underline{x}, \overline{x})$ *where agents report for* $x \geq x_I^*$ *and do not report for* $x < x_I^*$.

PROOF:

From Lemma OA.3, any potential equilibria must solve $G(x^*) = 0$, where:

$$G(x) = \pi^*(x, x) = x(\omega - \beta + \gamma E(x)(\alpha + \beta)) - c,$$
$$E(x) = \Phi\left(\frac{y - x}{\kappa}\right).$$

For notational brevity, we drop the I subscript in x_I^*.

First, we claim there exists a $x^* \in (\underline{x}, \overline{x})$ that is a solution to $G(x^*) = 0$ where $G(x^*)$ is increasing. Observe that $E(x)$ is continuous, which implies $G(x)$ is continuous. Notice that $G\left(\frac{c}{\omega - \beta}\right) > 0$, and $G\left(\frac{c}{\omega + \gamma\alpha - (1-\gamma)\beta}\right) < 0$, and $\frac{c}{\omega - \beta} > \frac{c}{\omega + \gamma\alpha - (1-\gamma)\beta}$. By the intermediate value theorem, there exists at least one solution $G(x^*) = 0$, where $G(x^*)$ is increasing.

Second, we show that any such solution x^* constitutes a symmetric threshold equilibrium. Recall from Equation 17 in Lemma OA.3 that the payoff from reporting when an agent's signal is $x > 0$, conditional on other players playing threshold strategies around k, equals:

$$\pi^*(x, k) = x\left(\omega - \beta + \gamma(\alpha + \beta)\Phi\left(\frac{\frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2} - k}{\sqrt{\sigma^2 + v^2}}\right)\right) - c.$$

This is a strictly increasing function in x. Since $G(x^*) = \pi^*(x^*, x^*) = 0$, we have $\pi^*(x, x^*) > 0$ for $x > x^*$ and $\pi^*(x, x^*) < 0$ for $x < x^*$, so that a threshold strategy around x^* is a best response to other players playing the same threshold strategy. If $x < 0 < x^*$, $\pi^*(x, x^*) < 0$. ■

Lemmas OA.5 and OA.6 substitute for Lemma 2 in the informed prior case.

LEMMA OA.5: *The following properties hold:*

[1] $\pi^*(x,k)$ is strictly increasing in x, weakly decreasing in k (strictly decreasing for $x > 0$), and continuous in both x and k. Furthermore, for any $k \geq 0$, $\pi^*(x,k)$ maps onto $\mathbb{R}$.

[2] Let ξ solve $\pi^*(\xi,\xi) = 0$. The sequence $\left\{\underline{\xi}^1, \underline{\xi}^2 ... \underline{\xi}^n ...\right\}$ defined as the solutions to the equations:

$$
\begin{aligned}
\pi^*\left(\underline{\xi}^1, 0\right) &= 0 \\
\pi^*\left(\underline{\xi}^2, \underline{\xi}^1\right) &= 0 \ldots \\
\ldots\ \pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^n\right) &= 0 \ldots
\end{aligned}
$$

is a well-defined increasing sequence, bounded from above by ξ and below by 0, with $\lim_{n\to\infty} \underline{\xi}^n = \underline{\xi}$, where $\underline{\xi}$ is the smallest solution to $\pi^*(\xi,\xi)$. Analogously, the sequence $\left\{\bar{\xi}^1, \bar{\xi}^2 ... \bar{\xi}^n ...\right\}$ defined as the solutions to:

$$
\begin{aligned}
\pi^*\left(\bar{\xi}^1, \infty\right) &= 0 \\
\pi^*\left(\bar{\xi}^2, \bar{\xi}^1\right) &= 0 \ldots \\
\ldots\ \pi^*\left(\bar{\xi}^{n+1}, \bar{\xi}^n\right) &= 0 \ldots
\end{aligned}
$$

is a well-defined decreasing sequence, bounded from below by $\xi > 0$, with $\lim_{n\to\infty} \bar{\xi}^n = \bar{\xi}$, where $\bar{\xi}$ is the largest solution to $\pi^*(\xi,\xi)$.

PROOF:

[1] These properties are evident from Equation 17 in Lemma OA.3.

