Online Appendix to accompany

"Active Investors, Passive Investors, and Common Ownership"

by Oz Shy and Rune Stenbacka

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Derivation of Table 1

Due to space constraints, with no significant loss of information, Table 1 in the paper omits the third row and third column. The complete Table 1 is:

		Investor B		
		$\langle s_1^B, s_2^B\rangle = \langle \frac{I}{2}, \frac{I}{2}\rangle$	$\langle s_1^B, s_2^B\rangle = \langle 0, I\rangle$	$\langle s_1^B, s_2^B\rangle = \langle I, 0\rangle$
	$\langle s_1^A, s_2^A\rangle = \langle \frac{I}{2}, \frac{I}{2}\rangle$	$q_1 = q_2 = \frac{\alpha}{4\beta}$, $p = \frac{\alpha}{2}$	$q_1 = 0$, $q_2 = \frac{\alpha}{2\beta}$, $p = \frac{\alpha}{2}$	$q_1 = \frac{\alpha}{2\beta}$ $q_2 = 0$, $p = \frac{\alpha}{2}$
A	$\langle s_1^A, s_2^A\rangle = \langle I, 0\rangle$	$q_1 = \frac{\alpha}{2\beta}$ $q_2 = 0$, $p = \frac{\alpha}{2}$	$q_1 = q_2 = \frac{\alpha}{3\beta}$, $p_* = \frac{\alpha}{3}$	$q_1 = q_2 = \frac{\alpha}{3\beta}$, $p_* = \frac{\alpha}{3}$
	$\langle s_1^A, s_2^A\rangle = \langle 0, I\rangle$	$q_1 = 0$, $q_2 = \frac{\alpha}{2\beta}$, $p = \frac{\alpha}{2}$	$q_1 = q_2 = \frac{\alpha}{3\beta}$, $p_* = \frac{\alpha}{3}$	$q_1 = q_2 = \frac{\alpha}{3\beta}$, $p_* = \frac{\alpha}{3}$

Table 1: Equilibrium production levels and market price (q_1, q_2, p) as functions of active investors' ownership shares in the producing firms. *Note*: Subscript * indicates lowest price.

The above table shows that all the entries of aggregate values (p and $q_1 + q_2$) on the third row and the third column are duplicated by some entries on the reduced 2×2 table presented in the paper. In addition, the second and third rows as well as the second and third columns correspond to symmetric outcomes. Therefore, no significant information is lost by focusing the analysis on the reduced 2×2 Table 1 presented in the paper.

To derive the first-row first-column entry in Table 1, substituting $s_1^A = s_2^A = s_1^B = s_2^B = \frac{I}{2}$ into (7), the two first-order conditions are: $0 = \frac{\partial \pi^A}{\partial q_1} = \frac{\partial \pi^B}{\partial q_2} = I(\alpha - 2\beta q_1 - 2\beta q_2)/2$. That is, the two first-order conditions are identical, which leaves us with one equation $\alpha = 2\beta(q_1 + q_2)$ and two variables. Focusing on the symmetric equilibrium only yields $q_1 = q_2 = \alpha/4\beta$. Substituting into (1) yields $p = \alpha/2$.

To derive the second-row first-column entry in Table 1, substituting $s_1^A = I$, $s_2^A = 0$ and $s_1^B = s_2^B = \frac{I}{2}$ into (7), the two first-order conditions are: $0 = \frac{\partial \pi^A}{\partial q_1} = I(\alpha - 2\beta q_1 - \beta q_2)$ and

$0 = \frac{\partial \pi^B}{\partial q_2} = I(\alpha - 2\beta q_1 - 2\beta q_2)/2$. Hence, $q_1 = \alpha/(2\beta)$ and $q_2 = 0$. Substituting into (1) yields $p = \alpha/2$. Note that the third-row first-column, first-row second-column, and first-row third-column are similarly derived by either exchanging firm 1 with firm 2, or investor A with investor B.

To derive the second-row second-column entry in Table 1, substituting $s_1^A = s_2^B = I$ and $s_2^A = s_1^B = 0$ into (7), the two first-order conditions are: $0 = \frac{\partial \pi^A}{\partial q_1} = I(\alpha - 2\beta q_1 - \beta q_2)$ and $0 = \frac{\partial \pi^B}{\partial q_2} = I(\alpha - \beta q_1 - 2\beta q_2)$. Hence, $q_1 = q_2 = \alpha/(3\beta)$. Substituting into (1) yields $p = \alpha/3$. Note that the third-row third-column entry in Table 1 can be similarly derived by exchanging investor A with investor B.

