Online Appendix
Crime, Broken Families, and Punishment

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A Some ethnographic studies

The narrative story of "Louisa and Robert" in Donald Braman's monography "Doing Time on the Outside" (pp. 169-175 in Braman, 2004)[A1], provides a good example illustrating the toll incarceration exerts in terms of isolation and reduced access to social resources for family support and parenting that the outside member (mostly a woman) faces in such situations. Louisa and Robert are a young couple in their mid-thirties with a son called Jimmy. Like many working-poor families, they are deeply religious and are well-integrated into the local church community. When Jimmy was three, Robert started using crack and was soon addicted, leaving Louisa and his son for the streets. After being incarcerated, completing a drug treatment program, and a period of reconciliation, the family was reunited. Although he tried to "keep on the straight and narrow", and working full-time at an entry-level job, Robert was arrested for a traffic violation, and put back into prison for a robbery he had committed during his addiction. For Louisa, her husband's reincarceration was particularly hard. In fear of the social stigma associated with her husband's arrest, she began to avoid friends and family, not wanting to talk about Robert's incarceration. She stopped discussing with her co-workers and church members, and, more importantly, with other family members as she did not want to open herself up to discussions about Robert. Importantly, Louisa's withdrawal from friends and family network (and the severance of crucial social ties in a moment of strong necessity) made it more difficult to cope with her increased parenting duties with her son Jimmy, leaving him in a state of higher economic and social vulnerability.

Alice Goffman's celebrated ethnographic study *On the Run* (Goffman, 2014)[A2] of a poor predominantly black neighborhood in Philadelphia is another example of the crucial connection between policing, family structure, and social reproduction of crime in the neighborhood. While a college and graduate student, Alice Goffman moved for six years (2002-2008) to the 6th street in Philadelphia (a neighborhood consisting essentially of a commercial strip and five residential blocks) to study, analyze, and chronicle how people lived there and how they interacted with the police and the legal system. She studied the life of Tim (youngest boy),

[A1]Braman, D. (2004), *Doing Time on the Outside: Incarceration and Family Life in Urban America*, Ann Arbor: University of Michigan Press.

[A2]Goffman, A. (2014), *On the Run. Fugitive Life in an American City*, Chicago: The University of Chicago Press.

Reggie (middle boy) and Chuck (oldest boy),[A3] three brothers, and their friends and neighbors, in particular, Mike. Tim, Reggie and Chuck had the same mother, Miss Linda, but not the same father. Miss Linda was a heavy crack user. The family had very little government assistance and Miss Linda never could hold a job for more than a few months at a time. When Tim was a baby, his dad had moved down to South Carolina and married a woman there; he did not keep in touch. Reggie's father was a man of no consequence or merit, in prison on long bids and then out for stints of drunken roberries. Reggie said he wouldn't recognize him in the street. By contrast, Chuck's father came around during his early years, but then disappears. If we now consider Mike, Chuck's friend, his father was a heavy crack user, and was in and out of jail during Mike's early years. By the time Mike was ten, Miss Regina (Mike's mother) told his father to stop coming around.

The family Alice Goffman lived with is a good example of a broken family in which the mother has no steady income and the father is either absent for the three boys (something that our set-up will incorporate) or is a bad role model for Chuck.

Importantly, Goffman's observations suggest that the boys reproduce the same family structure as their parents. None of them stayed with their wife once they had kids. One aspect, which is recurrent in the book, is the fact that the police is always chasing these boys. Indeed, in Philadelphia, between 1960 and 2000, the number of police officers increased by 69 percent, from 2.76 officers for every 1,000 citizens to 4.66 officers (Federal Bureau of Investigation, Uniform Crime Reports). Around 6th street, police helicopters circle overhead, police cameras monitor passerby, and police routinely stop, search, and arrest people in the streets.

Basically, these boys and most of the people living on the 6th street had some issues with the criminal justice. In the period Goffman lived in the sixth street, Chuck was in county jail awaiting trial. A month after he turned fifteen, Reggie was tested positive for marijuana. Alex, a friend of Chuck, was completing two years of parole after serving a year upstate for drugs. A cousin was out of bail and a neighbor was living under house arrest and another friend who was homeless and sleeping in his car had a warrant out for unpaid court fees. Between the ages of twenty-two and twenty-seven, Mike spent about three and half years in jail or prison. Going through his freshman yearbook years, when he turn twenty-two, Chuck identified roughly half the boys in the ninth-grade as currently sitting in jails or prisons.

[A3] All names are pseudonyms but the characters are real.

The main conclusion of Goffman (2014) is that, by increasing policing and incarceration in these poor (mostly black) neighborhoods, the community life is transformed in ways that are deep and enduring, not only for the young men who are their targets but also for their family member, partners and neighbors. In particular, even when these young men tried to work, they usually did not keep their jobs for long and then reverted to illegal activities. For example, a few weeks after his daughter was born, Mike lost his job at a pharmaceutical warehouse because of complications with his daughter's birth, which had caused him to miss work too many days in a row. Then, he persuaded a friend from another neighborhood to give him some crack to sell on credit. Moreover, black men with criminal records are so discriminated against in the labor market that the jobs for which they are legally permitted to apply are quite difficult to obtain (Pager, 2007)[A4]. Goffman (2014) thus concluded that increasing policing and incarceration may be counterproductive because of its negative impact on communities and because it forces young men to be on the run and therefore commit more crime. Even though we do not have an explicit racial analysis in our model, this is what we want to show by highlighting the possible negative impact of incarceration on crime because of its detrimental effect on the structure of the community and the family.

[A4]Pager, D. (2007), *Marked: Race, Crime, and Finding Work in an Era of Mass Incarceration*, Chicago: University of Chicago Press.

B Additional results

We provide here additional results that are mentioned in the main text.[A5]

B.1 Comparative statics results in the benchmark model (Section 3.3)

The following proposition provides some comparative statics results of the interior stable equilibrium $q^* = \overline{q}$, and of $\underline{q}$ that characterize the lower bound of the basin of attraction of $q^* = \overline{q}$.

Proposition B1 *Suppose that Assumption 1 holds and that $p \in [0, \widehat{p}_1] \cup [\widehat{p}_2, \overline{p}_2]$. Then, an increase in w or σ or a decrease in β, c^S, or c^B increases $\overline{q}$ (ie. the high steady state proportion of honest individuals in the population) and decreases $\underline{q}$ (ie. increase the size of the basin of attraction of the high steady state proportion of honest individuals in the population).*

B.2 Education versus incarceration (Section 3.4)

Let us start with the following definition:

Definition B.1

*(i) An incarceration policy is **repressive** if it minimizes short-run crime, i.e., $p = R$.*

*(ii) An incarceration policy is **permissive** if it does not minimize short-run crime, i.e., $p < R$.*

We have the following assumption:

Assumption B1

(i) $\quad -(\beta + \sigma) + w'(0) < 0$

(ii) $\quad \Delta^B > 1$ *and* $\left[\Delta^B - \frac{(-K+\beta)^2}{4(\beta+\sigma)}\left(\Delta^B - \Delta^S\right)\right] < 1.$

[A5] All the proofs of the results in this Section can be found in Section B.7.

Part (i) of Assumption B1 ensures that the policy $p = R$, where all the public budget is spent on repression, minimizes short-run crime. This allows us to concentrate on the trade-off between the short-run and long-run effects of repression on crime in what follows. Part (ii) of Assumption B1 is the counterpart of Assumption 1 when incarceration is costly. We consider the situation where the parent's socialization effort is insufficiently effective to maintain the culture of honesty when the rate of single-parent families is high.

Proposition B2 *Suppose that the public budget satisfies $R < \frac{\beta - K}{2(\beta+\sigma)}$ and that Assumption B1 holds. Then, there exist two thresholds $q^1_{\min} \in (0,1)$ and $q^2_{\min} \in (0,1)$ such that*

(i) When the initial proportion of honest individuals is low, i.e., $q_0 < q^1_{\min}$, incarceration policies have no effect on long-run honesty.

*(ii) When the initial proportion of honest individuals is sufficiently high, i.e., $q_0 > q^2_{\min}$, a **repressive policy** $p = R$ has a negative impact on long-run honesty q^*.*

When q_0, the initial proportion of honest individuals is sufficiently low and bad transmission by role models (peer effects) is so strong that whatever the incarceration policy, honesty always disappears in the long run, i.e., $q^* = 0$.

When the initial proportion of honest individuals is sufficiently high, incarceration policies matter for the long-run level of honesty q^*. As before, incarceration policies have a positive impact on honesty by decreasing crime at time t and a negative one by increasing the proportion of incarcerated parents. Compared with the case with no budget constraint, however, the positive impact of crime is reduced since it entails an additional opportunity cost, as it requires a reduction in spending on education. When public resources R are limited (i.e., $R < \frac{\beta - K}{2(\beta+\sigma)}$), whatever the incarceration rate, the crime rate is high; hence, any increase in p has a strong negative impact on the proportion of single-parent families. In that case, the *family disorganization effect* exceeds the *deterrence effect* and incarceration negatively impacts on long-run honesty.

Proposition B3 *Assume that Assumption B1 holds and that*

$$\frac{\beta - K}{2} < \frac{\Delta^B + \sqrt{\Delta^B(\Delta^B - 1)}}{2\Delta^B} K + w\left(\frac{\beta - K}{2(\beta + \sigma)}\right). \tag{B1}$$

Consider the two thresholds $q^1_{\min} \in (0,1)$ and $q^2_{\min} \in (0,1)$ in Proposition B2. Then, there exist $\tilde{R}$, such that, $\forall R \in [\tilde{R}, \frac{\beta - K}{2(\beta+\sigma)}]$:

(*i*) *When the initial proportion of honest individuals is low, i.e.,* $q_0 < q^1_{\min}$ *, a* ***repressive policy*** $p = R$ *is efficient.*

(*ii*) *When the initial proportion of honest individuals is high, i.e.,* $q_0 > q^2_{\min}$ *,* ***permissive policies*** $p \leq \overline{p} < R$ *are efficient.*

When the initial proportion of honest individuals is low, repressive policies are efficient because incarceration does not affect long-run honesty q^* (see Proposition B2). Indeed, whatever the family structure, honesty is initially so rare in the population that it does not persist in the long run. This is equivalent to the Beckerian framework. As explained above, when honesty does not matter, repressive policies are efficient because only the deterrence effect is at work.

When the initial proportion of honest individuals is sufficiently large, however, incarceration affects long-run honesty. In particular, when the budget R takes intermediate values (i.e., $R \in [\tilde{R}, \frac{\beta-K}{2(\beta+\sigma)}]$), incarceration policies may have deep (negative) consequences on community culture because both the crime rate and the incarceration rate are high, which maximizes family disorganization. When condition (B1) holds, the long-run crime rate when incarceration is minimal (i.e., $p = 0$) is low compared with the long-run crime rate when incarceration is maximal (i.e., $p_{\max} = \frac{-K+\beta}{2(\beta+\sigma)}$). Indeed, condition (B1) imposes that the punishment cost of crime when incarceration is maximal[A6] is lower than the expected cost of crime under minimal repression $p = 0$.[A7] As a result, when condition (B1) holds, the long-run expected incentives to commit crime are higher under maximal repression than under minimal repression and, therefore, any permissive policy with $p < R$ is efficient.[A8]

[A6]This is the left-hand side of (B1) with $q^* = 0$, which is equal to $p_{\max}(\beta + \sigma) = \frac{\beta-K}{2(\beta+\sigma)}(\beta + \sigma) = \frac{\beta-K}{2}$.

