Arrow Meets Hotelling: Modeling Spatial Innovation
— Online Appendix —

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Proof of Proposition 1

The utility of a consumer located at $s \in \mathbb{R}$ who consumes product j with known quality v_j and pays price p_{sj} is given by $u_{sj} = v_j - t\,|s - l_j| - p_{sj}$, where $t > 0$ are the "transportation costs."

In period 0, the incumbent's only product is located at $l_0 = 0$. The incumbent offers this product at $v_0 - t\,|s|$ to consumers $s \in [-v_0/t, v_0/t]$, each of whom accepts the offer. The incumbent's profits in period 0 are therefore given by

$$\pi_I^0 = \int_{-\frac{v_0}{t}}^{\frac{v_0}{t}} v_0 - t\,|s|\,\mathrm{d}s = \frac{v_0^2}{t}.$$

Suppose the incumbent owns the entrant in period 1 and that the location of product 1 satisfies $l_1 \leq 2\frac{v_0}{t}$. The incumbent now offers product 0 at $v_0 - t\,|s|$ to consumers $s \in \left[-\frac{v_0}{t}, \frac{l_1}{2}\right]$ and product 1 at $v_0 - t\,|s - l_1|$ to consumers $s \in \left[\frac{l_1}{2}, l_1 + \frac{v_0}{t}\right]$, where we used the fact that $\mathrm{E}[v_1] = v_0$. Each consumer accepts the offer. If $l_1 \leq 2\frac{v_0}{t}$ the incumbent's profits (gross of development costs) are therefore given by

$$\pi_I^1 = \int_{-\frac{v_0}{t}}^{\frac{1}{2}l_1} v_0 - t\,|s|\,\mathrm{d}s + \int_{\frac{1}{2}l_1}^{l_1+\frac{v_0}{t}} v_0 - t\,|s - l_1|\,\mathrm{d}s = \frac{2v_0^2}{t} - \frac{\left(v_0 - \frac{1}{2}tl_1\right)^2}{t}.$$

It is easy to verify that profits for $l_1 > 2\frac{v_0}{t}$ are the same as those for $l_1 = 2\frac{v_0}{t}$.

At the beginning of period 1, the incumbent's problem is then given by

$$\max_{l_1} \frac{2v_0^2}{t} - \frac{\left(v_0 - \frac{1}{2}tl_1\right)^2}{t} - c\,(l_1) \tag{1}$$

and its unique solution l_1^I is implicitly defined by the first order condition

$$v_0 - \frac{1}{2}tl_1 = c'\,(l_1)\,. \tag{2}$$

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Suppose now the entrant in period 1 is independent and suppose once again that $l_1 \le 2\frac{v_0}{t}$. The incumbent offers product 0 at $v_0 - t|s|$ to consumers $s \in \left[-\frac{v_0}{t}, l_1 - \frac{v_0}{t}\right]$, at $v_0 - t|s| - (v_0 - t|s - l_1|)$ to consumers $s \in \left[l_1 - \frac{v_0}{t}, \frac{l_1}{2}\right]$, and at 0 to anyone else. Similarly, the entrant sells product 1 at $v_0 - t|s - l_1|$ to consumers $s \in \left[\frac{v_0}{t}, l_1 + \frac{v_0}{t}\right]$, at $v_0 - t|s - l_1| - (v_0 - t|s|)$ to consumers $\left[\frac{l_1}{2}, \frac{v_0}{t}\right]$ and at 0 to anyone else. Given these prices, consumers $s \in \left[-\frac{v_0}{t}, \frac{l_1}{2}\right]$ buy product 0 and consumers $s \in \left[\frac{l_1}{2}, l_1 + \frac{v_0}{t}\right]$ buy product 1. If $l_1 \le 2\frac{v_0}{t}$, the entrant's profits are therefore given by

$$\pi_E^1 = \int_{\frac{1}{2}l_1}^{\frac{v_0}{t}} v_0 - t|s - l_1| - (v_0 - t|s|)\,\mathrm{d}s + \int_{\frac{v_0}{t}}^{l_1 + \frac{v_0}{t}} v_0 - t|s - l_1|\,\mathrm{d}s = \frac{v_0^2}{t} - \frac{\left(v_0 - \frac{1}{2}tl_1\right)^2}{t}.$$

It is easy to verify that the profits for $l_1 > 2\frac{v_0}{t}$ are the same as those for $l_1 = 2\frac{v_0}{t}$. The entrant's problem is therefore the same as the incumbent's problem (1) so that $l_1^E = l_1^I$. ■

Proof of Proposition 2

In period 1, the utility of a consumer who buys product 1 but consumed product 0 in period 0 is $u_{s1} - \gamma$, where $\gamma \in [0, v_0]$ are the switching costs. Switching costs are immaterial if the entrant is owned by the incumbent. In this case, the location of product 1 is still given by l_1^I.

