A Model of Occupational Licensing and Statistical Discrimination

Online Appendix

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A1. Proof of Theorem 1

To solve this sequential game, we use the solution concept of sub-game perfect equilibrium (SPE). In an SPE, we solve the model using backwards induction. First, workers in period 2 sort in to the sector that produces the highest net return, given wages and their preferences. Next in period 1, the representative firm in each sector chooses the corresponding wage to maximize firm profits, given the sorting of workers.

Period #2: Workers Choose Sector

Starting in period 2, the probability that a worker of ability a_i sorts into the licensed sector, $P(L = 1|a_i)$ is given by the probability that the net benefit of working in the licensed sector is greater than the net benefit of working in the unlicensed sector:

$$\begin{aligned} P(L_i = 1|a_i) = \text{Prob}(V_{L,i} > V_{U,i}) &= \text{Prob}(\omega_L - c_0 - \omega_U + \theta(a_i - \mu_a) > \epsilon_i) \\ &= \frac{1}{2} + \frac{\Delta\omega + \theta(a_i - \mu_a)}{2\sigma_\epsilon}, \end{aligned} \tag{A1}$$

where, $\Delta\omega \equiv (\omega_L - c_0) - (\omega_U + \mu_\epsilon)$ is the expected net benefit of licensing across workers of all types. The conditional probability of licensing is increasing in the expected net benefit of licensing. It is also increasing in worker ability for cases where worker ability lowers the cost of licensing $\theta > 0$ but decreasing in worker ability in cases where worker ability increases the cost of licensing $\theta < 0$.

Period #1: Firms Choose Wages

Next, we must compute firm profits given the sorting decisions of workers. In order to compute profits for the representative firms in both the licensed and unlicensed sectors, we first compute the fraction of workers who sort into the licensed profession and the unlicensed profession, *i.e.*, $E[P(L_i = 1|a_i)]$ and $E[P(L_i = 0|a_i)]$, because these quantities enter the expect labor cost of the firms.

$$\begin{aligned} E[P(L_i = 1|a_i)] = \frac{1}{2\sigma_a}\int_{\mu_a-\sigma_a}^{\mu_a+\sigma_a} P(L_i = 1|a_i)da_i &= \frac{1}{2\sigma_a}\int_{\mu_a-\sigma_a}^{\mu_a+\sigma_a}\left[\frac{1}{2} + \frac{\Delta\omega + \theta(a_i - \mu_a)}{2\sigma_\epsilon}\right]da_i \\ &= \frac{1}{2} + \frac{\Delta\omega}{2\sigma_\epsilon} \end{aligned} \tag{A2}$$

Given that we have a two-sector model, a worker is either employed in the licensed or in the unlicensed sector. Consequently:

$$\begin{aligned} E[P(L_i = 0|a_i)] &= 1 - E[P(L_i = 1|a_i)] \\ &= \frac{1}{2} - \frac{\Delta\omega}{2\sigma_\epsilon} \end{aligned} \tag{A3}$$

To compute firm profits, we must also compute the expected ability level of a worker given that she has a license $E(a_i|L_i = 1)$ and given that she does not have a license $E(a_i|L_i = 0)$ both of which contribute to firm revenue:

$$
\begin{aligned}
E[a_i|L_i=1] &= \int_{\mu-\sigma_a}^{\mu+\sigma_a} a_i \frac{P(L_i=1|a_i)P(a_i)}{P(L_i=1)} da_i = \frac{1}{2\sigma_a}\int_{\mu-\sigma_a}^{\mu+\sigma_a} a_i \frac{\left[\frac{1}{2}+\frac{\Delta\omega+\theta(a_i-\mu_a)}{2\sigma_\epsilon}\right]}{\frac{1}{2}+\frac{\Delta\omega}{2\sigma_\epsilon}} da_i \\
&= \mu_a + \frac{\theta\sigma_a^2}{3(\sigma_\epsilon+\Delta\omega)}
\end{aligned} \tag{A4}
$$

Similarly,

$$
\begin{aligned}
E[a_i|L_i=0] &= \int_{\mu-\sigma_a}^{\mu+\sigma_a} a_i \frac{P(L_i=0|a_i)P(a_i)}{P(L_i=0)} da_i = \frac{1}{2\sigma_a}\int_{\mu-\sigma_a}^{\mu+\sigma_a} a_i \frac{\left[\frac{1}{2}-\frac{\Delta\omega+\theta(a_i-\mu_a)}{2\sigma_\epsilon}\right]}{\frac{1}{2}-\frac{\Delta\omega}{2\sigma_\epsilon}} da_i \\
&= \mu_a - \frac{\theta\sigma_a^2}{3(\sigma_\epsilon-\Delta\omega)}
\end{aligned} \tag{A5}
$$