[A7]This is when there is a positive long-run proportion $q^* = \overline{q}(0) = \frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$ of honest individuals in the population, which is the right-hand side of (B1)), that is $\overline{q}(0)K + w\left(\frac{\beta-K}{2(\beta+\sigma)}\right)$. This encompasses a pecuniary part, $w\left(\frac{-K+\beta}{2(\beta+\sigma)}\right)$, which is the opportunity wage cost in the legal market when the maximal budget $\frac{\beta-K}{2(\beta+\sigma)}$ is spent on education, and a non-pecuniary moral part, $\overline{q}(0)K$, which is the expected psychological cost from engaging in criminal activities.

[A8]Condition (B1) holds when socialization rates are low (i.e., Δ^B is large), the psychological cost of crime K is high, the cost of punishment is low, and the proceeds from crime are low.

B.3 Subsidy to single-parent families

Incarceration policies are clearly not the only way in which to reduce crime. In this subsection, we examine a policy that provides financial help in the form of a subsidy to single-parent families.

We assume that the government can grant a subsidy δ to single-mother families, which reduces mothers' socialization cost to $c^S(1-\delta)\frac{(\tau_t)^2}{2}$. In this new framework, the optimal socialization effort of a family of type S is given by

$$\tau_t^S(\delta) = 2q_t(1-q_t)\frac{\Delta^S}{(1-\delta)}.$$

Denote by $C = C(q_t; p, \delta)$ the crime rate at time t as a function of q_t, p but also as a function of the subsidy δ. The government has a budget of R, which it trades off between subsidies for single-mother families, amounting to $pC\delta c^S\frac{\left(\tau_t^S(\delta)\right)^2}{2}$, and spending on education g. The budget constraint can now be written as

$$g(\delta, q_t, C) = R - pC\delta c^S\frac{\left[(\tau_t^S(\delta)\right]^2}{2} = R - 2pC\delta c^S\left[q_t(1-q_t)\frac{\Delta^S}{(1-\delta)}\right]^2. \tag{B2}$$

Suppose that $\delta \in [0, 1-\frac{c^B}{c^S}]$; hence, the maximal subsidy granted by the government is such that the socialization cost of single-mother families is equal to the socialization cost of biparental families. The proportions of honest and dishonest individuals who engage in criminal activities are now given by

$$\theta^h = \max\{\beta - K - p(\beta+\sigma) - w(g(\delta, q_t, C)), 0\}$$
$$\theta^d = \max\{\beta - p(\beta+\sigma) - w(g(\delta, q_t, C)), 0\},$$

and the crime rate $C(q_t; p, \delta)$ is implicitly determined by the following equation:

$$C = -q_tK + \beta - p(\beta+\sigma) - w\left(R - pC\frac{\delta}{(1-\delta)^2}c^S\left[q_t(1-q_t)\Delta^S\right]^2\right). \tag{B3}$$

Definition B.2 *A subsidy policy δ is efficient if it minimizes long-run crime.*

Formally, an efficient policy δ solves

$$\min_\delta C(q^*(p); p, \delta). \tag{B4}$$

Again, we cannot solve this program since we do not know the form of the function $q^*(p)$. Instead, we define two cases: the *no-subsidy policy* (i.e., $\delta = 0$) and the *subsidy policy* (i.e., $\delta > 0$). In particular, for the positive subsidy case, we focus on $\delta = 1 - \frac{c^B}{c^S}$ (because this is convenient for analytical tractability). Whenever the crime rate at $\delta = 1 - \frac{c^B}{c^S}$ is lower than that at $\delta = 0$, we deduce that

$$\text{argmin}_\delta\, C(q^*(p); p, \delta) \neq 0,$$

suggesting that the subsidy policy is efficient.

Assumption B2 *Suppose that:*

(*i*) $-K + \beta - p(\beta + \sigma) - w(R) > 0$.

(*ii*) $p\frac{(c^S - c^B)}{c^B}\frac{1}{16}\Delta^2 w'(0) < 1$.

(*iii*) $\Delta^B - \frac{(\beta - K - w(1 - \frac{c^B}{c^S}))^2}{4(\beta+\sigma)}(\Delta^B - \Delta^S) - 1 < 1$.

If R and p are such that part (*i*) of Assumption B2 does not hold, then the repression policy removes all incentives to engage in criminal activities. In that case, the subsidy case is inefficient because whatever the long-run honesty, the crime rate is equal to zero. Part (*i*) excludes this trivial case. Moreover, parts (*i*) and (*ii*) together ensure the uniqueness of the crime rate implicitly given by (B3). Finally, part (*iii*) of Assumption B2 is the counterpart of Assumption 1. By considering such situations when social disorganizing forces are strong, if p takes intermediate values (i.e., the proportion of single-parent families is high), honesty cannot be maintained whatever the initial proportion of honest agents.

Proposition B4 *Suppose that Assumption B2 holds and that*

$$R > \frac{\beta^2}{4(\beta + \sigma)}(c^S - c^B). \tag{B5}$$

(*i*) *When the initial proportion of honest individuals is low, i.e.,* $q_0 < \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B - 1)}}{2\Delta^B}$, *subsidy policies have no effect on long-run honesty.*

(ii) When the initial proportion of honest individuals is high, i.e., $q_0 > \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$*, the subsidy policy* ($\delta > 0$) *maximizes long-run honesty.*

When R satisfies condition (B5), the government budget is sufficiently large to provide a subsidy that cancels out the negative impact of family disruption on the transmission of honesty. In other words, in that case, monoparental and biparental families both have the same cost of transmitting the honesty trait. As a result, when part (i) of Assumption B2 holds, that is $-K+\beta-p(\beta+\sigma)-w(R) > 0$, meaning that the crime rate is positive whatever the value of the subsidy and strong subsidies can be implemented to ensure that (B5) holds, the incentives to commit crime are low. Therefore, the proportion of single-parent families is low and the socialization cost of single-parent families c^S is low, and thus the positive effect on socialization outweighs the negative impact on crime.[A9] Therefore, if the initial proportion of honest agents is sufficiently high (so that bad peer effects are low), a sufficiently strong subsidy has a positive impact on long-run honesty.

Proposition B5 *Suppose that Assumption B2 holds,* $p \in]\hat{p}^1(R), \min\{\hat{p}^2(R), 1\}[$*, and*

$$R \in \left] \frac{\beta^2(c^S - c^B)}{4(\beta+\sigma)}, w^{-1}\left(K\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}\right)\right[. \tag{B6}$$

Then,

(i) If $q_0 < \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$*, the subsidy policy has no impact on long-term crime.*

(ii) If $q_0 > \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$*, the subsidy case is efficient, i.e.,* $\delta > 0$.

Let us summarize Propositions B4 and B5. When the proportion of honest agents is initially low (i.e., society is culturally disorganized), transmission by bad role models is so strong that whatever the effort of parents and, therefore, whatever the subsidy, the culture of honesty cannot expand. Since long-run honesty converges to zero, owing to cultural complementarity, the socialization efforts of both types of parents decrease to zero. This fact

[A9]Indeed, subsidies affect long-run honesty in two ways. On the one hand, they have a positive impact on the dynamics of the honesty trait by increasing the socialization efforts of single-parent families. On the other hand, they have a negative short-run (static) impact by increasing incentives to commit crime (i.e., an opportunity cost in terms of education).

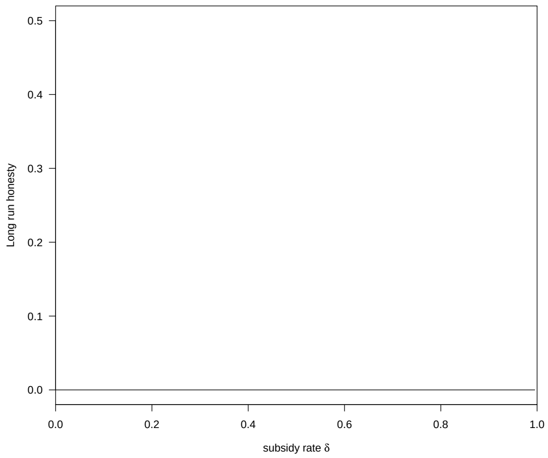
implies that spending on the subsidy policy is zero (since this depends on the socialization effort; see (B2)). As a result, the subsidy has no impact on long-term crime. Hence, in this case, a subsidy policy should not be implemented.

When the initial proportion of honest individuals is sufficiently high, socialization by parents affects long-run honesty and thus subsidies matter. In particular, when p takes intermediate values (i.e., family disorganization is maximal), some subsidies have deep (positive) consequences on community culture. Indeed, without any policy, socialization rates of honesty are too low to overcome bad peer transmission and the honesty trait disappears in the long run. When R is sufficiently high (see (B6)), although family disorganization is high, some subsidies have a sufficiently positive impact on socialization rates to overcome bad peer transmission and thus allow honesty to persist in the long run. The subsidy reduces the negative impact of family disorganization on honesty and long-run crime. Finally, whenever R is not too high (see (B6)), the opportunity cost of crime is high, and thus the strong positive impact on long-run honesty outweighs the negative short-run impact on crime (in the short run, raising the subsidy increases crime because it decreases the legal wage).

Again, to provide more general results on the efficient subsidy value, namely the subsidy that solves (B4), we run some numerical simulations. Figure A1 displays long-run honesty and the crime rate as a function of the subsidy rate δ for an initially low proportion of honest agents (i.e., $q_0 = 0.1$). Figure A2 depicts long-run honesty and the crime rate as a function of the subsidy rate δ for an initially high proportion of honest agents (i.e., $q_0 = 0.6$).

Figure A1 shows that when q_0 is low, a subsidy policy has no impact on long-run crime. This is because independent of the value of the subsidy, long-run honesty will converge to zero (see Proposition B4). In this case, the socialization efforts of both types of parents are zero and so is the spending on the subsidy policy. Therefore, this policy affects neither community culture nor the opportunity cost of committing crime (through the decrease in legal income). In other words, it is neutral for long-run crime.

By contrast, when q_0 is sufficiently high (Figure A2), the effect of the subsidy δ on long-run crime is negative. Above a certain threshold ($\delta = 0.8$ in the figure), the effect on long-run honesty starts to be different from zero and this leads to a sharp increase in honesty. As a result, long-run crime also strongly decreases when δ is above the threshold value of 0.8 and reaches its lower value when $\delta = 1$, i.e., when the cost of the socialization of single-mother families is totally subsidized and thus equal to that of biparental families. This is because the short-run effect of the subsidy, which increases crime through a reduction in the legal wage, is outweighed by the long-run effect of the subsidy, which reduces crime through an increase in honesty.



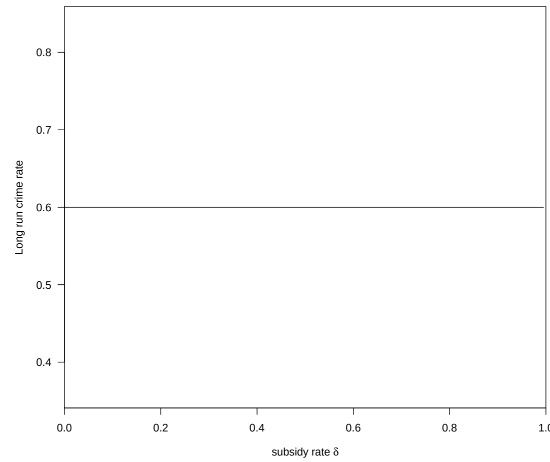


Figure A1: Long-run honesty (left) and the crime rate (right) as a function of the subsidy rate δ for $q_0 = 0.1$. We set $w(g) = ag/(1+g)$ and the parameters such that $a = 0,02$ $\beta = 0.07$, $K = 1/7$, $\Delta = 0.01$, $c^B = 0.01$, $c^S = 2$, $\sigma = 0.05$, $p = 0.15$, and $R = 0.7$.

