# Online Appendix to
# "Competition for Attention and News Quality"

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## A. Supplementary Materials for Section IV.A

**Proof of Proposition 7.** Aggregate action is

$$Q = \beta_0 + \sum_k 0.5(\overline{\beta}_k + \underline{\beta}_k)\hat{y}_k.$$

Let $\beta = 0.5(\overline{\beta}_k + \underline{\beta}_k)$. Substituting $Q$ into the objective function of news outlet $j$ and performing the same exercise as in the base model, we obtain:

$$\alpha\beta = \frac{1-H}{\frac{1-\gamma}{\gamma} + \frac{\phi}{\gamma\beta^2}}, \tag{OA1}$$

where $H = n\alpha\beta$.

On the reader's side, consider a consumer who prefers type-$R$ outlets. This consumer exhibits reliance $\overline{\beta}$ on stories from type-$R$ outlets. We have

$$\alpha\overline{\beta} = \frac{\overline{\tau}}{1 + 0.5(\overline{\tau} + \underline{\tau})n}, \tag{OA2}$$

where

$$\overline{\tau} = \frac{1}{\frac{1-\gamma}{\gamma} + \frac{\chi}{\overline{z}\alpha^2 v_\theta}}, \tag{OA3}$$

$$\underline{\tau} = \frac{1}{\frac{1-\gamma}{\gamma} + \frac{\chi}{\underline{z}\alpha^2 v_\theta}}. \tag{OA4}$$

For stories from outlets with the opposite bias, this consumer exhibits reliance $\underline{\beta}$ on their stories, where

$$\alpha\underline{\beta} = \frac{\underline{\tau}}{1 + 0.5(\overline{\tau} + \underline{\tau})n}. \tag{OA5}$$

The case for the opposite consumer who prefers type $L$ outlets is analogous. If we let $\tau = 0.5(\overline{\tau} + \underline{\tau})$, then these equations can be combined to obtain:

$$\alpha\beta = \frac{\tau}{1 + n\tau}.$$

Because $H = n\alpha\beta$, we have

$$1 - H = \frac{1}{1 + 0.5(\overline{\tau} + \underline{\tau})n}. \tag{OA6}$$

Now, consider the determination of $\overline{z}$ and $\underline{z}$. We use the new cost function, i.e., the marginal cost of attention is $p - \Delta$ for aligned media or $p + \Delta$ for misaligned media. The first-order conditions for $\overline{z}$ and $\underline{z}$ are:

$$\begin{aligned}
\left(\frac{\overline{\tau}}{1 + J\tau}\right)^2 \frac{1}{\overline{z}^2\alpha^2} - \frac{p - \Delta}{\chi} &= 0, \\
\left(\frac{\underline{\tau}}{1 + J\tau}\right)^2 \frac{1}{\underline{z}^2\alpha^2} - \frac{p + \Delta}{\chi} &= 0.
\end{aligned}$$

These expressions can also be written as

$$\frac{\overline{\beta}}{\overline{z}} = \sqrt{\frac{p - \Delta}{\chi}}, \tag{OA7}$$

$$\frac{\underline{\beta}}{\underline{z}} = \sqrt{\frac{p + \Delta}{\chi}}. \tag{OA8}$$

Equations (OA1) to (OA8) give eight equations in eight unknowns, $(\alpha, \underline{\beta}, \overline{\beta}, \underline{\tau}, \overline{\tau}, \underline{z}, \overline{z}, H)$.

We use $H = n\alpha\beta$ to substitute $\beta$ from equation (OA1) and obtain

$$\phi\alpha^2 = \gamma\frac{H(1 - H)}{n} - (1 - \gamma)\frac{H^2}{n^2}. \tag{OA9}$$

From equation (OA6), equation (OA2) can be written as $\alpha\overline{\beta} = (1 - H)\overline{\tau}$. Using the definition of $\overline{\tau}$ in equation (OA3), we have

$$1 - H = \alpha\overline{\beta}\frac{1 - \gamma}{\gamma} + \frac{\chi}{\alpha v_\theta}\frac{\overline{\beta}}{\overline{z}}.$$

Similarly,

$$1 - H = \alpha\underline{\beta}\frac{1 - \gamma}{\gamma} + \frac{\chi}{\alpha v_\theta}\frac{\underline{\beta}}{\underline{z}}.$$

Summing these two equations and using (OA7) and (OA8), we have

$$1 - H = \alpha\beta\frac{1 - \gamma}{\gamma} + \frac{\sqrt{\chi}}{\alpha v_\theta}k,$$

where $k \equiv 0.5(\sqrt{p-\Delta}+\sqrt{p+\Delta})$. Using $\alpha\beta = H/n$, this equation further reduces to

$$H = \frac{1-\frac{\sqrt{\chi}}{\alpha v_\theta}k}{1+\frac{1}{n}\frac{1-\gamma}{\gamma}}. \tag{OA10}$$