Suppose that the entrant is independent and that $l_1 \le 2\frac{v_0}{t} - \frac{\gamma}{t}$. The incumbent offers product 0 at $v_0 - t|s|$ to consumers $s \in \left[-\frac{v_0}{t}, l_1 - \frac{1}{t}(v_0 - \gamma)\right]$, at $v_0 - t|s| - (v_0 - \gamma - t|s - l_1|)$ to consumers $s \in \left[l_1 - \frac{1}{t}(v_0 - \gamma), \frac{1}{2}l_1 + \frac{1}{2t}\gamma\right]$, and at 0 to anyone else. Similarly, the entrant offers product 1 at $v_0 - t|s - l_1|$ to consumers $s \in \left[\frac{v_0}{t}, l_1 + \frac{v_0}{t}\right]$, at $v_0 - \gamma - t|s - l_1| - (v_0 - t|s|)$ to consumers $\left[\frac{1}{2}l_1 + \frac{1}{2t}\gamma, \frac{v_0}{t}\right]$ and at 0 to anyone else. Given these prices, consumers $s \in \left[-\frac{v_0}{t}, \frac{1}{2}l_1 + \frac{1}{2t}\gamma\right]$ buy product 0 and consumers $s \in \left[\frac{1}{2}l_1 + \frac{1}{2t}\gamma, l_1 + \frac{v_0}{t}\right]$ buy product 1. If $l_1 \le 2\frac{v_0}{t} - \frac{\gamma}{t}$, the entrant's profits are therefore given by

$$\begin{aligned}\pi_E^1(\gamma) &= \int_{\frac{1}{2}l_1 + \frac{1}{2t}\gamma}^{\frac{v_0}{t}} v_0 - \gamma - t|s - l_1| - (v_0 - t|s|)\,\mathrm{d}s + \int_{\frac{v_0}{t}}^{l_1 + \frac{v_0}{t}} v_0 - t|s - l_1|\,\mathrm{d}s \\ &= \frac{v_0^2}{t} - \frac{\left(v_0 - \frac{1}{2}tl_1\right)^2}{t} - \frac{1}{4t}\gamma(-\gamma + 4v_0 - 2tl_1)\end{aligned}$$

Similar reasoning shows that if $2\frac{v_0}{t} - \frac{\gamma}{t} \le l_1 \le 2\frac{v_0}{t}$, profits are given by

$$\pi_E^1(\gamma) = \int_{\frac{v_0}{t}}^{l_1} (v_0 - t(l_1 - s))\,\mathrm{d}s + \int_{l_1}^{l_1 + \frac{v_0}{t}} (v_0 - t(s - l_1))\,\mathrm{d}s = \frac{v_0^2}{t} - \frac{2\left(v_0 - \frac{1}{2}tl_1\right)^2}{t}.$$

and that profits for $l_1 > 2\frac{v_0}{t}$ are the same as those for $l_1 = 2\frac{v_0}{t}$.

At the beginning of period 1, the entrant's problem is given by

$$\max_{l_1} \pi_E^1(\gamma) - c(l_1)$$

The unique solution $l_E^1(\gamma)$ to this problem is implicitly defined by the first order conditions.

$$\begin{aligned} v_0 - \frac{1}{2} t l_1 + \frac{1}{2}\gamma &= c'(l_1) \text{ if } \gamma \leq c'\left(2\frac{v_0}{t} - \frac{\gamma}{t}\right) \\ 2v_0 - t l_1 &= c'(l_1) \text{ if } \gamma \geq c'\left(2\frac{v_0}{t} - \frac{\gamma}{t}\right). \end{aligned}$$

Comparing these conditions to the first order condition for l_1^E in (2) shows that $l_E^1(0) = l_E^1$ and $l_E^1(\gamma) > l_E^1$ if $\gamma > 0$. ■