B.4 "Good" versus "bad" fathers as role models (Section 3.5)

In this section, we modify the socialization technology to include the mechanism of bad role models. Indeed, suppose that a child who is born in a bi-parental family *now* mimics his father's trait with some probability γ. That is, a child who is born with an honest (resp. dishonest) father who is still at home becomes honest (resp. dishonest) with probability γ. If the child does not imitate the father (with probability $1-\gamma$), then he is subject to his parental socialization effort and peer effects within the society just as in the baseline model.[A10] Denote by $P_t^{hB,h}$ the probability that a child born in a bi-parental family with

[A10] Another modeling strategy would have been to assume three different socialization costs:

$$c^{Bh} < c^S < c^{Bd}$$

where c^{Bh} (resp. c^{Bd}) is the cost incurred by bi-parental families with an honest (resp. dishonest) father. The results would be qualitatively the same.

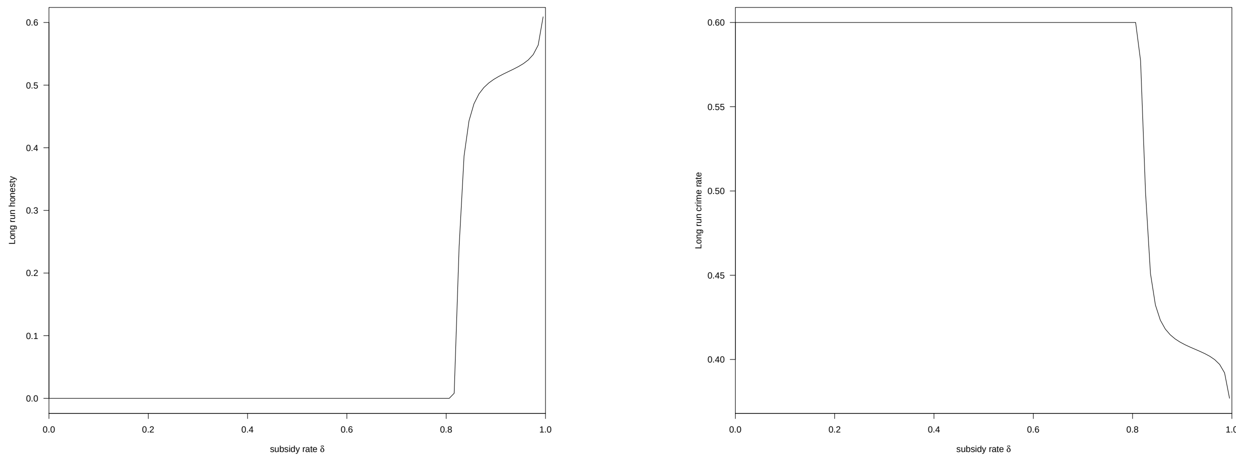


Figure A2: Long-run honesty (left) and the crime rate (right) as a function of the subsidy rate δ for $q_0 = 0.6$. We set $w(g) = ag/(1+g)$ and the parameters such that $a = 0,02$ $\beta = 0.07$, $K = 1/7$, $\Delta = 0.01$, $c^B = 0.01$, $c^S = 2$, $\sigma = 0.05$, $p = 0.15$, and $R = 0.7$.

an *honest* father adopts the trait h and by $P_t^{dB,h}$ the probability that a child born in a bi-parental family with a *dishonest* father adopts the trait h. We have:

$$P_t^{hB,h} = \gamma + (1-\gamma) q_t \left[2\tau^B(1-q_t) + q_t\right]$$
$$P_t^{dB,h} = (1-\gamma) q_t \left[2\tau^B(1-q_t) + q_t\right],$$

We see that, with probability γ, the child of an honest father will become honest ("good" role model *inside the family*) while, with probability $1-\gamma$, he will be exposed to his parents' socialization effort and role models *outside the family*.

To understand how we model the fact that a "bad" father can be a negative role model, consider $P_t^{dB,d}$, the probability that a child born in a bi-parental family with a dishonest father adopts the dishonest trait d. We have:

$$P_t^{dB,d} = 1 - P_t^{dB,h} = \gamma + (1-\gamma)\left[\tau^B(1-q_t)^2 + (1-\tau^B)(1-q_t) + (1-\tau^B)q_t(1-q_t)\right]$$

Indeed, with probability γ, a child whose father is dishonest becomes dishonest ("bad" role model *inside the family*). If this does not work (with probability $1-\gamma$), then we go back to the benchmark model where the child becomes dishonest if parent's honesty socialization succeeds but he meets "twice" a dishonest peer, or parent's socialization did not succeed and he meets a dishonest peer or parent's socialization did not succeed and he meets first an honest peer and then a dishonest peer.

Observe that, in this model, the "bad" father has a negative influence on his child (with probability γ) but, still, because of the mother, the household puts effort in order for the child to become honest. Observe also that the probability to adopt trait h when born in a *single-parent family* is the same as in the baseline model and given by (5) since, in that case, there is no more "bad" or "good" fathers since they are in prison. Of course, when $\gamma = 0$, we are back to our benchmark model where these quantities are given by (5).

The optimal socialization effort of a bi-parental family τ^B is now given by

$$\tau^B = 2(1-\gamma)q_t(1-q_t)\Delta^B,$$

instead of (7). The fraction of honest agents at time $t+1$ is then equal to:

$$q_{t+1} = \underbrace{q_t\left[1-p\theta^h(p)\right]}_{\text{non-incarcerated honest fathers}} P_t^{hB,h} + \underbrace{(1-q_t)\left(1-p\theta^d(p)\right)}_{\text{non-incarcerated dishonest fathers}} P_t^{hB,d} + \underbrace{pC(q_t,p)}_{\text{single-parent families}} P_t^{hS} \tag{B7}$$

In the following, we assume that $\beta - w < K$, which implies that $\theta^h = 0$, $\forall p \in [0, \bar{p}_2]$. This means that the psychological cost incurred by honest agents when they commit crime is so high that whatever the probability of being arrested they never engage in criminal activities. With this assumption, we focus on situations in which a father's imprisonment necessarily reduces a child's exposure to a "bad" role model and therefore reduce his likelihood to commit crime (a feature which runs against the family disruption effect).[A11]

With this assumption, the dynamic equation of q_t can be written as:

$$\begin{aligned}\Delta q_t = q_t(1-q_t)\big[&(1-\gamma)\left(4q_t(1-q_t)\Delta^B(1-\gamma)-1\right)\\ &-p\theta^d 4q_t(1-q_t)^2\left(\Delta^B(1-\gamma)^2-\Delta^S\right)+p\theta^d\gamma q_t\big].\end{aligned}$$

Assumption B3 *Suppose that: (i)* $\Delta^B(1-\gamma)-1>0$ *and (ii)* $0<\Delta^B(1-\gamma)^2-\Delta^S<\gamma$.

[A11] Assuming that $\theta^h \neq 0$ would not only introduce a positive effect of incarceration on criminal attitudes (going through the role modeling of "bad" fathers) but also a negative effect since higher incarceration decreases the fraction of bi-parental families with an "honest" father, which negatively affects the honesty-trait transmission. In this case, we would obtain that incarceration may raise crime not only through the family disruption channel but also because of the incarceration of honest fathers. We remove this second force as it would only strengthen our previous results.

Part (i) of Assumption B3 allows us to consider situations in which bi-parental families with an honest father are efficient enough to ensure the development of a culture of honesty whenever the initial level of honesty is not too low. Part (ii) allows us to compare the efficiency of bi-parental families with a dishonest father to the efficiency of single-parent families in the transmission of honesty. The right-hand side (RHS) of the inequality amounts to assume that, when oblique transmission of honesty is quite likely (i.e., for high enough values of q_t), then single-parent families are more efficient in transmitting the trait h than bi-parental families with a dishonest father. By contrast, the left-hand side (LHS) of the inequality implies that, in situations where oblique transmission of honesty is pretty low (i.e., for low values of q_t), then the children raised in a bi-parental families, even with a dishonest father, are more likely to adopt the honesty trait than the children raised by a single mother. In other words, we consider the interesting situations in which the relative efficiency of single-parent families is affected by the population composition effects.[A12]

Note that, in order to satisfy parts (i) and (ii) of Assumption B3, the direct role model exposure effect of honest and dishonest fathers, γ, has to take intermediate values. Indeed, for values of γ close to 0, one would get back to the cultural dynamics similar to our benchmark model, while for values of γ close to 1, the socialization technology of honest fathers would be so efficient that a complete steady-state culture of honesty (i.e., $q = 1$) would always prevail in the society, independently of criminal behavior.

Proposition B6 *Suppose that Assumption B3 holds. Then, there exist* $\underline{q}_2(p) \in]0,1[$ *and* $\overline{q}_2(p) \in]0,1]$ *such that, for any* $q_0 \in [0, \underline{q}_2(p)[$*, the sequence* q_t *converges to 0, while, for any* $q_0 \in [\underline{q}_2(p), 1]$*, the sequence* q_t *converges to* $\overline{q}_2(p)$*.*

Proposition B6 studies the long-run behavior of honesty with "bad" fathers as possible role models. As in the benchmark model, we find that the long-run level of honesty depends on the initial population composition. However, we now find that, when the initial level of honesty is sufficiently high, the population may end up fully honest (i.e., $\overline{q}_2(p)$ can now be equal to 1), which was not possible in the benchmark model.

[A12] If this was not the case, we would have the same cultural dynamics as in the benchmark model when bi-parental families with dishonest fathers are always more efficient at socialization than monoparental families, and a full honesty ($q_t = 1$) long-run steady state if monoparental families are always more efficient at socialization than bi-parental families with dishonest fathers.

To see this more clearly, we can rewrite the cultural dynamics equation as follows:

$$\Delta q_t = \underbrace{(1-q_t)P_t^{hB,d} + q_t P_t^{hB,h}}_{\text{average bi-parental transmission}} + \underbrace{pC(q_t,p) \times \left[P_t^{hS} - P_t^{hB,d}\right]}_{\text{crime repression and family structure change}} - q_t$$

The first term of the RHS describes the average rate of honesty transmission across bi-parental families with "honest" and "dishonest" fathers, while the second term reflects the effect of crime repression on family structure from biparental families with "bad" fathers to monoparental families, and how this affects the transmission rate of honesty in the society. When we are close to $q_t = 1$, the cultural dynamics can be approximated by:[A13]

$$\Delta q_t \simeq (1-q_t)\left[\gamma(1+p\theta^d(p)) - 1\right]$$

It should be clear that $q_t = 1$ can now become a stable cultural steady state when γ is large enough, i.e., when $\gamma > \frac{1}{1+p\theta^d(p)}$. Note, as well, that, under Assumption B3, contrary to the benchmark case, there is no value of p such that, for any initial level of honesty q_0, the steady-state distribution of moral attitudes converges to zero. This result is due to the new socialization technology where honesty can also be transmitted by honest fathers. When role modeling by honest father is sufficiently effective (i.e., γ high enough), then, whenever the fraction of honest agents is sufficiently high (i.e., q_0 high), the positive role modeling effect of fathers outweighs the negative effect due to single-parenthood (even though the fraction of single-parent families is high).

