Online Appendix

Misbehavior in Common-Value Auctions: Bidding Rings and Shills

Dan Levin and James Peck

In this Online Appendix, we analyze the Sophi Auction when the existence of the ring is common knowlege and when potential ring members can decide whether or not to join the ring.

1. Common Knowledge of the Ring in the Sophi Auction

For the Sophi auction, as in the English auction, sometimes the ring must stand ready to outbid a bidder of type $\overline{x}$ when the ring is common knowledge. That is, if r_1 were to bid only $\overline{x}$, and if the bidders know about the ring, a bidder with signal near $\overline{x}$ would want to bid $\overline{x}$, which is inconsistent with equilibrium. When the ring is common knowledge, the equilibrium involves r_1 bidding more than $\overline{x}$ for some signal realizations, and the extension of the pricing rule becomes important. When r_1 submits the highest bid, t^*, and the bidder with signal y^1 is the only bidder bidding more than $\overline{x}$, r_1 wins and pays the price $h^R(\overline{x}, \underline{x} - \overline{x} + t^*, \overline{x}, y^2, ..., y^{n-1})$.[1]

Proposition A1: *Suppose that x-separability holds. Consider (i) the optimal strategy of the ring, as characterized by r_2 bidding $\underline{x}$ and r_1 bidding t^*, and (ii) the "sincere" strategies of the bidders, as characterized earlier. For the Sophi auction in which the existence of the ring is common knowledge, this strategy profile is an equilibrium profile, except that t^* may have to be greater than $\overline{x}$; the outcomes are as characterized earlier. Furthermore, the equilibrium is without regret.*

Proof. From Proposition 6, $t^* = t^{*SophiR}$ holds.

For signal realizations in Cases 1-3 of the proof of Proposition 2, r_1 is bidding $t^* \leq \overline{x}$. The argument for Sophi is identical to the argument for English in the proof of Proposition 2, due to the result that t^* in English does not depend on the

[1] If we have $\underline{x} - \overline{x} + t^* > \overline{x}$, then the price would be $h^R(\overline{x}, \overline{x}, \overline{x}, y^2, ..., y^{n-1})$, but r_1 would never want to submit a bid that high.

exit history and outcomes are the same for the two formats. For Case 4, when r_1 in English is willing to bid more than $h^R(\overline{x}, \underline{x}, \overline{x}, y^2, ..., y^{n-1})$, then r_1 needs to bid greater than $\overline{x}$ in Sophi. The bid is such that r_1 would receive zero profits upon winning the object, assuming that $y^1 = \overline{x}$ holds, and the bidder with signal y^1 bids more than $\overline{x}$. This condition satisfies

$$h^R(s_1, s_2, \overline{x}, y^2, ..., y^{n-1}) = h^R(\overline{x}, \underline{x} - \overline{x} + t^*, \overline{x}, y^2, ..., y^{n-1}), \tag{1.1}$$

and by x-separability, the same t^* satisfies (1.1) for any $y^2, ..., y^{n-1}$. At $t^* = \overline{x}$, the left side of (1.1) is weakly greater than the right side, because we are in Case 4. If $t^* = 2\overline{x} - \underline{x}$ holds, then the right side of (1.1) equals $h^R(\overline{x}, \overline{x}, \overline{x}, y^2, ..., y^{n-1})$, so the left side of (1.1) is less than the right side. Because h^R is continuous, (1.1) must have a solution for t^*.

A bidder with signal y^j (for $j \geq 1$) does not want to obtain the object. To see this, by (1.1), the price a bidder with signal y^1 would have to pay is $h^R(s_1, s_2, \overline{x}, y^2, ..., y^{n-1})$, and a different bidder would have to pay more. However, the value of the object to the bidder is only $h^R(s_1, s_2, y^1, y^2, ..., y^{n-1})$, so the profits from winning are non-positive.

■

2. Stability of the Ring in the Sophi Auction

Returning to the model in which bidders are not aware of the ring and bid sincerely, consider the following strategy profile for the Sophi auction:

The two potential ring members say "yes," and along the equilibrium path, the game continues as analyzed above.

If one potential ring member (w.l.o.g., r_1) says "yes" and the other potential ring member (r_2) says "no," then r_1 believes that r_2's signal is $\overline{x}$. In the auction, r_1 bids $\overline{x}$ and r_2 bids $\underline{x}$.

If both potential ring members say "no," then r_1 and r_2 each believe that the other's signal is $\overline{x}$. In the auction, both bid their types.

Proposition A2. *The strategy profile described above constitutes a PBE for the Sophi auction.*

Proof. If r_1 and r_2 both say "yes," then truthfully revealing their signals to each other, and bidding as specified in the previous section, is sequentially rational.

Suppose r_1 says "yes" and r_2 says "no." Then, given the beliefs of r_1, for arbitrary $y^1, y^2, ..., y^{n-1}$, r_1 wins the auction and receives expected profit

$$h^R(s_1, \overline{x}, y^1, y^2, ..., y^{n-1}) - h^R(y^1, \underline{x}, y^1, y^2, ..., y^{n-1}).$$

By symmetry, the expected profit equals

$$h^R(s_1, \overline{x}, y^1, y^2, ..., y^{n-1}) - h^R(\underline{x}, y^1, y^1, y^2, ..., y^{n-1}).$$

Therefore, since $s_1 > \underline{x}$ and $\overline{x} > y^1$ hold with probability one, the expected profit is positive for every s_1 and every realization of $y^1, y^2, ..., y^{n-1}$. Since expected profit from winning is positive and the price paid by r_1 does not depend on his bid (as long as he wins), bidding $\overline{x}$ is sequentially rational. For r_2, bidding $\underline{x}$ is sequentially rational, since the only way r_2 can win is to bid $\overline{x}$, in which case r_2's profit, when he wins the tie-break coin flip, is

$$h^R(s_1, s_2, y^1, y^2, ..., y^{n-1}) - h^R(\overline{x}, \overline{x}, y^1, y^2, ..., y^{n-1}), \tag{2.1}$$

which is non-positive.

If r_1 and r_2 both say "no," then each bidder believes that the other bidder is type $\overline{x}$, in which case they will lose the auction unless they bid $\overline{x}$. However, r_1's profit from bidding $\overline{x}$ is

$$h^R(s_1, \overline{x}, y^1, y^2, ..., y^{n-1}) - h^R(\overline{x}, \overline{x}, y^1, y^2, ..., y^{n-1}),$$

which is non-positive, and r_2's profit from bidding $\overline{x}$ is

$$h^R(\overline{x}, s_2, y^1, y^2, ..., y^{n-1}) - h^R(\overline{x}, \overline{x}, y^1, y^2, ..., y^{n-1}),$$

which also is non-positive. Thus, sincere bidding, after r_1 and r_2 both say "no," is sequentially rational.

It is sequentially rational for r_1 and r_2 to both say "yes." The reason is that, after saying "no," the best continuation play leads to zero profit, while saying "yes" leads to positive expected profit.

Clearly, the beliefs are consistent. ■

Note: The construction in Proposition A2 uses a continuation strategy for r_2, after r_1 says "yes" and r_2 says "no," that might be weakly dominated. If we impose x-separability, then an alternate construction is available with strategies that cannot be weakly dominated. The idea is that r_2 bids x', determined such that r_1 receives zero expected profits by winning when $(s_1, s_2) = (\underline{x}, \overline{x})$. r_1 submits a bid such that expected profits from winning are zero when tying with the bidder with the highest signal, based on the belief that r_2 has signal $\overline{x}$. This ensures sequential rationality, and that r_2 could receive negative profits by bidding more.