[2] Property [1] implies that all $\underline{\xi}^n$ and $\bar{\xi}^n$ are well-defined. We know that not reporting is dominant for $x < \underline{x}$, so $\pi^*(x,0) < 0$ for all $x < \underline{x}$. But we also know that $\pi^*(x,0) > 0$ for all $x > \bar{x}$. Define $\underline{\xi}^0 \equiv 0$. By continuity in x, there exists at least one solution x with $\pi^*\left(x, \underline{\xi}^0\right) = 0$, where $x \in [\underline{x}, \bar{x}]$. Call $\underline{\xi}^1$ the smallest such solution. Note that $\bar{\xi}^1 > 0$. Furthermore, note that $\underline{\xi}^1 < \xi \in (\underline{x}, \bar{x})$: if not, then $0 = \pi^*\left(\underline{\xi}^1, 0\right) \geq \pi^*(\xi, 0) > \pi^*(\xi,\xi) = 0$, a contradiction.

To show that $\underline{\xi}^n$ is an increasing sequence, proceed by induction. Our starting point is to show that, because $\pi^*\left(\underline{\xi}^1, 0\right) = \pi^*\left(\underline{\xi}^2, \underline{\xi}^1\right) = 0$, we have $\underline{\xi}^1 < \underline{\xi}^2$. To see why, proceed by contradiction. Suppose $\underline{\xi}^1 \geq \underline{\xi}^2$. Then $\pi^*\left(\underline{\xi}^1, 0\right) \geq \pi^*\left(\underline{\xi}^2, 0\right)$ because π^* is increasing in x, but $\pi^*\left(\underline{\xi}^2, 0\right) > \pi^*\left(\underline{\xi}^2, \underline{\xi}^1\right)$ because π^* is decreasing in k. Thus, $\pi^*\left(\underline{\xi}^1, 0\right) > \pi^*\left(\underline{\xi}^2, \underline{\xi}^1\right)$, a contradiction. Note that $\underline{\xi}^2 < \xi$: if not, then $0 = \pi^*\left(\underline{\xi}^2, \underline{\xi}^1\right) \geq \pi^*\left(\xi, \underline{\xi}^1\right) > \pi^*(\xi,\xi) = 0$, a contradiction.

The inductive hypothesis is that $\underline{\xi}^{n-1} < \underline{\xi}^n$ with $\underline{\xi}^n < \xi$; we claim $\underline{\xi}^n < \underline{\xi}^{n+1}$ with $\underline{\xi}^{n+1} < \xi$. Proceed again by contradiction. By definition, $\pi^*\left(\underline{\xi}^n, \underline{\xi}^{n-1}\right) =$

$\pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^{n}\right)$. Suppose that $\underline{\xi}^{n} \geq \underline{\xi}^{n+1}$. Then $\pi^*\left(\underline{\xi}^{n}, \underline{\xi}^{n-1}\right) > \pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^{n-1}\right)$ because π^* is increasing in x, but $\pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^{n-1}\right) > \pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^{n}\right)$ because π^* is decreasing in k. Thus, $\pi^*\left(\underline{\xi}^{n}, \underline{\xi}^{n-1}\right) > \pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^{n}\right)$, a contradiction. Note that $\underline{\xi}^{n+1} < \xi$: if not, then $0 = \pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^{n}\right) \geq \pi^*\left(\xi, \underline{\xi}^{n}\right) > \pi^*\left(\xi, \xi\right) = 0$, a contradiction.

Because $\{\underline{\xi}^{n}\}$ is a bounded increasing sequence, there exists a $\underline{\xi}$ with $\lim_{n\to\infty} \underline{\xi}^{n} = \underline{\xi}$. Note that $\lim_{n\to\infty} \pi^*\left(\underline{\xi}^{n+1}, \underline{\xi}^{n}\right) = 0$ so by construction and continuity of π^*, we must have $\pi^*\left(\underline{\xi}, \underline{\xi}\right) = 0$ and that $\underline{\xi}$ is the smallest such solution to $\pi^*\left(\xi, \xi\right) = 0$.

An analogous argument works identically to show that $\left\{\bar{\xi}^{n}\right\}$ is a bounded decreasing sequence, that there exists a $\bar{\xi}$ with $\lim_{n\to\infty} \bar{\xi}^{n} = \bar{\xi}$, and that $\bar{\xi}$ is the largest such solution to $\pi^*\left(\xi, \xi\right) = 0$. ■

LEMMA OA.6: *Uniqueness of equilibrium (allowing for either strategy to be played at x^*):*

[a] *The equilibrium x^* is a unique threshold equilibrium if:*

$$\kappa > \frac{1}{\sqrt{2\pi}} \frac{c\gamma\left(\alpha+\beta\right)}{\left(\omega-\beta\right)^2}.$$