Proposition B7 *Suppose that*

$$\left(1+\sqrt{\frac{(\Delta^B(1-\gamma)-1)}{\Delta^B(1-\gamma)}}\right)^2 \left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma > 0,$$

[A13]Indeed, the dynamics equation can be written as:

$$\begin{aligned}\Delta q_t &= (\gamma-1)q_t + (1-\gamma)\left[4q_t^2(1-q_t)^2(1-\gamma)^2\Delta^B + q_t^2\right] \\ &\quad +(1-q_t)p\theta^d(p)\left[4q_t^2(1-q_t)^2\left[\Delta^S - (1-\gamma)^2\Delta^B\right] + q_t^2\gamma\right]\end{aligned}$$

Close to $q = 1$, this dynamics equation can be approximated by the following Taylor expansion:

$$\Delta q_t \simeq (1-q_t)\left[\gamma(1+p\theta^d(p)) - 1\right]$$

and

$$\frac{(\beta - w)^2}{4(\beta + \sigma)}\gamma - (1 - \gamma) < 0.$$

Then, $\forall p \in [0, (\beta - w)/(2(\beta + \sigma))]$, *we have:*

$$\frac{\partial \underline{q}_2(p)}{\partial p} > 0, \qquad \frac{\partial \overline{q}_2(p)}{\partial p} > 0.$$

An important implication of Proposition B7 is that the effect of repressive policies on honesty and crime may now depend on the initial fraction q_0 of honest agents.[A14] To see this, consider, first, the case when $q_0 \in]\underline{q}_2(0), \underline{q}_2((\beta - w)/(2(\beta + \sigma)))[$. When the incarceration rate is zero, the fraction of honest agents converges to $\overline{q}_2(0) > 1/2$. Suppose, now, that the government implements a tougher incarceration policy $p = (\beta - w)/(2(\beta + \sigma))$. Then, given that $q_0 < \underline{q}_2((\beta - w)/(2(\beta + \sigma)))$, the long-run fraction of honest individuals falls now to zero. In the first case with $p = 0$, the long-run crime rate is $(1 - \overline{q}_2(0))(\beta - w)$ while, in the second case, the long run crime rate is $(1/2)(\beta - w) > (1 - \overline{q}_2(0))(\beta - w)$. The incarceration policy $p = (\beta - w)/(2(\beta + \sigma))$ therefore increases the long term crime rate.

Now consider a situation where the initial condition satisfies $q_0 \in]\underline{q}_2((\beta - w)/(2(\beta + \sigma))), 1[$. Again, when the incarceration rate is zero, the fraction of honest individuals converges to $\overline{q}_2(0)$. Because q_0 is now larger than $\underline{q}_2((\beta - w)/(2(\beta + \sigma)))$, under the incarceration policy $p = (\beta - w)/(2(\beta + \sigma))$, the long-run fraction of honest individuals converges to $\overline{q}_2((\beta - w)/(2(\beta + \sigma)) > \overline{q}_2(0)$. In such a case, the incarceration policy still reduces short-run crime due the standard deterrence effect, but, now, also increases long-run honesty. The overall effect on long-run crime is therefore negative.

This result differs from the conclusions obtained in our benchmark model where fathers did *not* act as role models. In that case, "bad" transmission from peer effects were so strong that, when q_0 was sufficiently low, whatever the incarceration policy, honesty was always disappearing in the long run, i.e., $q^* = 0$.

When there is room for fathers to be role models, the impact of repressive policies varies with the initial culture of honesty. Typically, a rise in p has two distinct impacts on the dynamics of honesty. On the one hand, it (negatively) affects the transmission of honesty by increasing the proportion of single-parent families. On the other hand, it has an impact on

[A14] The assumption on γ is not necessary for this result.

the long-run honesty by reducing the proportion of bi-parental families with "bad" fathers. The overall impact of p on steady-state honesty then depends on which type of family is more efficient in transmitting honesty.

The relative efficiency of these two different types of family depends, in turn, on q_t, the fraction of honest individuals in the population, because children born in single-parent families are much more subject to population composition effects (i.e., role models outside the family). Indeed, bi-parental families exert a higher socialization effort (because of their lower socialization cost). However, they have a lower chance of transmitting honesty through role models outside the family (i.e., peers) since the child does not only mimic individuals from his neighborhood but also his father who is dishonest.

When q_t is high, the transmission of honesty *outside the family* is strong (which is also implied by the fact that socialization efforts are low because of cultural substitution). In this case, single-parent families are more efficient in transmitting honesty than bi-parental families with a dishonest father. As a consequence, a rise in incarceration, even though it increases the percentage of single-parent families in the population, has a positive impact on the transmission of honesty.

When q_t is low, the transmission of honesty *outside the family* is weak. Thus, provided that incentives to socialize children are not too low (i.e., because of cultural complementary, q_t is higher than some threshold), exerting a high socialization effort is better for the cultural diffusion of the honesty trait. In other words, bi-parental families with a "bad" father are more efficient in transmitting honesty than single-parent families. In that case, a rise in p, which decreases the fraction of bi-parental families with a "bad" father, has a negative impact on the long-run level of honesty.

It is worth noting that the population composition effects that we just highlighted are specific to our cultural approach of crime. In particular, these results are very different from what we could obtain in a model in which cultural traits are replaced by human capital. While both approaches would increase the cost of crime (psychological cost for culture and economic opportunity wage cost for education), they have different transmission effects. In the education case, there would not be any direct negative effect of the role model aspect of fathers.[A15] Therefore, in such a framework, the transmission technology of single-parent families will always be worse than with any other family socialization technology.

[A15] Unless one assumes that there is a transmission of human capital for crime, which might occurs in some specific cases such as the mafia type of criminal activities.

Thus, contrary to what we obtain in the present framework, in a model of human capital accumulation, the effect of repressive policies on long-run crime will never be qualitatively affected by the population composition.

B.5 Broken families, segregation, and crime: Efficiency versus equity (Section 4)

We have the following result:[A16]

Proposition B8 *Suppose that the probability of being apprehended, p, is such that* $p \in [0, \widehat{p}_1[\cup]\widehat{p}_2, \overline{p}_2]$. *Define* $q_{max} = \max\{\underline{q}, \frac{1}{2}\}$.

- (i) $\forall Q_0 \in \left[0, 2\underline{q}\right[\cup [2q_{max}, 2]$, *the market solution is always efficient. In particular, when* $Q_0 \in [\underline{q}, 2\underline{q}[$, *segregation is efficient but conflicts with equity. When* $Q_0 \in [0, \underline{q}[\cup]2q_{max}, 2[$, *there is spatial integration and efficiency matches with equity.*
- (ii) $\forall Q_0 \in \left[2\underline{q}, 2q_{max}\right[$, *the market solution (i.e., segregation) is not efficient because the planner would choose integration (first best), i.e.,* $q^1 = q^2 = \overline{q}$. *Efficiency matches with equity.*

Figure A3 illustrates case (i) of Proposition B8. Each rectangle represents a neighborhood. The upper-panel rectangles show the spatial equilibrium in the city (case (i) of Proposition 4). As shown in Proposition 4, the unique steady-state equilibrium is such that the city is segregated with $q_1^* = \overline{q}$ and $q_2^* = 0$. The lower-panel rectangles depict the spatial distribution of agents by type if the planner chooses to reallocate people. When such a policy is implemented, the outcome is worse than the equilibrium one since the proportion of honest agents falls below $\underline{q}$ in both neighborhoods. This leads to a long-run outcome where $q_1^* = q_2^* = 0$. As a result, it is better that the planner does not try to integrate agents in the urban space, implying that, in this case, *segregation is efficient*.

Now, consider a planner who cannot reallocate people between the two locations but who rather implements a policy that affects the model's parameters. In particular, assume

[A16]Given Proposition 4 and its proof, the proofs of Propositions B8 and B9 are straightforward and thus omitted.

Figure A3: Spatial equilibrium under segregation where $q_1^* = \overline{q}$ and $q_2^* = 0$ (upper panel) and under integration where $q_1^* = q_2^* = 0$ (lower panel).

that the planner can reduce the socialization cost of single-parent families (through a non-costly subsidy).[A17] From Proposition B5, we can deduce that whenever $Q_0 > \underline{q}^\delta$, where $\underline{q}^\delta := \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$, this policy is efficient. Indeed, a sufficiently high subsidy (so that the cost of transmitting the honesty trait is the same for single and biparental families) reduces $\underline{q}$ to $\underline{q}^\delta < \underline{q}$ and increases the value of honesty in the steady state to $\overline{q}^\delta > \overline{q}$. We now compare the efficiency (in terms of a reduction in total crime) of a reallocation policy (as above) with a subsidy policy.

Proposition B9 *Suppose that the probability of being apprehended, p, is such that* $p \in [0, \widehat{p}_1[\cup]\widehat{p}_2, \overline{p}_2]$. *Define* $q^\delta_{max} = \max\{\underline{q}^\delta, \frac{1}{2}\}$.

- *(i)* $\forall Q_0 \in \left[0, \underline{q}^\delta\right]$, *neither the subsidy policy nor the reallocation policy is efficient.*
- *(ii)* $\forall Q_0 \in \left]\underline{q}^\delta, 2\underline{q}^\delta\right[\cup [2q^\delta_{max}, 2]$, *the subsidy policy is more efficient than the reallocation policy.*
- *(iii)* $\forall Q_0 \in \left[2\underline{q}^\delta, 2q^\delta_{max}\right[$, *the reallocation policy is more efficient than the subsidy policy.*

Figure A4 illustrates part *(iii)* of this proposition. In this figure, the upper-panel rectangles display the long-run spatial equilibrium when a subsidy policy is implemented. As stated above, the spatial equilibrium after the subsidy policy is implemented is segregated because $Q_0 < 1$. Compared with the long-run equilibrium without a policy, the only change is that the proportion of honest families in neighborhood 1 increases from $\overline{q}$ to $\overline{q}^\delta$. As a result, the subsidy policy is efficient because it reduces total crime.

The lower-panel rectangles in Figure A4 show the spatial distribution of agents when the planner chooses to optimally reallocate the agents to force integration. In such a case, given that $Q_0 > 2\underline{q}$, the proportion of honest families in both neighborhoods is higher than $\underline{q}$, meaning that it converges to $\bar{q}$. Since $2\overline{q} > \overline{q}^\delta$ (because $2\overline{q} > 1$), the reallocation policy is more efficient than the subsidy policy because, in the latter, the proportion of honest families is $\overline{q}^\delta$ compared with $2\overline{q}$ in the former.

[A17] Suppose that the maximal subsidy is such that $\Delta^S = \Delta^B$, which means that, at the maximal subsidy, the planner completely removes the negative effect of family disruption on crime. We could equivalently consider an increase in w, increase in β, or increase in σ. Nevertheless, we do not consider changes in p since p is restricted to a given interval.

Figure A4: Spatial equilibrium under a subsidy policy (upper panel) and under a reallocation policy (lower panel) when $Q_0 \in \left[2\underline{q}, 2q_{max}\right[$.

B.6 Extension: Wrongful convictions and the dynamics of crime

In our paper, criminals may or may not be apprehended, but arrests of innocent individuals do not happen. In a statistical sense, type II error is allowed but type I error is not. Yet, wrongful arrests and convictions are well documented. Substantiated or unsubstantiated examples of wrongful convictions abound in the popular news media, one prominent example being the Central Park Five (Duru, 2003;[A18] Dwyer, 2019[A19]).[A20] Gross et al. (2017)[A21] states that: "African Americans are only 13% of the American population but a majority of innocent defendants wrongfully convicted of crimes and later exonerated. They constitute 47% of the 1,900 exonerations listed in the National Registry of Exonerations (as of October 2016), and the great majority of more than 1,800 additional innocent defendants who were framed and convicted of crimes in 15 large-scale police scandals and later cleared in group exonerations."