Putting this all together, we get that profits in the licensed sector are given by:

$$
\pi_1 = \underbrace{\left((1+h)\bar{\omega}\left[\mu_a + \frac{\theta\sigma_a^2}{3(\sigma_\epsilon+\Delta\omega)}\right] - \omega_L\right)}_{\text{Expected Profit per. licensed worker}} \times \underbrace{\left[\frac{1}{2}+\frac{\Delta\omega}{2\sigma_\epsilon}\right]}_{\text{Frac. Licensed workers}}, \tag{A6}
$$

Firm profits in the unlicensed sector are given by:

$$
\pi_2 = \left(\bar{\omega}\left[\mu_a - \frac{\theta\sigma_a^2}{3(\sigma_\epsilon-\Delta\omega)}\right] - \omega_U\right)\left[\frac{1}{2}-\frac{\Delta\omega}{2\sigma_\epsilon}\right] \tag{A7}
$$

Firm 1 chooses ω_L to maximize its profits, π_1. This results in the following first order condition, $\frac{\partial\pi_1}{\partial\omega_L}=0$:

$$
\begin{aligned}
&\underbrace{-\left(1+\left[\frac{(1+h)\bar{\omega}\theta\sigma_a^2}{3(\sigma_\epsilon+\Delta\omega)^2}\right]\right)\left[\frac{1}{2}+\frac{\Delta\omega}{2\sigma_\epsilon}\right]}_{\text{Decrease in Unit Profit}} + \underbrace{\frac{1}{2\sigma_\epsilon}\left((1+h)\bar{\omega}\left[\mu_a+\frac{\theta\sigma_a^2}{3(\sigma_\epsilon+\Delta\omega)}\right]-\omega_L\right)}_{\text{Increase in Volume}} = 0 \\
&\implies \omega_L = -\sigma_\epsilon - \Delta\omega + (1+h)\bar{\omega}\mu_a
\end{aligned} \tag{A8}
$$

To get the best response function of the firm in the licensed sector, we re-arrange the expression above and substitute in the definition for the net benefit of licensing $\Delta\omega = (\omega_L - c_0) - (\omega_U + \mu_\epsilon)$:

$$
\boxed{\omega_L(\omega_U) = \frac{1}{2}[(1+h)\bar{\omega}\mu_a + \omega_U + c_0 + (\mu_\epsilon - \sigma_\epsilon)]} \tag{A9}
$$

The best response function for the wages in the licensed sector is increasing in the level of human capital that is bundled with the license h and with the quality of the firm's technology $\bar{\omega}$. It is also increasing in the wage offered by the unlicensed firm, the cost of licensing and the minimum taste for the unlicensed sector, $\mu_\epsilon - \sigma_\epsilon$.

To find the best response function for firm 2, we assert that firm 2 chooses ω_U to maximize its profits, π_2. When we take the first order condition $\frac{\partial\pi_2}{\partial\omega_U}=0$, we get:

$$
\omega_U = -\sigma_\epsilon + \Delta\omega + \bar{\omega}\mu_a \tag{A10}
$$

To get the best response function of the firm 2, we re-arrange the expression above and use the definition for the net benefit of licensing $\Delta\omega = (\omega_L - c_0) - (\omega_U + \mu_\epsilon)$:

$$\boxed{\omega_U(\omega_L) = \frac{1}{2}[\bar{\omega}\mu_a + (\omega_L - c_0) - (\mu_\epsilon + \sigma_\epsilon)]} \tag{A11}$$

The best response function for the wages in the unlicensed sector is increasing with the quality of the firm's technology $\bar{\omega}$, the average ability of all workers, and the competing wages in the licensed sector. It is decreasing in the cost of obtaining a license and the maximum taste for the unlicensed sector by workers, $\mu_\epsilon + \sigma_\epsilon$. At the Nash equilibrium both firms' wages are mutual best responses. Substituting the best response of the firm in the licensed sector into the best response function for the firm in the unlicensed sector, we solve for the equilibrium wage in the unlicensed sector ω_U^*.

$$\begin{aligned} \omega_U &= \frac{1}{2}[\bar{\omega}\mu_a + -c_0 - (\mu_\epsilon + \sigma_\epsilon)] + \frac{1}{2}\left[\frac{1}{2}[(1+h)\bar{\omega}\mu_a + \omega_U + c_0 + (\mu_\epsilon - \sigma_\epsilon)]\right] \\ \implies & \boxed{\omega_U^* = \left(1 + \frac{1}{3}h\right)\bar{\omega}\mu_a - \frac{1}{3}c_0 - \frac{1}{3}\mu_\epsilon - \sigma_\epsilon} \end{aligned} \tag{A12}$$