[b] *Whenever $y = \frac{c}{\omega-\beta+\frac{1}{2}\gamma(\alpha+\beta)}$, a sufficient condition for non-unique equilibria is:*

$$\kappa < \frac{1}{\sqrt{2\pi}} \frac{\gamma\left(\alpha+\beta\right)c}{\left(\omega-\beta+\frac{1}{2}\gamma\left(\alpha+\beta\right)\right)^2}.$$

[c] *For $y \to \infty$, there is a unique threshold equilibrium x^* with $x^* \to \underline{x}$. For $y \to -\infty$, there is a unique threshold equilibrium x^* with $x^* \to \bar{x}$.*

[d] *If there is a unique equilibrium in threshold strategies, then the equilibrium strategy is the only strategy that satisfies the iterated deletion of strictly dominated strategies, and in particular, the unique threshold strategy equilibrium is the globally unique equilibrium.*

PROOF:

Because $G\left(\underline{x}\right) < G\left(\bar{x}\right)$ with at least one solution in between where G is increasing from Lemma OA.4, and because G is differentiable for all x, a necessary and sufficient condition for uniqueness is that $G'\left(x^*\right) > 0$ for all $x^* \in \left(\underline{x}, \bar{x}\right)$ such that $G\left(x^*\right) = 0$. We have

$$\begin{aligned} G'\left(x\right) &= \gamma\left(\alpha+\beta\right)xE'\left(x\right) + \left(\omega-\beta+\gamma\left(\alpha+\beta\right)E\left(x\right)\right) \\ &= \omega-\beta+\gamma\left(\alpha+\beta\right)\left(xE'\left(x\right)+E\left(x\right)\right), \end{aligned}$$

and also:

$$G'\left(x\right) = \gamma\left(\alpha+\beta\right)xE'\left(x\right) + \frac{c}{x}$$

Substituting in $E'(x) = -\frac{1}{\kappa}\phi\left(\frac{y-x}{\kappa}\right)$ into Equation 18, a necessary and sufficient condition for uniqueness is, for all solutions x^*:

$$\gamma(\alpha+\beta)(x^*)^2\phi\left(\frac{y-x^*}{\kappa}\right)\frac{1}{\kappa} \quad < \quad c.$$

Recall from Lemmas OA.3 and OA.4 that any solution to $G(x)$ constitutes a symmetric threshold equilibrium and that there exists at least one such equilibrium. We now provide conditions under which such an equilibrium is unique or not unique.

[a] Using the fact that $\phi(z) < \frac{1}{\sqrt{2\pi}}$ and $x^* < \bar{x} = \frac{c}{\omega-\beta}$, a sufficient condition for uniqueness from Equation 18 is:

$$\gamma(\alpha+\beta)\frac{1}{\sqrt{2\pi}}\frac{c}{(\omega-\beta)^2}\frac{1}{\kappa} < 1 \quad \Leftrightarrow \quad \frac{1}{\sqrt{2\pi}}\frac{c\gamma(\alpha+\beta)}{(\omega-\beta)^2} < \kappa.$$

[b] For $y = \frac{c}{\omega-\beta+\frac{1}{2}\gamma(\alpha+\beta)}$, note that $x^* = y$, $E(y) = 1/2$, $E'(y) = -\frac{1}{\kappa}\frac{1}{\sqrt{2\pi}}$, and we have the following expression for $G'(y)$ from Equation 18:

$$G'(y) \quad = \quad \gamma(\alpha+\beta)\,yE'(y) + (\omega-\beta+\gamma(\alpha+\beta)E(y))$$

But $y = \frac{c}{\omega-\beta+\frac{1}{2}\gamma(\alpha+\beta)}$, so $G'(y) < 0$ if and only if $\frac{1}{\sqrt{2\pi}}\frac{\gamma(\alpha+\beta)c}{\left(\omega-\beta+\frac{1}{2}\gamma(\alpha+\beta)\right)^2} > \kappa$.

[c] For $y \to \infty$, $\lim_{y\to\infty} E(x) = 1$, so $\lim_{y\to\infty} G(x) = x(\omega-\beta+\gamma(\alpha+\beta)) - c$. By continuity, any solutions x^* are arbitrarily close to $\underline{x}$.

For $y \to -\infty$, $\lim_{y\to-\infty} E(x) = 0$, so $\lim_{y\to-\infty} G(x) = x(\omega-\beta) - c$. By continuity, any solutions x^* are arbitrarily close to $\overline{x}$.

To show uniqueness in both cases, observe from Equation 18 that:

$$\lim_{y\to\infty} G'(x) \quad = \quad \omega-\beta+\gamma(\alpha+\beta) > 0,$$
$$\lim_{y\to-\infty} G'(x) \quad = \quad \omega-\beta > 0.$$

Continuity of $G'(x)$ in y and x implies $G'(x) > 0$ for arbitrarily large (positive or negative) y, in particular $G'(x^*) > 0$ for any solution x^*.