In the economics literature, recent work on discrimination in arrests and convictions hinges on the possibility of such wrongful arrests and convictions (Antonovics and Knight, 2009;[A22] Fryer, 2019[A23]). For example, Fryer (2019) finds that, on nonlethal uses of force, blacks and Hispanics are more than 50 percent more likely to experience some form of force in interactions with police.

We could introduce this in our model. In that case, equation (9), which describes the dynamics of q_t, will change. Let p^w be the probability that an innocent individual is wrongfully convicted. Then, equation (9) becomes:

[A18] Duru, N.J. (2003), "The central park five, the scottsboro boys, and the myth of the bestial black man," *Cardozo Law Review* 25, 1315

[A19] Dwyer, J. (2019), "The true story of how a city in fear brutalized the central park five," The New York Times.

[A20] Five juvenile males: Antron McCray, Kevin Richardson, Raymond Santana, Yusef Salaam and Kharey Wise, were convicted for rape and served their sentences. Their convictions were vacated in 2002 when Matias Reyes, a convicted rapist and murderer serving a life sentence for other crimes, confessed to committing the crime and DNA evidence confirmed his involvement in the rape.

[A21] Gross, S.R., Possley, M. and Stephens, K. (2017), "Race and wrongful convictions in the United States,"University of Michigan Law School.

[A22] Antonovics, K. and B.G. Knight (2009), "A new look at racial profiling: Evidence from the Boston police department," *The Review of Economics and Statistics* 91, 163–177.

[A23] Fryer Jr, R.G. (2019), "An empirical analysis of racial differences in police use of force," *Journal of Political Economy* 127, 1210–1261.

$$q_{t+1} = \underbrace{(1-p^w)[1-C(q_t,p,p^w)]}_{\text{fraction of non-criminals}} P_t^{hB} + \underbrace{(1-p)C(q_t,p,p^w)}_{\text{fraction of non-caught criminals}} P_t^{hB} + \underbrace{p^w[1-C(q_t,p,p^w)]}_{\text{fraction of non-criminals}} P_t^{hS} + \underbrace{pC(q_t,p,p^w)}_{\text{fraction of caught criminals}} P_t^{hS}$$

Compared to (9), there is a new term, which describes the non-criminals that are wrongfully convicted and thus cannot form a bi-parental family. Of course, when $p^w = 0$, we are back to our benchmark model. As a result, among the mass 1 of men, $(1 - p^w)[1-C(q_t,p,p^w)] + (1-p)C(q_t,p,p^w)$ form biparental families, while $p^w[1-C(q_t,p,p^w)] + pC(q_t,p,p^w)$ form single-mother families.

It should be clear that this mechanism will, in fact, reinforce our results since, now, there are even more single-mother families than in the benchmark model. This implies that high levels of incarceration can break even more families and yield higher crime rates.

B.7 Proofs of the additional results provided in this Section

Proof of Proposition B1: Let us first focus on the interior stable equilibrium $\overline{q}$ of Proposition 1 and, therefore, assume that $p \in [0, \widehat{p}_1] \cup [\widehat{p}_2, \overline{p}_2]$. In the proof of Proposition 1, we showed that the steady-state proportion of honest individuals in the population, $\overline{q}$, is implicitly given by

$$f(\overline{q}) = 0,$$

where $f(.)$ is defined by (13). Hence,

$$\frac{\partial \overline{q}}{\partial w} = -\frac{\partial f(.)/\partial w \mid_{\overline{q}}}{\partial f(.)/\partial q \mid_{\overline{q}}}.$$

In the proof of Proposition 1, we showed that $\partial f(.)/\partial q \mid_{\overline{q}} < 0$, which means that the sign of $d\overline{q}/dw$ is the same as the sign of $\partial f(.)/\partial w$. Totally differentiating $f(.)$ in (13) and using (4) lead to

$$\frac{\partial f(q_t)}{\partial w} \mid_{q_t=\overline{q}} = \overline{q}(1-\overline{q})4\overline{q}\,(1-\overline{q})\,p\left(\Delta^B - \Delta^S\right) > 0.$$

As a result,

$$\frac{\partial \overline{q}}{\partial w} > 0.$$

By using the same approach, we can show that $\frac{\partial \overline{q}}{\partial \beta} < 0$ and $\frac{\partial \overline{q}}{\partial \sigma} > 0$, $\frac{\partial \overline{q}}{\partial c^S} < 0$, $\frac{\partial \overline{q}}{\partial c^B} < 0$.

Now if we focus on the unstable interior equilibrium $\underline{q}$ that defines the boundary of the basin of attraction of $q = 0$, we know that it is defined by $f(\underline{q}) = 0$ with $\partial f(.)/\partial q \mid_{\underline{q}} > 0$. Thus the sign of which means that the sign of $d\overline{q}/dw$ is the opposite of the sign of $\partial f(.)/\partial w$. Again totally differentiating $f(.)$ in (13) and using (4) lead to

$$\frac{\partial f(q_t)}{\partial w} \mid_{q_t=\underline{q}} = \underline{q}(1-\underline{q})4\underline{q}\left(1-\underline{q}\right) p\left(\Delta^B - \Delta^S\right) > 0.$$

As a result,

$$\frac{\partial \underline{q}}{\partial w} < 0.$$

By using the same approach, we can show that $\frac{\partial \underline{q}}{\partial \beta} > 0$ and $\frac{\partial \underline{q}}{\partial \sigma} < 0$, $\frac{\partial \underline{q}}{\partial c^S} > 0$, $\frac{\partial \underline{q}}{\partial c^B} > 0$. ∎

Proof of Proposition B2:

Following the proof of Proposition 1, the stationary equilibria of the economy are 0, 1, and q such that

$$h^*(q_t) = \frac{1}{4},$$

where $h^* : [0,1] \to [0,1]$ is given by

$$h^*(q_t) = q_t(1-q_t)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right],$$
$$\text{with} \quad C(q_t) = -q_t K + \beta - p(\beta+\sigma) - w(R-p).$$

Step 1. From the proof of Proposition 1, we deduce that h^* admits a unique maximum q_m^*.

Step 2. Define

$$u^*(p) \equiv h^*(q_m^*(p), p).$$

We show that for any $R < \frac{-K+\beta}{2(\beta+\sigma)}$, $\frac{du^*}{dp} < 0$.

Let us perform the derivative of the function u^*,

$$\begin{aligned}\frac{du^*}{dp} &= -\left(\Delta^B - \Delta^S\right)(-q_t K + \beta - 2p(\beta+\sigma) - w(R-p)) - pw^{'}(R-p)\\ &< -\left(\Delta^B - \Delta^S\right)(-K + \beta - 2p(\beta+\sigma) - w(R-p))\\ &< -\left(\Delta^B - \Delta^S\right)(-K + \beta - 2R(\beta+\sigma)).\end{aligned}$$

Note that $\Xi(p) = -(-K + \beta - 2p(\beta+\sigma) - w(R-p))$ is increasing in p. Indeed, $\Xi'(p) = 2(\beta+\sigma) - w^{'}(R-p)$ and $\Xi''(p) = w''(R-p) < 0$. Hence, $\Xi'(p)$ is decreasing in p and positive as $\Xi'(R) = 2(\beta+\sigma) - w^{'}(0) > (\beta+\sigma) - w^{'}(0) > 0$ from Assumption B1. Hence, $\Xi(p)$ is increasing in p and

$$\begin{aligned}\frac{du^*}{dp} &< -\left(\Delta^B - \Delta^S\right)(-K + \beta - 2p(\beta+\sigma) - w(R-p)) = \left(\Delta^B - \Delta^S\right)\Xi(p)\\ &< \left(\Delta^B - \Delta^S\right)\Xi(R) = -\left(\Delta^B - \Delta^S\right)(-K + \beta - 2R(\beta+\sigma)).\end{aligned}$$

Consequently, for any $R < \frac{-K+\beta}{2(\beta+\sigma)}$, $\frac{du^*}{dp} < 0$.

Step 3. We show that there exists $\tilde{R}$ such that
- for any $R \in [0, \tilde{R}]$, $u^*(p) > \frac{1}{4}$ $\forall p \in [0, R]$ (equivalent to $h^*(q_t) = \frac{1}{4}$ admits two solutions $\underline{q}(p)$ and $\bar{q}(p)$),
- for any $R \in]\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$, there exists $\hat{p}^{1*}$ such that $u^*(p) > \frac{1}{4}$ $\forall p \in [0, \hat{p}^{1*}]$ (equivalent to $h^*(q_t) = \frac{1}{4}$ admits two solutions), and $u^*(p) < \frac{1}{4}$ $\forall p \in]\hat{p}^{1*}, R]$ (equivalent to $h^*(q_t) = \frac{1}{4}$ admits no solution).

First, we study $u^*(p)$ at $p = R$. Define

$$\begin{aligned}&v(R) \equiv u^*(R) = q_m^*(R)(1 - q_m^*(R))\left[\Delta^B - R(-q_m^*(R)K + \beta - R(\beta+\sigma))\left(\Delta^B - \Delta^S\right)\right],\\ &\text{and} \quad v^1(R) \equiv q_m^*(R)(1 - q_m^*(R))\left[\Delta^B - R(-K + \beta - R(\beta+\sigma))\left(\Delta^B - \Delta^S\right)\right],\end{aligned}$$

with $v^1(R) > v(R)$ $\forall R$.

Note that $v^1(\frac{-K+\beta}{2(\beta+\sigma)}) < 1/4$. Indeed, since

$$v^1(R) < \frac{1}{4}\left[\Delta^B - R(-K + \beta - R(\beta+\sigma))\left(\Delta^B - \Delta^S\right)\right], \quad \forall R,$$

we have

$$v^1(\frac{-K+\beta}{2(\beta+\sigma)}) < \frac{1}{4}\left[\Delta^B - \frac{(-K+\beta)^2}{4(\beta+\sigma)}\left(\Delta^B - \Delta^S\right)\right] < \frac{1}{4},$$

where the latter inequality comes from Assumption B1. We deduce

$$v(\frac{-K+\beta}{2(\beta+\sigma)}) < v^1(\frac{-K+\beta}{2(\beta+\sigma)}) < \frac{1}{4}.$$

Following Step 2, we have $\frac{\partial v}{\partial R} < 0$. Further, we have $v(0) = q_m^*(0)(1-q_m^*(0))\Delta^B = (1/4)\Delta^B > 1/4$ (recall that from Assumption 1, $\Delta^B > 1$). Hence, we deduce that there exists $\tilde{R}$ such that

$$(i)\ v(R) > \frac{1}{4} \quad \forall R < \tilde{R},$$
$$(ii)\ v(R) \leq \frac{1}{4} \quad \forall R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}].$$

Returning to the function $u^*(p)$. We have $u^*(0) > 1/4$, $\partial u^*/\partial p < 0$. When item (i) holds, that is when $R < \tilde{R}$, $u^*(R) > 1/4$. We deduce that $u^*(p) > 1/4 \;\; \forall p \in [0, R]$. When item (ii) holds, that is when $R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$, we have $u^*(R) < 1/4$. We deduce that there exists $\hat{p}^{1*} \geq \tilde{R}$ such that $u^*(p) \geq \frac{1}{4}\ \forall p \in [0, \hat{p}^{1*}]$, and $u^*(p) < \frac{1}{4}\ \forall p \in]\hat{p}^{1*}, R]$.

The function u^* is represented in Figures A5 (for $R = \tilde{R}$) and A6 (for $R > \tilde{R}$).