Combining equations (OA9) and (OA10), we have

$$(n\gamma+1-\gamma)v_\theta^2\phi\alpha^4 - \gamma^2 v_\theta\sqrt{\chi}k\alpha + \gamma^2\chi k^2 = 0. \tag{OA11}$$

The left-hand side of (OA11) decreases and then increases in $\alpha$. There are two roots, and we focus on the larger one because the equilibrium corresponding to the larger root is locally stable. This focus corresponds to the convention in the main text of picking the larger root for the $D_j(\cdot)$ function when there are two solutions to the key equation (16).

Differentiating equation (OA11) with respect to $n$, we have

$$\left[4(n\gamma+1-\gamma)v_\theta^2\phi\alpha^3 - \gamma^2 v_\theta\sqrt{\chi}k\right]\frac{\partial\alpha}{\partial n} = -\gamma v_\theta^2\phi\alpha^4.$$

The term in brackets is positive since the left-hand-side of equation (OA11) is increasing in $\alpha$ at the larger root. Thus, $\partial\alpha/\partial n < 0$. □

## B. Supplementary Materials for Section IV.B

Using the generalized utility function for news outlet $j$, we can perform the same exercise as in the benchmark model to derive its reporting strategy. This process leads to

$$\alpha_j\beta_j = \frac{\gamma_j\beta_j(\beta_j(1-H)+\lambda\phi_j)}{(1-\gamma_j)\beta_j^2+\phi_j} =: F_1(\beta_j). \tag{OA12}$$

On the reader's side, the optimal action rule is derived from Bayes' rule and implies

$$\alpha_j\beta_j = \frac{1-H}{\frac{1-\gamma_j}{\gamma_j}+\frac{\chi}{z_j\alpha_j^2 v_\theta}} =: F_2(\alpha_j; z_j). \tag{OA13}$$

We drop the subscript $j$ whenever it is unlikely to cause confusion. We first show that $F_1' \in (0, 2\alpha)$ if $\phi > 1-\gamma$. Taking the derivative,

$$F_1' = \frac{2\beta\gamma(1-H)+\gamma\lambda\phi}{(1-\gamma)\beta^2+\phi} - \frac{2\beta(1-\gamma)}{(1-\gamma)\beta^2+\phi}F_1 < 2\left(\frac{\beta\gamma(1-H)+\gamma\lambda\phi}{(1-\gamma)\beta^2+\phi}\right) = \frac{2}{\beta}F_1 = 2\alpha.$$

We can also express the derivative explicitly as:

$$F_1' = \frac{\gamma\phi\left(2\beta(1-H)+\lambda\phi-(1-\gamma)\lambda\beta^2\right)}{\left((1-\gamma)\beta^2+\phi\right)^2}.$$

A sufficient condition for $F_1' > 0$ is $\phi > 1-\gamma$.

We next show that $F_2' \in (0, 2\beta)$. Taking the derivative,

$$F_2' = \frac{1-H}{\left(\frac{1-\gamma}{\gamma}+\frac{\chi}{z\alpha^2 v_\theta}\right)^2}\frac{\chi}{z v_\theta}\frac{2}{\alpha^3} = \frac{F_2}{\alpha}\frac{2\frac{\chi^2}{z\alpha^2\sigma_\theta^2}}{\frac{1-\gamma}{\gamma}+\frac{\chi^2}{z\alpha^2\sigma_\theta^2}} < 2\frac{F_2}{\alpha} = 2\beta.$$

The fact that $F_2' > 0$ is obvious.

**Proof of Lemma 5.** For given $z_j$ and $H$, equations (OA12) and (OA13) give two equations with two unknowns. Let $\hat{\alpha}_j(z_j;H)$ and $\hat{\beta}_j(z_j;H)$ represent the solution to this equation system. Implicit differentiation gives

$$\frac{\partial\hat{\beta}_j(z_j;H)}{\partial z_j} = \frac{\beta_j F_{2z}}{\alpha_j\beta_j-(\alpha_j-F_1')(\beta_j-F_2')}.$$

From equation (OA13), it is clear that $F_{2z} > 0$. Furthermore, since $F_1' \in (0, 2\alpha_j)$ (when $\phi_j > 1-\gamma_j$) and $F_2' \in (0, 2\beta_j)$, the denominator is also positive. This result establishes that $\hat{\beta}_j(\cdot;H)$ is increasing.