[d] If $G(x)$ has a unique solution x^*, then from Lemma OA.5 we have $x^* = \xi = \underline{\xi} = \overline{\xi}$. By Lemma 3, the only strategy which survives the iterated deletion of dominated strategies is the x^*-threshold strategy. This implies that the x^*-threshold equilibrium is the globally unique equilibrium. ■

LEMMA OA.7: *The marginal agent has beliefs over r summarized by the cumulative distribution function* $\Phi\left(\frac{t}{\sqrt{t+u}}(x-y) + \frac{\sqrt{t+u}}{\sqrt{u}}\Phi^{-1}(r)\right)$, *with expectation* $E_I^*[r] = \Phi\left(\frac{y-x^*}{\kappa}\right)$.

PROOF:

Given an agent's signal x, what is her assessment of the cumulative distribution function of r when others are playing cutoff strategies around k, $\Psi(\tilde{r}; x, k)$? We can follow the same logic as in Lemma 2. For any $\tilde{r}$, the probability that $r < \tilde{r}$ equals the probability that $\theta < k - \sigma F^{-1}(1 - \tilde{r})$. In words, the probability $\Psi(\tilde{r}; x, k) \equiv \Pr(r < \tilde{r} \mid x)$ that the true proportion of players reporting is less than $\tilde{r}$ equals the probability that the true θ satisfies $r(\theta; k) = 1 - F\left(\frac{k-\theta}{\sigma}\right) < \tilde{r}$, or equivalently that θ is such that fewer than $\tilde{r}$ players observe a signal greater than k; in turn, this equals the probability that the true θ is less than $k - \sigma F^{-1}(1 - \tilde{r})$, integrated against the conditional density $f(\theta \mid x)$. With some abuse of notation,

$$\begin{aligned}
\Psi(r; x, k) &= \int_{-\infty}^{k-\sigma\Phi^{-1}(1-r)} f(\theta \mid x)\, d\theta \\
&= \int_{-\infty}^{k-\sigma\Phi^{-1}(1-r)} \frac{1}{v}\phi\left(\frac{\theta-\lambda,}{v}\right) d\theta \\
&= \int_{-\infty}^{z=\frac{k-\lambda}{v}-\frac{\sigma}{v}\Phi^{-1}(1-r)} \phi(z)\, dz \text{ for } z = \frac{\theta-\lambda}{v},\ dz = \frac{1}{v}d\theta \\
&= \Phi\left(\frac{k-\lambda}{v} - \frac{\sigma}{v}\Phi^{-1}(1-r)\right).
\end{aligned}$$

For the marginal agent with $x = k$,

$$\frac{k-\lambda}{v} = \frac{(t+u)x - ty - ux}{\sqrt{t+u}} = \frac{t}{\sqrt{t+u}}(x-y).$$

Combining this insight with $\sigma/v = \sqrt{t+u}/\sqrt{u}$ yields:

$$\Psi(r; x, x) = \Phi\left(\frac{t}{\sqrt{t+u}}(x-y) + \frac{\sqrt{t+u}}{\sqrt{u}}\Phi^{-1}(r)\right),$$

where we use $\Phi^{-1}(r) = -\Phi^{-1}(1-r)$ in the derivation.

Let $\psi(r; x, x)$ denote the probability density function associated with $\Psi(r; x, x)$. Because there is a one-to-one mapping of r and θ, we can re-write $\pi^*(x, k)$ as $\pi^*(x, k) = \int_0^1 \psi(r; x, k)\, \pi(r, x)\, dr$, so for the marginal agent (assuming $x > 0$, which we verify):

$$\pi^*(x, x) = \int_0^1 \psi(r; x, x)\left[x(\omega - \beta + \gamma r(\alpha + \beta)) - c\right] dr.$$

Because the marginal agent must be indifferent between reporting and not reporting, the equilibrium condition is then $\pi^*(x, x) = 0$. This gives the implicit function:

$$x^* = \frac{c}{\omega - \beta + \gamma E_I^* [r] (\alpha + \beta)},$$

where $E_I^*[r] = \int_0^1 r \, \psi(r; x, x) \, dr$. But then by Lemma OA.3, $E_I^*[r] = \Phi\left(\frac{y - x^*}{\kappa}\right)$. ■

PROOF OF PROPOSITION OA.2:

Part 1 follows from Lemma OA.4. Part 2 follows from Lemma OA.6. Part 3 follows from Lemma OA.7. ■

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