Step 4. We show that $\forall q_0 < \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$, q_t converges to 0. Indeed, $u^*(p) < u^*(0)$ $\forall p$ implies $\underline{q}(p) > \underline{q}(0)$. We know that $\forall q_0 < \underline{q}(p)$, the sequence q_t converges to 0. We deduce that $\forall q_0 < \underline{q}(0) = \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$, the sequence q_t converges to 0.

Step 5. We then show that

(i) When $R < \tilde{R}$, $\forall q_0 > \bar{q}(\tilde{R})$, the population converges to $\bar{q}(p)$, with $\partial \bar{q}/\partial p < 0$.

(ii) When $R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$, $\forall q_0 > \underline{q}(\hat{p}^{1*})$, the population converges to $\bar{q}(p)$, with $\partial \bar{q}/\partial p < 0$ (if $p \leq \hat{p}^{1*}$), or to zero (if $p > \hat{p}^{1*}$).

First, from proof of Proposition 1, we know that when $h^*(q)$ admits two solutions $\underline{q}(p)$ and $\bar{q}(p)$, for any $q_0 > \underline{q}(p)$, the sequence q_t converges to $\bar{q}(p)$.

Hence, when $R < \tilde{R}$, $\forall p$, for any $q_0 > \underline{q}^*(p)$, the sequence q_t converges to $\bar{q}(p)$. When $R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$, $\forall p < \hat{p}^{1*}$, for any $q_0 > \underline{q}^*(p)$, the sequence q_t converges to $\bar{q}^*(p)$ and

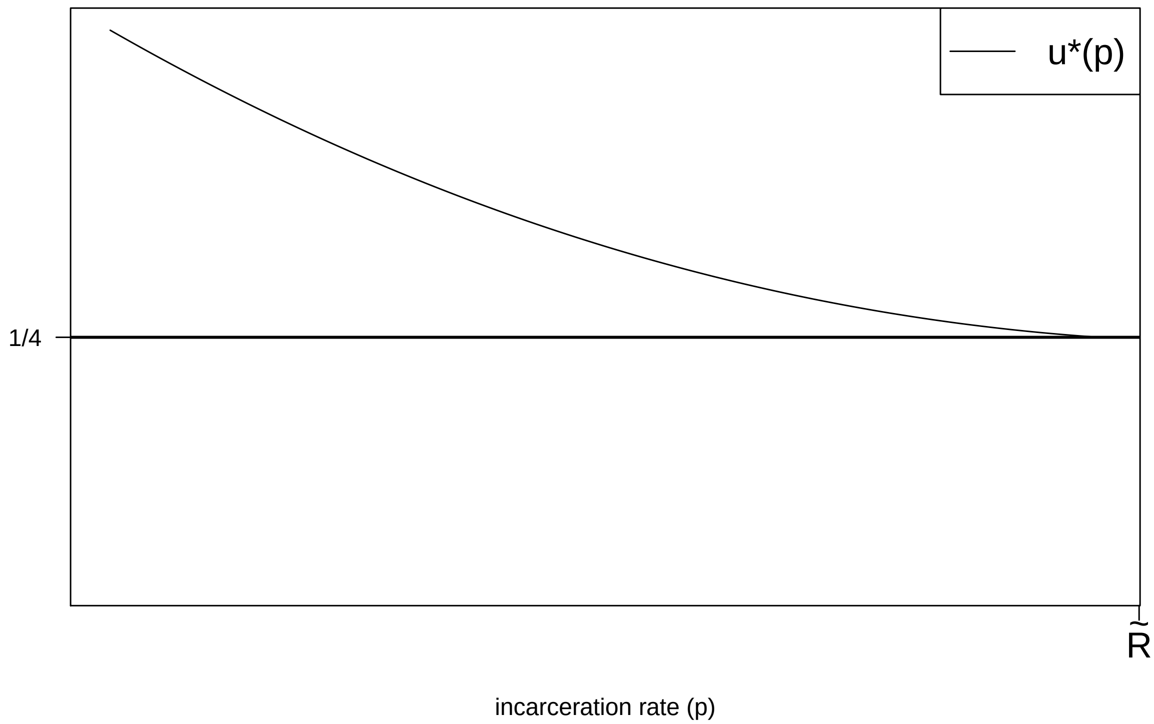


Figure A5: The function u when $R = \tilde{R}$. Whenever $R < \tilde{R}$, then $u(p) > \frac{1}{4}$.

$\forall p \in \hat{p}^{1*}, R]$, for any $q_0 \in [0, 1]$, the sequence q_t converges to 0.

The convergence condition above depends on p. We must find a condition that holds for any p to compare convergence under different incarceration rates. To do so, we find an upper bound for $\underline{q}^*(p)$. When $R < \tilde{R}$, the upper bound for $\underline{q}^*(p)$ is $\underline{q}(\tilde{R})$ (this can be shown by using the fact that $\partial u^*/\partial p < 0$ and $u^*(\tilde{R}) = 1/4$). When $R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$, the upper bound is $\underline{q}(\hat{p}^{1*})$ (this is shown by using similar arguments).

We deduce that when $R < \tilde{R}$, and $\forall q_0 > \underline{q}(\tilde{R})$, the sequence q_t converges to $\bar{q}(p)$. Then, we easily show that $\partial \bar{q}/\partial p < 0$ (i.e., the higher the incarceration rate p, the lower is the steady-state level of honesty).

When $R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$, and $\forall q_0 > \underline{q}(\hat{p}^{1*})$, the sequence q_t converges to $\bar{q}(p)$, with $\partial \bar{q}/\partial p < 0$ for $p < \hat{p}^{1*}$ and to zero for p higher than $\hat{p}^{1*}$. Again, an increase in incarceration has a negative impact on long-run honesty.

Figures A7 and A8 depict cases (i) and (ii) of Step 5. Figure A7 shows that when

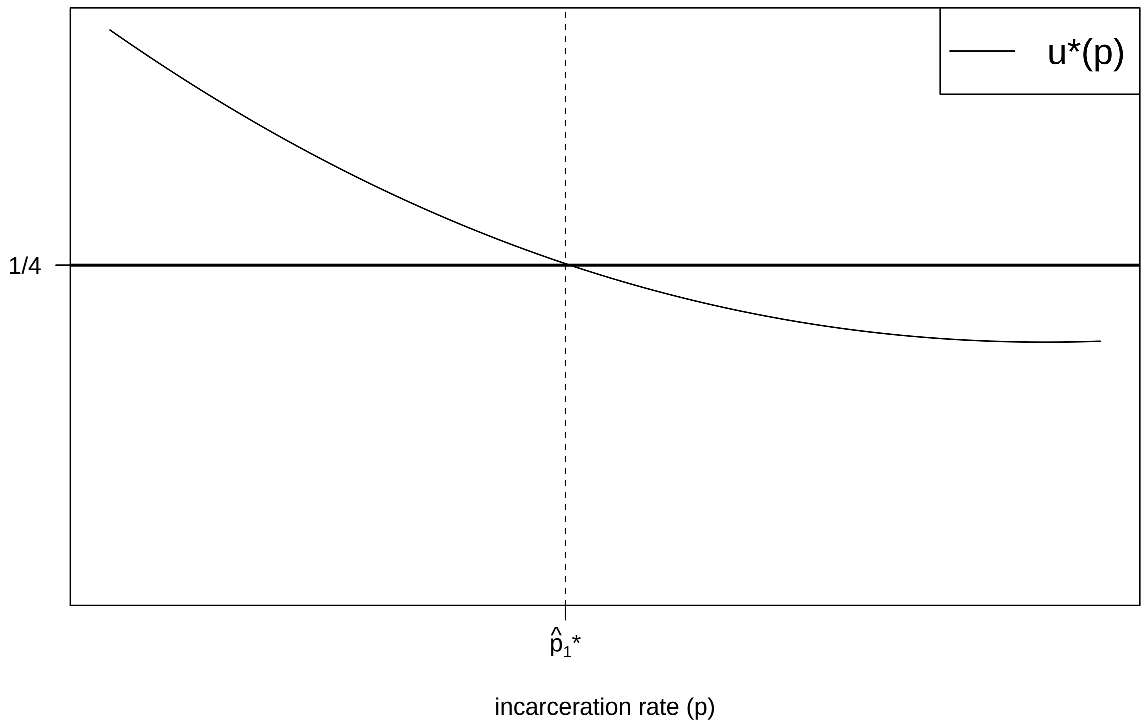


Figure A6: The function u when $R > \tilde{R}$. Here, we have $u(p) > \frac{1}{4} \Leftrightarrow p > \hat{p}_1^*$.

$R < \tilde{R}$, for any $q_0 > \underline{q}(\tilde{R})$, $\forall p$, the sequence q_t converges to $\bar{q}(p)$. Figure A8 shows that for $R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$, $\forall q_0 > \underline{q}(\hat{p}^{1*})$, the sequence q_t converges to $\bar{q}(p)$.

Finally, part (i) of Proposition B2 follows from Step 4, posing $q^1_{\min} = \underline{q}(0) = \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B - 1)}}{2\Delta^B}$.

Part (ii) of Proposition B2 follows from Step 5, posing $q^2_{\min} = \underline{q}(\tilde{R})$ for $R < \tilde{R}$, and $q^2_{\min} = \underline{q}(\hat{p}^{1*})$ for $R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$. ∎

Proof of Proposition B3:

In this proof, we compare the long-run crime rate (denoted by $\bar{C}$) for $p = R$ with the long-run crime rate for $p < R$.

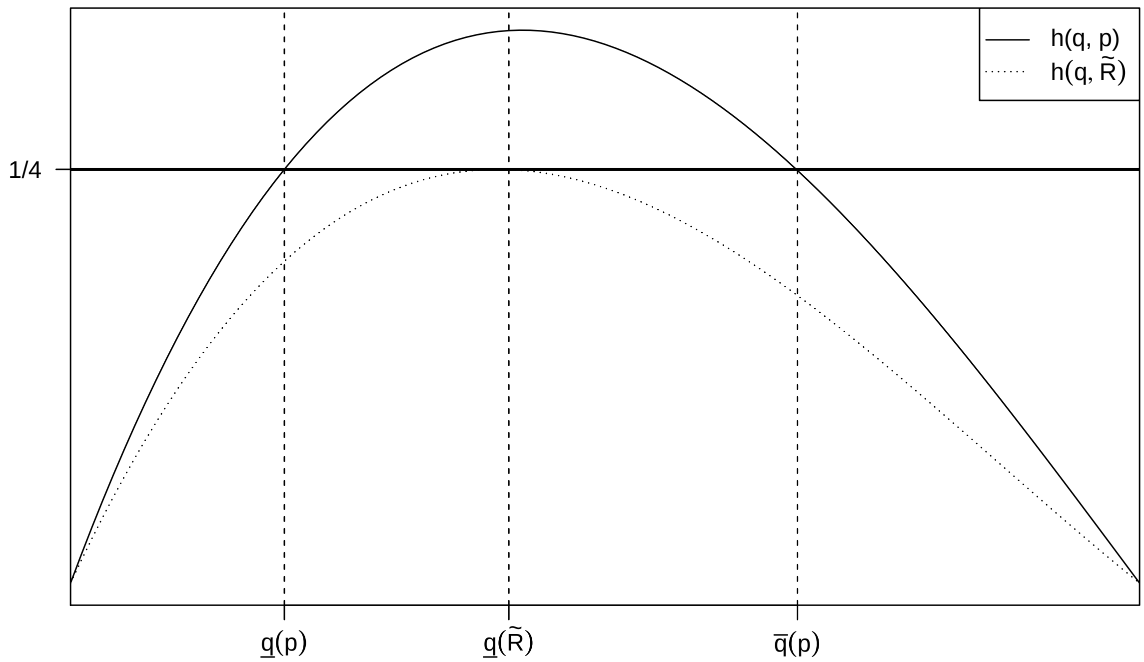


Figure A7: Case $R = \tilde{R}$: functions $h^*(q,p)$ and $h^*(q,\tilde{R})$, with $h^*(q,p) > h^*(q,\tilde{R})$.