The derivations of the second-row third-column $\langle s_1^A, s_2^A\rangle = \langle s_1^B, s_2^B\rangle = \langle I, 0\rangle$ and the third-row second-column or $\langle s_1^A, s_2^A\rangle = \langle s_1^B, s_2^B\rangle = \langle 0, I\rangle$ are slightly different because both investor A and B concentrate their portfolios on one producing firm while leaving the other producing firm with no active investors. Consider the case $\langle s_1^A, s_2^A\rangle = \langle s_1^B, s_2^B\rangle = \langle I, 0\rangle$. Either investor A (or investor B as there is no conflict of interest) chooses q_1 to maximize π^A given in (7). Under Assumption (b), firm 2 with only passive investors chooses q_2 to maximize π_2 given in (2). The two first-order conditions are: $0 = \frac{\partial \pi^A}{\partial q_1} = \frac{\partial \pi^B}{\partial q_1} = I(\alpha - 2\beta q_1 - \beta q_2)$ and $0 = \frac{\partial \pi_2}{\partial q_2} = I(\alpha - \beta q_1 - 2\beta q_2)/2$. Hence, $q_1 = q_2 = \alpha/(3\beta)$. Substituting into (1) yields $p = \alpha/3$.

Result 1

When investors specialize in only one producing firms (instead of diversifying between the two producing firms), aggregate industry output is the highest because

$$q_1 + q_2 = 2\,\frac{\alpha}{3\beta} > 2\,\frac{\alpha}{4\beta} = \frac{\alpha}{2\beta} + 0 = 0 + \frac{\alpha}{2\beta}.$$

The resulting market price is the lowest because $\alpha/3 < \alpha/2$.

Table 2

Due to space constraints, with no significant loss of information, Table 2 in the paper omits the third row and and the third column. The complete Table 2 is:

		Investor B		
		$\langle s_1^B, s_2^B\rangle = \langle \frac{I}{2}, \frac{I}{2}\rangle$	$\langle s_1^B, s_2^B\rangle = \langle 0, I\rangle$	$\langle s_1^B, s_2^B\rangle = \langle I, 0\rangle$
	$\langle s_1^A, s_2^A\rangle = \langle \frac{I}{2}, \frac{I}{2}\rangle$	$\frac{I\alpha^2}{8\beta}, \frac{I\alpha^2}{8\beta}$	(e) $\frac{I\alpha^2}{8\beta}, \frac{I\alpha^2}{4\beta}$	(e) $\frac{I\alpha^2}{8\beta}, \frac{I\alpha^2}{4\beta}$
Investor A	$\langle s_1^A, s_2^A\rangle = \langle I, 0\rangle$	(e) $\frac{I\alpha^2}{4\beta}, \frac{I\alpha^2}{8\beta}$	$\frac{I\alpha^2}{9\beta}, \frac{I\alpha^2}{9\beta}$	$\frac{I\alpha^2}{9\beta}, \frac{I\alpha^2}{9\beta}$
	$\langle s_1^A, s_2^A\rangle = \langle 0, I\rangle$	(e) $\frac{I\alpha^2}{4\beta}, \frac{I\alpha^2}{8\beta}$	$\frac{I\alpha^2}{9\beta}, \frac{I\alpha^2}{9\beta}$	$\frac{I\alpha^2}{9\beta}, \frac{I\alpha^2}{9\beta}$

Table 2: Profits earned by investors A and B (π^A, π^B) as functions of their ownership shares in the producing firms. *Note*: (e) denotes equilibrium profits of investors A and B.

The above table shows that the second and third rows as well as the second and third columns correspond to symmetric outcomes. Therefore, no significant information is lost by focusing the analysis on the reduced 2×2 Table 2 presented in the paper.

Result 2

Consider the outcome with the payoffs (investors' profits) displayed on the first-row and second-column in Table 2. Then, investor A will not deviate because

$$\pi^A = \frac{I\alpha^2}{8\beta} > \frac{I\alpha^2}{9\beta}.$$

Investor B will not deviate because

$$\pi^B = \frac{I\alpha^2}{4\beta} \geq \frac{I\alpha^2}{4\beta} > \frac{I\alpha^2}{8\beta}.$$

The proofs for the other 3 Nash equilibria are basically the same.

Result 3

Recall that $I \leq 1/2$. Comparing π_*^P displayed in the first-row and first-column to other π^P in Table 3 yields:

$$\frac{\alpha^2(1-I)}{4\beta} - \frac{\alpha^2(2-3I)}{8\beta} = \frac{I\alpha^2}{8\beta} > 0 \quad \text{and} \quad \frac{\alpha^2(1-I)}{4\beta} - \frac{2\alpha^2(1-I)}{9\beta} = \frac{\alpha^2(1-I)}{36\beta} > 0.$$

Result 4

Comparing CS_* displayed in the second-row and second-column to other CS in Table 3 yields:

$$\frac{2\alpha^2}{9\beta} - \frac{\alpha^2}{8\beta} = \frac{7\alpha^2}{72\beta} > 0.$$

Comparing W_* displayed in the second-row and second-column to other W in Table 3 yields:

$$\frac{4\alpha^2}{9\beta} - \frac{3\alpha^2}{8\beta} = \frac{5\alpha^2}{72\beta} > 0.$$