To solve for the equilibrium wages in the licensed sector, we insert equilibrium wages from the unlicensed sector into the best response function for the licensed sector:

$$\begin{aligned} \omega_L &= \frac{1}{2}[(1+h)\bar{\omega}\mu_a + c_0 + (\mu_\epsilon - \sigma_\epsilon)] + \frac{1}{2}\left[\left(1 + \frac{1}{3}h\right)\bar{\omega}\mu_a - \frac{1}{3}c_0 - \frac{1}{3}\mu_\epsilon - \sigma_\epsilon\right] \\ \implies & \boxed{\omega_L^* = \left(1 + \frac{2}{3}h\right)\bar{\omega}\mu_a + \frac{1}{3}c_0 + \frac{1}{3}\mu_\epsilon - \sigma_\epsilon} \end{aligned} \tag{A13}$$

To solve for the fraction of licensed workers, we substitute equilibrium wages into the expression for the fraction of licensed workers in equation (A14):

$$f^* = \frac{1}{2} + \frac{\bar{\omega}\mu_a h - c_0 - \mu_\epsilon}{6\sigma_\epsilon}. \tag{A14}$$

Defining $\underline{c} \equiv h\bar{\omega}\mu_a - \mu_\epsilon - 3\sigma_\epsilon$, it is straight forward to show that if the average cost of licensing, c_0, is lower than $\underline{c}$ that licensing is sufficiently cheap. Then, all workers obtain a license and work in the licensed sector ($f = 1$). Likewise, defining $\bar{c} \equiv h\bar{\omega}\mu_a - \mu_\epsilon + 3\sigma_\epsilon$. If the average cost of licensing, c_0, is higher than $\bar{c}$, licensing is sufficiently onerous. Hence, all workers prefer not to obtain a license ($f = 0$). It is only for intermediate value $c_0 \in (\underline{c}, \bar{c})$, that we observe a non-zero fraction of workers in both the licensed and unlicensed sectors.

We further simplify the expression for the fraction of licensed workers in equation (A14) and the equilibrium wages for workers in equations using the definitions for $\bar{c}$ and $\underline{c}$:

$$\boxed{f^* = \left(\frac{\bar{c} - c_0}{6\sigma_\epsilon}\right)}, \tag{A15}$$

$$\boxed{\omega_U^* = \bar{\omega}\mu_a - \frac{1}{3}(c_0 - \underline{c})}, \tag{A16}$$

$$\boxed{\omega_L^* = \omega_U^* + \frac{1}{3}h\bar{\omega}\mu_a + \frac{2}{3}(c_0 + \mu_\epsilon)}. \tag{A17}$$

Corollary 1. *Wages are unambiguously higher in the licensed sector than in the unlicensed sector, and the wedge between these two wages is increasing in the cost of licensing. In equilibrium, unlicensed workers also experience a wage benefit from the human capital that is bundled with the licensing. This wage benefit is half the human capital benefit experienced by licensed workers.*

The fact that licensing is bundled with human capital h increases the market return to licensed labor and, in doing so, increases the value of the outside option of workers who opt not to become licensed. Consistent with this prediction of the model, **?** provide evidence that workers in a licensed occupation who do not possess a license but can practice because of *grandfathering* provisions experience a 5% increase in wages as a result of their occupation becoming licensed, when compared to similar unlicensed workers in occupations with no licensing requirements. By contrast, the wage premium to licensed workers in the occupation, when compared to similar unlicensed workers in occupations with no licensing requirements, is 12 percentage points higher than the wage premium experienced by grandfathered workers.

Corollary 2. *Given two distinct groups of workers B and W such that the average cost of licensing is greater for group B than for group W (*i.e., *$c_{o,B} > c_{0,W}$), unlicensed B workers earn less than unlicensed W workers. By contrast, licensed B workers earn more than licensed W workers,* ceteris paribus*. This follows from the fact that wages are decreasing in c_0 for unlicensed workers (equation* **??***) but increasing in c_0 for licensed workers (equation* **??***).*

The result of this corollary offers testable predictions. First, unlicensed black men earn less, on average, than unlicensed white men. Second, licensed black men working in occupations with felony restrictions earn, on average, slightly more than licensed white men in similar occupations. The presumption here is that the felony restriction imposes a higher average cost of licensing on black men relative to white men. Using data from the Bureau of Justice Statistics, **?** documents that black men are six times more likely to be incarcerated than white men, which is consistent with this assumption.