To show case (i), note first that $\forall q_0 < q^1_{\min} = \underline{q}(0) = \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$, q_t converges to 0. We deduce that for any incarceration policy, the crime rate is given by $\bar{C} = \beta - w(R-p) - p(\beta+\sigma)$, which is a decreasing function of p. Hence, repressive policies are those where $p = R$ minimizes long-run crime.

(ii) We know from the proof of Proposition B2 that when $R \in [\tilde{R}, \frac{-K+\beta}{2(\beta+\sigma)}]$ and $q_0 > q^2_{\min} = \underline{q}(\hat{p}^{1*})$, for $p = R$, q_t converges to 0, while for $p < \hat{p}^{1*}$, q_t converges to $\bar{q}(p) > 0$. In particular, for $p = 0$, q_t converges to $\bar{q}(0) = \frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$.

We deduce that at $p = R$, the long-run crime rate $\bar{C}$ is given by $\bar{C}(R) = \beta - R(\beta+\sigma)$. At $p = 0$, we have $\bar{C}(0) = -\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B} K + \beta - w(R)$.
A repressive policy $p = R$ does not minimize $\bar{C}$ and is dominated by continuity by a

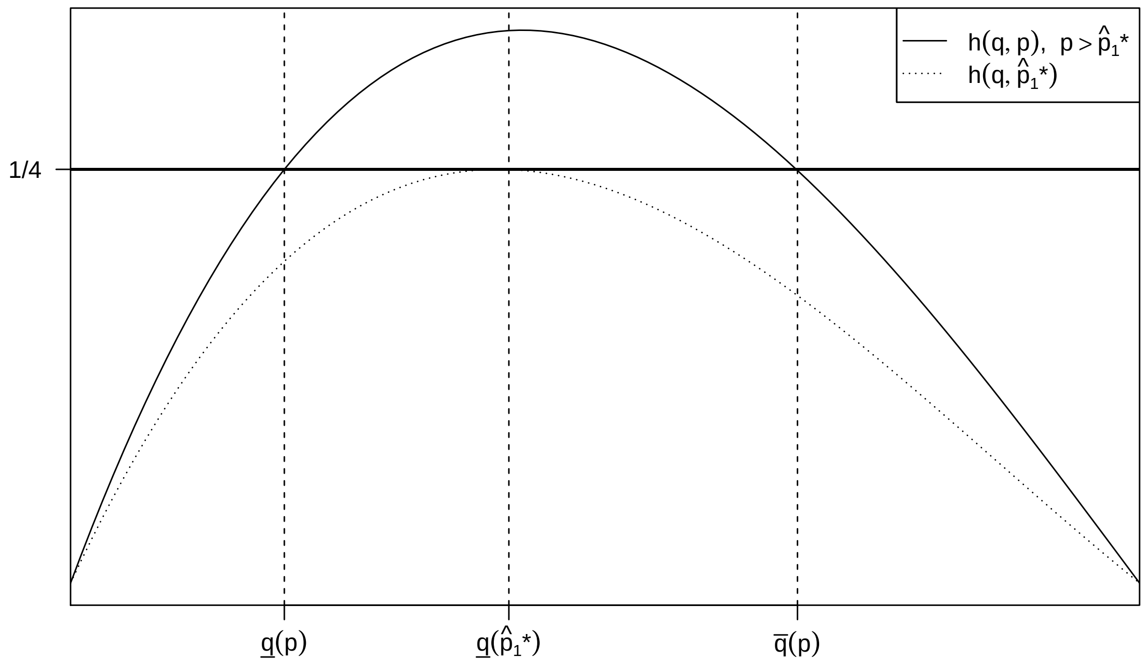


Figure A8: Case $R > \tilde{R}$: functions $h^*(q,p)$ and $h^*(q,\hat{p}_1^*)$, with $h^*(q,p) > h^*(q,\hat{p}_1^*)$.

permissive policy $p \leq \overline{p} < R$ if $\bar{C}(0) < \bar{C}(R)$ or

$$-\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B - 1)}}{2\Delta^B}K + \beta - w(R) < \beta - R(\beta + \sigma),$$
$$\Leftrightarrow -\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B - 1)}}{2\Delta^B}K - w(R) + R(\beta + \sigma) < 0.$$

Since $-w(R) + R(\beta + \sigma)$ is increasing in R by assumption, the above inequality holds if

$$-\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B - 1)}}{2\Delta^B}K - w(\frac{-K + \beta}{2(\beta + \sigma)}) + \frac{\beta - K}{2} < 0$$

or

$$\frac{\beta - K}{2} < \frac{\Delta^B + \sqrt{\Delta^B(\Delta^B - 1)}}{2\Delta^B}K + w(\frac{-K + \beta}{2(\beta + \sigma)}),$$

which is condition (B1) of Proposition B3. ∎

Proof of Proposition B4:

Let us first state the following lemma.

Lemma B1 *There exist* $\widehat{p}^1(R)$ *and* $\widehat{p}^2(R)$ *such that:*

(i) For any $p \in]\widehat{p}^1(R), \min\{\widehat{p}^2(R), 1\}[$, $\forall q_0 \in [0,1]$, *the sequence* q_t *converges to zero.*

(ii) For any $p \leq \widehat{p}^1(R)]$ *or* $p \geq \min\{\widehat{p}^2(R), 1\}$, *there exists* $\underline{q}(p)$, $\bar{q}(p)$ *such that* $\forall q_0 \in [0, \underline{q}(p)[$, *the sequence* q_t *converges to zero,* $\forall q_0 \in [\underline{q}(p), 1]$, *the sequence* q_t *converges to* $\bar{q}(p)$.

Proof of Lemma B1: The proof uses the same arguments as in the proof of Proposition 1.

Let us now prove Proposition B4.

Step 1. We show that when R is sufficiently high, the government can grant a subsidy $\delta = 1 - \frac{c^B}{c^S}$.

The government can implement a subsidy $\delta = 1 - \frac{c^B}{c^S}$ if and only if

$$R \geq pC(q_t; p, 1 - \frac{c^B}{c^S})(1 - \frac{c^B}{c^S})c^S(\tau^S)^2/2.$$

The left-hand side is bounded above by[A24]

$$\frac{\beta^2}{4(\beta+\sigma)}(c^S - c^B).$$

A sufficient condition for the above inequality is

$$R > \frac{\beta^2}{4(\beta+\sigma)}(c^S - c^B).$$

Step 2. We study the dynamics when $\delta = 1 - \frac{c^B}{c^S}$.

[A24]This inequality comes from the fact that $pC(q_t; p, 1 - \frac{c^B}{c^S}) < p(\beta - p(\beta+\sigma)) < \frac{\beta^2}{4(\beta+\sigma)}$.

If the government implements the subsidy $\delta = 1 - \frac{c^B}{c^S}$, following the proof of Proposition 1, the stationary equilibria are 0, 1, and q such that

$$h^{**}(q_t) = q_t(1-q_t)\Delta^B = \frac{1}{4}.$$

From the proof of Proposition 1, we know that the above equation admits two solutions: $\frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$ and $\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$. We also know that for any $q_0 > \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$, q_t converges to $\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$.

Step 3. We study the dynamics when $\delta = 0$.

When $\delta = 0$, the stationary equilibria are 0, 1, and q such that

$$h(q_t, R) = q_t(1-q_t)\left(\Delta^B - p\left(-q_t K + \beta - p(\beta+\sigma) - w(R)\right)\left(\Delta^B - \Delta^S\right)\right) = \frac{1}{4}.$$

Depending on p, this equation admits zero or two solutions $\underline{q}(p, R)$ $\bar{q}(p, R)$.

Step 4. Let us compare the dynamics under $\delta = 1 - \frac{c^B}{c^S}$ and $\delta = 0$. We have

$$q_t(1-q_t)\Delta^B > q_t(1-q_t)\left(\Delta^B - p\left(-q_t K + \beta - p(\beta+\sigma) - w(R)\right)\left(\Delta^B - \Delta^S\right)\right),$$

which implies $\frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B} < \underline{q}(p, R)$ and $\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B} > \bar{q}(p, R)$. For any $q_0 > \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$, when a subsidy $\delta = 1 - \frac{c^B}{c^S}$ implemented, the sequence q_t converges to $\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$. For any $q_0 > \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$, when $\delta = 0$, the sequence q_t converges to $\bar{q}(p, R) < \frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$ or $0 < \frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$. Hence, $\forall q_0 > \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$ strong subsidies have a positive impact on long-run honesty. ▮

Proof of Proposition B5 :

Step 1. Express the long-run crime rate under $\delta = 0$.

Under $p \in]\hat{p}^1(R), \min\{\hat{p}^2(R), 1\}[$, the long-run crime rate under $\delta = 0$, which we denote by $\bar{C}_0$, is given by $\bar{C}_0 = \beta - p(\beta+\sigma) - w(R)$.

Step 2. Let us express the long-run crime rate under $\delta = 1 - \frac{c^B}{c^S}$ and $q_0 > \frac{\Delta^B - \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}$. This is implicitly given by

$$\bar{C}_\delta - \left(-K\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B} + \beta - p(\beta+\sigma) - w(R - p\bar{C}_\delta \frac{\delta}{(1-\delta)^2} c^S \left[q_t(1-q_t)\Delta^S\right]^2) \right) = 0.$$

Step 3. Let us compare the crime rate without a subsidy with the crime rate with a strong subsidy. Note first that

$$\bar{C}_\delta < -K\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B} + \beta - p(\beta+\sigma).$$

A sufficient condition for $\bar{C}_\delta < \bar{C}_0$ is

$$\begin{aligned}
&-K\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B} + \beta - p(\beta+\sigma) < \beta - p(\beta+\sigma) - w(R) \\
\Leftrightarrow &K\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B} > w(R), \\
\Leftrightarrow &w^{-1}(K\frac{\Delta^B + \sqrt{\Delta^B(\Delta^B-1)}}{2\Delta^B}) > R,
\end{aligned}$$

whenever the inverse function w^{-1} exists. ∎

Proof of Proposition B6:

Stationary equilibria are such that: $\Delta q_t := q_{t+1} - q_t = 0$. Clearly, 0 and 1 are stationary equilibria. If other equilibria exist they are such that

$$h_2(q_t, p) = 0,$$

where the function $h_2 : [0,1] \times [0,1] \to [0,1]$ is given by:

$$h_2(q_t, p) = (1-\gamma)\left[4q_t(1-q_t)\Delta^B(1-\gamma) - 1\right] - p\theta^d 4q_t(1-q_t)^2\left[\Delta^B(1-\gamma)^2 - \Delta^S\right] + p\theta^d\gamma q_t.$$

The function $h_2(q_t, p)$ is a polynomial of order three. We have:

$$h_2(0,p) = -(1-\gamma) < 0 \quad \forall p \in [0,1],$$
$$h_2(1,p) = p\theta^d\gamma - (1-\gamma).$$

Observe that the term $p\theta^d$ is bounded above by $(\beta - w)^2/(4(\beta+\sigma))$.