We next show that $\hat{\beta}_j(z_j;H)/z_j$ is single-crossing from above. Substituting equation (OA12) into (OA13) yields

$$\left(\frac{1-\gamma_j}{\gamma_j}+\frac{\chi}{z_j\alpha_j^2 v_\theta}\right)\gamma_j\beta_j(\beta_j(1-H)+\lambda\phi_j) = (1-H)\left((1-\gamma_j)\beta_j^2+\phi_j\right).$$

This equation simplifies to:

$$\chi\gamma_j\beta_j(\beta_j(1-H)+\lambda\phi_j) = z_j\alpha_j^2 v_\theta\phi_j\left(1-H-(1-\gamma_j)\beta_j\lambda\right)$$

Multiply both sides by $\beta_j^2$ and use equation (OA12) again:

$$\chi\gamma_j\beta_j^3(\beta_j(1-H)+\lambda\phi_j) = z_j v_\theta\phi_j\left(1-H-(1-\gamma_j)\beta_j\lambda\right)\left(\frac{\gamma_j\beta_j(\beta_j(1-H)+\lambda\phi_j)}{(1-\gamma_j)\beta_j^2+\phi_j}\right)^2$$

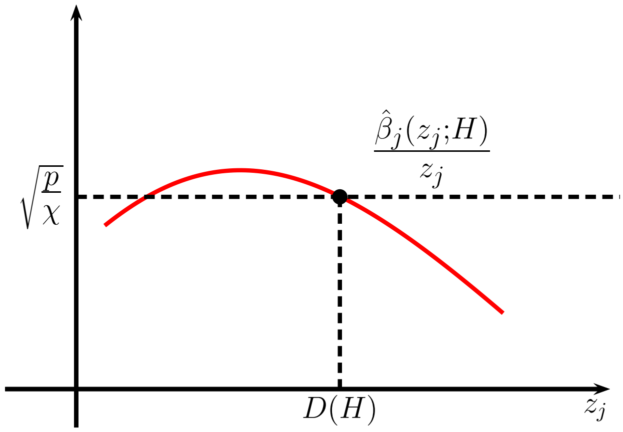


***Figure 1.*** *When the objective function takes the general form (i.e., equation (18)), the reduced-form marginal benefit of paying attention increases and then decreases in $z_j$. The set of parameters used in this numerical example are $\gamma = 0.5$, $\chi = 0.3$, $v_\theta^2 = 1$, $\phi_j = 0.5$, $\lambda = 0.7$, $H = 0.5$ and $p = 1.2$.*

This equation simplifies to

$$\frac{\beta_j}{z_j} = \frac{\gamma_j v_\theta \phi_j}{\chi} \left( \frac{(1 - H - (1-\gamma_j)\beta_j\lambda)\,(\beta_j(1-H) + \lambda\phi_j)}{((1-\gamma_j)\beta_j^2 + \phi_j)^2} \right). \tag{OA14}$$

Denote the term in parentheses on the right-hand side of (OA14) by $\Omega$. We have

$$\frac{\partial\Omega}{\partial\beta_j} = \frac{\gamma_j v_\theta \phi_j}{\chi((1-\gamma_j)\beta_j^2 + \phi_j)^2} \times \left( (1-H)^2 - (1-\gamma_j)\lambda^2\phi_j - 4(1-\gamma_j)\beta_j \left( (1-\gamma_j)\beta_j^2 + \phi_j \right) \Omega \right). \tag{OA15}$$

At the point where $\partial\Omega/\partial\beta_j = 0$, the second derivative is

$$\frac{\partial^2\Omega}{\partial\beta_j^2} = \frac{\gamma_j v_\theta \phi_j}{\chi((1-\gamma_j)\beta_j^2 + \phi_j)^2} \left( -4(1-\gamma_j) \left( 3(1-\gamma_j)\beta_j^2 + \phi_j \right) \Omega \right) < 0.$$

This result shows that $\partial\Omega/\partial\beta_j$ is single-crossing from above in $\beta_j$, which means that $\Omega$ is quasi-concave in $\beta_j$. Since we have already established that $\hat{\beta}_j(z_j;H)$ increases in $z_j$, $\Omega$ is quasi-concave in $z_j$. Therefore, $\hat{\beta}_j(z_j;H)/z_j$ is quasi-concave in $z_j$. □

Suppose that the term $(1-\gamma_j)\lambda^2\phi_j$ is small. An inspection of equation (OA15) shows that the right-hand side of equation (OA15) is positive when $\beta_j$ is small and negative when $\beta_j$ is large. That is, $\beta_j/z_j$ is increasing and then decreasing. Figure 1 illustrates an example where $\lambda = 0.7$.