A2. Proof of Proposition 3

PROOF:

By definition the license premium is:

$$\alpha \equiv \frac{\omega_L^* - \omega_U^*}{\omega_U^*} = \frac{\frac{1}{3}\bar{\omega}\mu_a h + \frac{2}{3}(c_0 + \mu_\epsilon)}{\left(1 + \frac{1}{3}h\right)\bar{\omega}\mu_a - \frac{1}{3}(c_0 + \mu_\epsilon) - \sigma_\epsilon}. \tag{A18}$$

The license premium increases in c_0 because the wage gap (numerator) increases in c_0 and the wage in the unlicensed sector (denominator) is decreasing in c_0. In particular, the derivative of the licensing premium with respect to c_0 is:

$$\frac{d\alpha}{dc_0} = \frac{1}{3}\left(\frac{\omega_L - \omega_U}{\omega_U^2}\right) > 0. \tag{A19}$$

The derivative of the licensing premium with respect to the mean ability is:

$$\frac{d\alpha}{d\mu_a} = -\frac{\bar{\omega}[h(\mu_\epsilon + \sigma_\epsilon + c_0) + 2(c_0 + \mu_\epsilon)]}{3{\omega_U^*}^2} \implies \frac{d\alpha}{d\mu_a} < 0. \tag{A20}$$

The derivative of the licensing premium with respect to h is:

$$\frac{d\alpha}{dh} = \frac{\bar{\omega}\mu_a[2\omega_U^* - \omega_L^*]}{3{\omega_U^*}^2} \tag{A21}$$

Therefore $\frac{d\alpha}{dh} > 0 \implies 2\omega_U^* - \omega_L^* > 0$, which holds when $\frac{\omega_L^* - \omega_U^*}{\omega_U^*} < 1$ (*i.e.*, $\alpha < 1$).

The positive relationship between the licensing premium and the dispersion in sector taste comes from the fact that wages in the unlicensed sector (denominator) fall with σ_ϵ.

A3. Proof of Proposition 4

The total social surplus is the sum of the firm's revenue minus the expected cost of licensing. Since the expected wages of employees is a cost to firms and a benefit to workers, it nets out in the social surplus calculation, in the case where we place an equal weighting on firm profits and net worker wages:

$$\begin{aligned}
SS =& \underbrace{(1+h)\bar{\omega}\left(\mu_a + \frac{\theta\sigma_a^2}{3(\sigma_\epsilon + \Delta\omega)}\right)\left(\frac{1}{2} + \frac{\Delta\omega}{2\sigma_\epsilon}\right)}_{\text{Firm 1 Revenue}} + \underbrace{\bar{\omega}\left[\mu_a - \frac{\theta\sigma_a^2}{3(\sigma_\epsilon - \Delta\omega)}\right]\left(\frac{1}{2} - \frac{\Delta\omega}{2\sigma_\epsilon}\right)}_{\text{Firm 2 Revenue}} \\
& - \underbrace{\left[c_0 - \frac{\theta^2\sigma_a^2}{3(\sigma_\epsilon + \Delta\omega)}\right]\left(\frac{1}{2} + \frac{\Delta\omega}{2\sigma_\epsilon}\right)}_{\text{Expected Licensing Costs}} \\
=& \frac{1}{2\sigma_\epsilon}(1+h)\bar{\omega}\left(\mu_a(\sigma_\epsilon + \Delta\omega) + \frac{1}{3}\theta\sigma_a^2\right) + \frac{1}{2\sigma_\epsilon}\bar{\omega}\left(\mu_a(\sigma_\epsilon - \Delta\omega) - \frac{1}{3}\theta\sigma_a^2\right) \\
& - \frac{1}{2\sigma_\epsilon}\left(c_0(\sigma_\epsilon + \Delta\omega) - \frac{1}{3}\theta\sigma_a^2\right)
\end{aligned} \tag{A22}$$

To find the social optimal cost of licensing, we take the derivative of the social surplus with respect to the cost, c_0. Recall the following:

$$\Delta\omega = \frac{1}{3}(\bar{\omega}\mu_a h - c_0 - \mu_\epsilon) \implies \frac{d\Delta\omega}{dc_0} = -\frac{1}{3} \tag{A23}$$

Therefore

$$\begin{aligned}
&\frac{d(SS)}{dc_0} = 0 \\
&\implies -\frac{1}{6\sigma_\epsilon}(1+h)\bar{\omega}\mu_a + \frac{1}{6\sigma_\epsilon}\bar{\omega}\mu_a - \frac{1}{2\sigma_\epsilon}(\sigma_\epsilon + \Delta\omega) + \frac{1}{6\sigma_\epsilon}c_0 = 0 \\
&\implies \boxed{c_0^* = \frac{1}{2}(\bar{c} + h\bar{\omega}\mu_a)}
\end{aligned} \tag{A24}$$