Supplementary Appendix to "On the Heterogeneity in Family Earnings and Income Dynamics in the PSID"

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First-stage controls.

The full set of first-stage controls in a regression used to extract income residuals is: head's year-of-birth dummies, a dummy for outside dependents, a dummy for extra earners other than the head and wife, year dummies; and race, region of residence, proximity to a big city, unemployment, employment, temporary leave, family size, and the number of kids dummies, all interacted with year dummies.

Details on estimation of the income process with heterogenous parameters.

The income process with heterogeneity is:

$$\begin{aligned} y_{it} =&(1-\rho_i)\alpha_i + \rho_i\gamma_i + \rho_i y_{it-1} + \\ &(1-\rho_i)\gamma_i t + \varepsilon_{it} + \theta_i\varepsilon_{it-1}, \;\; t>0, \end{aligned}$$

where y_{it} is individual i's income at age t.

The parameters are assumed to be generated by the following factor structure:

$$\begin{aligned} \sigma_i^2 &= \exp(\psi_1 + \phi_{11}\eta_{i1}), \\ \sigma_{it}^2 &= \sigma_i^2 \exp\left(v_1 \cdot (\text{time trend}) + v_2 \cdot (\text{time trend})^2\right), \\ \theta_i &= \frac{\psi_2 + \sum_{k=1}^{2}\phi_{2k}\eta_{ik}}{1+|\psi_2 + \sum_{k=1}^{2}\phi_{2k}\eta_{ik}|}, \\ \gamma_i &= \psi_3 + \sum_{k=1}^{3}\phi_{3k}\eta_{ik}, \\ \rho_i &= l(\psi_4 + \sum_{k=1}^{4}\phi_{4k}\eta_{ik}), \\ \alpha_i &= \psi_5 + \sum_{k=1}^{5}\phi_{5k}\eta_{ik}, \\ y_{i0} &= \psi_6 + \alpha_i + \sum_{k=1}^{4}\phi_{6k}\eta_{ik} + \phi_{66}\eta_{i6}, \end{aligned}$$

where $\theta_i \in (-1,1)$, $l(x) = \frac{\exp(x)}{1+\exp(x)} \in (0,1)$, so that $\rho_i \in (0,1)$, and $\eta_{ik} \sim iidN(0,1)$, $k = 1,\ldots,6$. Since our residual incomes average out to zero in a cross-section, we normalized ψ_3 and ψ_5 to zero; ψ_6 is allowed to be nonzero, to capture potentially nonzero intercepts in individual regressions of incomes on their lags and an age trend that are used for fitting the model. In a model with codependent heterogeneity we therefore estimated 26 parameters. To identify the model parameters, we used the moments in Browning, Ejrnæs and Alvarez (2010) but dropped one moment used to identify the variance of measurement error as we don't model measurement error separately from an income innovation (this choice was also made in Browning and Ejrnæs (2013) and Blundell, Pistaferri and Preston (2008)). We also added a moment used to identify the variance of permanent shocks in Meghir and Pistaferri (2004), $E\left[\Delta y_{it}(y_{it+2} - y_{it-3})\right]$, the third, fourth and fifth-order autocovariances of income growth rates, and the first ten autocorrelations of income levels. This gave us 58 moments for identification of the model (44 moments from Browning, Ejrnæs and Alvarez (2010) plus 14 extra moments added by us).

In a restricted model with independent heterogeneity, we used 37 moments (dropping the "correlation" moments from the full set of 58 moments) to identify 12 parameters: ψ_k, $i = 1,2,4,6$, and ϕ_{kk}, $k = 1,\ldots,6$, and v_k, $k = 1,2$.

REFERENCES

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Table 1—Cross-sectional characteristics for sample and nonsample families.

	Means		Same means, p-value
	Sample sons	Nonsample	
Head's age	36.58	36.02	14%
Wife's age	34.28	33.43	2%
Net fam. income	38476	38711	84%
Head's earnings	27394	27623	81%
Wife's earnings	9199	9234	95%
Transfers, head and wife	997	1147	32%
Transfers, family	1095	1197	52%
If others have inc.	0.24	0.20	4%
Head's hours	2204	2225	44%
Wife's hours	1144	1164	64%
Head works	0.98	0.98	22%
Wife works	0.78	0.80	24%
No. children	1.62	1.60	85%
Fam. size	3.74	3.70	52%
Northeast	0.19	0.22	37%
Midwest	0.31	0.31	99%
South	0.31	0.30	73%
West	0.20	0.18	60%
MSA: largest city more than 100,000	0.39	0.43	19%
Percent tot. fam. inc., major assign.	2.50	2.72	75%
Percent tot. fam. inc., minor assign.	3.13	3.30	82%
Head changed occ.	0.35	0.35	90%
Head changed industry	0.29	0.32	16%
Head becomes disabled	0.09	0.08	20%
Head displaced	0.05	0.06	22%
Fam. owns business	0.19	0.22	7%
Owns house	0.82	0.80	29%
No. tax exemptions, head and wife	3.73	3.70	67%
No college	0.40	0.43	25%
College	0.61	0.57	25%
White	0.94	0.93	29%
Black	0.05	0.05	73%
Region grew: Northeast	0.21	0.24	30%
Region grew: Midwest	0.35	0.35	81%
Region grew: South	0.27	0.26	86%
Region grew: West	0.16	0.13	19%
Region grew: Foreign	0.02	0.01	32%
No. obs. in inc. spell	12.84	12.59	10%

Note: p-values are calculated by bootstrap. Number of sample (nonsample) families 514 (468).