- First, suppose that:

$$\frac{(\beta-w)^2}{4(\beta+\sigma)}\gamma - (1-\gamma) < 0 \Leftrightarrow \gamma < \frac{1}{1+\frac{(\beta-w)^2}{4(\beta+\sigma)}}$$

Then, we have: $h_2(1,p) < 0$, $\forall p \in [0, \bar{p}_2]$. Furthermore,

$$h_2(1/2,p) = (1-\gamma)\left[\Delta^B(1-\gamma)-1\right] - \frac{1}{2}p\theta^d\left[\left[\Delta^B(1-\gamma)^2 - \Delta^S\right] - \gamma\right].$$

Due to part (ii) of Assumption B3, we have $h_2(1/2,p) > (1-\gamma)\left(\Delta^B(1-\gamma)-1\right) > 0$, $\forall p \in [0, \bar{p}_2]$.

Summarizing, under Assumption B3 and $\frac{(\beta-w)^2}{4(\beta+\sigma)}\gamma - (1-\gamma) < 0$, then $\forall p \in [0, \bar{p}_2]$. We have:

$$h_2(0,p) < 0,$$
$$h_2(1/2,p) > 0,$$
$$h_2(\overline{q}_2(0),p) < 0.$$

Given that $h_2(.)$ is a continuous polynomial function of order three, we deduce that there exists $\underline{q}_2(p)$ and $\overline{q}_2(p)$ with $\underline{q}_2(p) < 1/2 < \overline{q}_2(p) < 1$ implicitly defined by

$$h_2(\underline{q}_2(p),p) = 0 \quad \text{and} \quad h_2(\overline{q}_2(p),p) = 0.$$

Equivalently, the dynamic system admits two additional equilibria: $\underline{q}_2(p)$ and $\overline{q}_2(p)$. One can easily infers that, in this case, for any $q_0 \in [0, \underline{q}_2(p)[$, the sequence q_t converges to 0 while for any $q_0 \in]\underline{q}_2(p), 1]$, the sequence q_t converges to $\overline{q}_2(p)$.

- Now suppose that
$$\frac{(\beta - w)^2}{4(\beta+\sigma)}\gamma - (1-\gamma) > 0$$

We easily deduce that there exist

$$p_1 = \frac{(\beta - w)\gamma - \sqrt{(\beta-w)^2\gamma^2 - 8(\beta+\sigma)\gamma(1-\gamma)}}{4(\beta+\sigma)\gamma},$$
$$p_2 = \frac{(\beta - w)\gamma + \sqrt{(\beta-w)^2\gamma^2 - 8(\beta+\sigma)\gamma(1-\gamma)}}{4(\beta+\sigma)\gamma},$$

such that

$$\forall p \in [p_1, p_2], \quad h_2(1,p) \geq 0,$$
$$\forall p \in [0, p_1[\cup]p_2, 1], \quad h_2(1,p) < 0.$$

Finally, when $p \in [0, p_1[\cup]p_2, 1]$, the same conclusions as for the case $\frac{(\beta-w)^2}{4(\beta+\sigma)}\gamma - (1-\gamma) < 0$ hold.

When $p \in [p_1, p_2]$, we easily deduce that there exists $\overline{q}^2(p) > 1$ implicitly defined by $\widetilde{h}_2(\overline{q}^2(p), p) = 0$ where the function $\widetilde{h}_2$ is the function h_2 considered on $\mathbb{R}^+ \times [0,1]$. Hence, one can conclude that for any $q_0 \in [0, \underline{q}_2(p)[$, the sequence q_t converges to 0 while for any $q_0 \in]\underline{q}_2(p), 1]$, the sequence q_t converges to 1. ∎

Proof of Proposition B7:

Let us examine the sign of the derivatives:

$$\frac{\partial \underline{q}_2}{\partial p} \quad \text{and} \quad \frac{\partial \overline{q}_2}{\partial p}.$$

Using the implicit function theorem, we obtain

$$\frac{\partial \overline{q}_2}{\partial p} = -\frac{\partial h_2/\partial p|_{\overline{q}_2}}{\partial h_2/\partial q_t|_{\overline{q}_2}},$$
$$\frac{\partial \underline{q}_2}{\partial p} = -\frac{\partial h_2/\partial p|_{\underline{q}_2}}{\partial h_2/\partial q_t|_{\underline{q}_2}}.$$

We know from the proof of Proposition B6 that $\partial h_2/\partial q_t|_{\overline{q}_2} < 0$ and $\partial h_2/\partial q_t|_{\underline{q}_2} > 0$. Hence, the sign of the first derivative is the sign of $\partial h_2/\partial p|_{\underline{q}_2}$ while the sign of the second derivative is the opposite sign of $\partial h_2/\partial p|_{\underline{q}_2}$. We have

$$\frac{\partial h_2}{\partial p} = -q_t \left[\beta - w - 2p(\beta+\sigma)\right] \left[4(1-q_t)^2 \left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma\right]$$

Consider the first derivative $\partial\overline{q}_2/\partial p$. Due to Assumption B3, we know that the last term in brackets is negative at $q_t = 1/2$. Furthermore, this term is decreasing in q_t. Hence it is negative for any $q_t \geq 1/2$ and in particular at $q_t = \overline{q}_2$. Given that the first term in brackets is positive $\forall p \in [0, (\beta - w)/(2(\beta+\sigma))]$ and negative $\forall p \in [(\beta - w)/(2(\beta+\sigma)), 1]$, we deduce that

$$\forall p \in [0, (\beta-w)/(2(\beta+\sigma))], \quad \frac{\partial h_2}{\partial p}|_{\overline{q}_2} > 0, \Leftrightarrow \frac{\partial \overline{q}_2}{\partial p} > 0,$$
$$\forall p \in [(\beta-w)/(2(\beta+\sigma)), 1], \quad \frac{\partial h_2}{\partial p}|_{\overline{q}_2} < 0, \Leftrightarrow \frac{\partial \overline{q}_2}{\partial p} < 0.$$

Now, consider the second derivative. Suppose that $4(1-\underline{q}_2(0))^2\left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma > 0$, which is equivalent to

$$\left(1 + \sqrt{\frac{(\Delta^B(1-\gamma)-1)}{\Delta^B(1-\gamma)}}\right)^2 \left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma > 0.$$

Given the above, it is equivalent to

$$\frac{\partial \underline{q}_2}{\partial p}|_0 > 0.$$

One can show that it implies that:

$$\frac{\partial \underline{q}_2}{\partial p} > 0 \quad \forall p \in \left[0, \frac{(\beta - w)}{2(\beta+\sigma)}\right].$$

Let us prove this by contradiction. Suppose that there exists some $p^* < (\beta - w)/(2(\beta+\sigma))$ such that $\partial\underline{q}_2/\partial p|_{p^*} < 0$. The last inequality is equivalent to

$$4(1-\underline{q}_2(p^*))^2\left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma < 0. \tag{B8}$$

Given that $\partial \underline{q}_2/\partial p|_0 > 0$ and the function $\underline{q}_2$ and its derivative are continuous, it implies that there exists some $p < p^*$ such that $\partial \underline{q}_2/\partial p|_p = 0$, which is equivalent to

$$4(1 - \underline{q}_2(p))^2 \left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma = 0.$$

Denote by p^{**}, the value of p which globally maximizes $\underline{q}_2$ on $[0, p^*]$. We necessarily have that $\partial \underline{q}_2/\partial p|_{p^{**}} = 0$, or equivalently that

$$4(1 - \underline{q}_2(p^{**}))^2 \left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma = 0.$$

and $\underline{q}_2(p^{**}) > \underline{q}_2(p^*)$. Therefore

$$4(1 - \underline{q}_2(p^{*}))^2 \left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma > 0,$$

which contradicts the above equation (B8). We deduce that $4(1-\underline{q}_2(0))^2 \left(\Delta^B(1-\gamma)^2 - \Delta^S\right) - \gamma > 0$ implies

$$\frac{\partial \underline{q}_2}{\partial p} > 0, \quad \forall p \in \left[0, \frac{(\beta - w)}{2(\beta + \sigma)}\right].$$

This complete the proof. ∎

C Urban equilibrium

Let us first calculate the bid rent of each parent of type $i = h, d$ residing in neighborhood $n = 1, 2$. Consider utility (12). Then, by using (5) and (7), we have

$$u_n^k = 4q_{n,t}^2(1-q_{n,t})^2\frac{(\Delta V)^2}{c^k} + q_{n,t}^2\Delta V + V^d - c^k\frac{\left(\tau_t^k\right)^2}{2}, \tag{C9}$$

where $\Delta V = V^h - V^d$.

To calculate each bid rent in the land market, we need to compute the expected utility of a worker of type (i, n) before the revelation of θ. To simplify the presentation, we skip the time index. We have

$$U_n^i = \int_0^{\theta^i}\left[(1-p)\,\beta - p\sigma - K\,\mathbf{1}_{i=h} - \theta\right]d\theta + \int_{\theta^i}^1 w d\theta - \rho_{n,t}^i + \int_0^{\theta^i}\left[pu_n^S + (1-p)\,u_n^B\right]d\theta + \int_{\theta^i}^1 u_n^B d\theta,$$

where $\mathbf{1}_{i=h}$ is an indicator function equal to one if the parent is of type d and zero otherwise. This utility can be written as

$$U_n^i = \left[(1-p)\,\beta - p\sigma - K\,\mathbf{1}_{i=h}\right]\theta^i - \frac{\theta^2}{2} + \left(1-\theta^i\right)w - \rho_{n,t}^i + p\theta^i u_n^S + \left(1-p\theta^i\right)u_n^B. \tag{C10}$$

We can now define the bid rent for a parent of type $i = h, d$ residing in neighborhood $n = 1, 2$. We have

$$\rho_{n,t}^i = \left[(1-p)\,\beta - p\sigma - K\,\mathbf{1}_{i=h}\right]\theta^i - \frac{\theta^2}{2} + \left(1-\theta^i\right)w + p\theta^i u_n^S + \left(1-p\theta^i\right)u_n^S - U_n^i, \tag{C11}$$

where u_n^S and u_n^B are defined in (C9).

To determine the urban equilibrium, we need to determine the bid rent differential $\Delta\rho \equiv \rho^h - \rho^d$ between honest and dishonest families. Again, we skip the time index. The bid rent $\rho^i = \rho_1^i$ that makes both neighborhoods equally attractive to a trait$-i$ parent is such that $U_1^i = U_2^i$ for $i = h, d$. Given that $\rho_2^i = 0$, we obtain

$$\rho^i = p\theta^i\left(u_1^S - u_2^S\right) + \left(1-p\theta^i\right)\left(u_1^B - u_2^B\right).$$

The bid rent differential, $\Delta\rho \equiv \rho^h - \rho^d$, is then given by

$$\Delta\rho = p\left(\theta^d - \theta^h\right)\left[\left(u_1^B - u_2^B\right) - \left(u_1^S - u_2^S\right)\right].$$

By using (C9), it is easily verified that

$$\left(u_1^B - u_2^B\right) - \left(u_1^S - u_2^S\right) = 4\left(\Delta V\right)^2\left(\frac{1}{c^B} - \frac{1}{c^S}\right)\left[q_1^2(1-q_1)^2 - q_2^2(1-q_2)^2\right].$$

As a result, by using (1) and (2), we obtain

$$\Delta\rho = 4pK\left(\Delta V\right)^2\left(\frac{1}{c^B} - \frac{1}{c^S}\right)\left[q_1^2(1-q_1)^2 - q_2^2(1-q_2)^2\right]. \tag{C12}$$

Since $q_2 = Q - q_1$, this equation can be written as

$$\Delta\rho\left(q_1\right) = 4pK\left(\Delta V\right)^2\left(\frac{1}{c^B} - \frac{1}{c^S}\right)\left[q_1^2(1-q_1)^2 - \left(Q - q_1\right)^2\left(1 - Q + q_1\right)^2\right]. \tag{C13}$$