Discrimination, Segregation, Integration and Expropriation Online Appendix

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Mathematical Appendix

A1. Model

As in Becker, I assume Cobb-Douglas production

$$Y_e = A{K_e}^{\alpha}{L_e}^{1-\alpha}$$

where $\alpha = 2/3$, for any economic sector e where capital and labor are combined (integrated, in the context of racial discrimination).

It will be helpful to have general rules for comparing the returns to capital and/or labor in different economies, so we set up a few basic ratios. Consider two economies, 1 and 2, with $L_2 = \gamma L_1$ and $K_2 = \beta K_1$ we can see that

$$r_2 = r_1 \left(\frac{\gamma}{\beta}\right)^{1-\alpha}$$

$$w_2 = w_1 \left(\frac{\beta}{\gamma}\right)^{\alpha}$$

$$\frac{r_2/r_1}{w_2/w_1} = \frac{\gamma}{\beta}$$

Because of the complementarity of capital and labor, and diminishing marginal returns to each factor, an increase in one factor quantity greatly increases the return to the other factor while actively reducing own return.

A2. Integration

In the case of complete integration and perfect competition in factor markets, overall output is given by

$$Y_{b+w} = AK^{\alpha}_{b+w}{L_{b+w}}^{1-\alpha}$$

The return to holders of capital, Black or white, per unit, is the marginal product of capital:

$$R_{b+w} = \alpha\left(\frac{L_{b+w}}{K_{b+w}}\right)^{1-\alpha}$$

The wage for labor, Black or white, is the marginal product

$$W_{b+w} = (1-\alpha)\left(\frac{K_{b+w}}{L_{b+w}}\right)^{\alpha}$$

Since each factor is dealt with uniformly, returns to white and Black labor are identical.

A3. Complete Segregation

In the opposite case of complete racial segregation, assuming perfect competition within both sectors, overall output is

$$Y_{b+w} = A{K_b}^{\alpha}L_b^{1-\alpha} + A{K_w}^{\alpha}L_w^{1-\alpha}$$

In this case, all four factor × ethnicity combinations earn very different rates:

$$r_i = \alpha\left(\frac{L_i}{K_i}\right)^{1-\alpha}$$

$$w_i = (1-\alpha)\left(\frac{K_i}{L_i}\right)^{\alpha}$$

Given the general range of values to be found in many urban southern counties, discussed above, the integrated economy would have roughly 50% more labor than the segregated white economy, but only about 4.5% more capital. Under segregation, white capital earns roughly $(1.045/1.5)^0.66 \approx 89\%$ relative to its earnings in segregation, while white labor earns $(1.5/1.045)^0.33 \approx 113\%$, a noticeable increase.

A4. Partial Integration and Expropriated Wages

I now summarize one possible bargaining solution available to white capital and labor that allows the two factors to realize close to their best return under integration and segregation, respectively.

In this version, capital is completely segregated, but labor is mobile between sectors. The assumption of segregated capital/partially integrated labor is justified by the stylized facts for much of the 100 years after the civil war of (a) enterprise capital largely held in sole proprietorships or small partnerships, (b) low levels of Black wealth, (c) interracial cooperation among coequal business partners almost non-existent while most black workers were employed by white owners/managers.

Some portion of Black labor, $L_{\tilde{b}}$ moves to work within the white economy. Black labor earns the same return in both sectors; white labor earns its marginal return plus some amount of money expropriated from Black labor in the white sector, restoring some of the compensation it gains under total segregation. Because there is a substantial fraction of Black labor that moves to the white sector, white capital earns close to its returns under full integration if it simply paid its marginal return.

Total production is set by

$$Y = Y_w + Y_b = A{K_w}^{\alpha}(L_w + L_{\tilde{b}})^{1-\alpha} + A{K_b}^{\alpha}(L_b - L_{\tilde{b}})^{1-\alpha}$$

The returns to Black capital and Black labor in the Black sector $(L_b - L_{\tilde{b}})$ are

$$r_b = \alpha\left(\frac{L_b - L_{\tilde{b}}}{K_b}\right)^{1-\alpha}$$

$$w_{b-\tilde{b}} = (1-\alpha)\left(\frac{K_b}{L_b - L_{\tilde{b}}}\right)^{\alpha}$$

respectively.

The return to White capital, White labor, and Black labor in the White sector are

$$r_w = \alpha\left(\frac{L_w + L_{\tilde{b}}}{K_w}\right)^{1-\alpha}$$

$$w_w = (1-\alpha)\left(\frac{K_w}{L_w + L_{\tilde{b}}}\right)^{\alpha} + t_w$$

$$w_{\tilde{b}} = (1-\alpha)\left(\frac{K_w}{L_w + L_{\tilde{b}}}\right)^{\alpha} - t_b$$

where

$$t_w \times L_w = t_b \times L_b$$

In this model, whites are able to stonewall Black workers in the white sector and only pay them the wage they would earn working for Black capital. Thus, marginal product of labor is different in each sector, and below the optimal level. Effectively, whites are trying to maximize the difference between the overall productivity of the white sector (Y_w) and how much must be paid to Black laborers in the sector - at the salary of the Black sector ($w_b \times L_{\tilde{b}}$) - as follows:

$$\underbrace{AK_w^\alpha(L_w + L_{\tilde{b}})^{1-\alpha}}_{Y_w} - \underbrace{(1-\alpha)AK_b^\alpha(L_b - L_{\tilde{b}})^{-\alpha}L_{\tilde{b}}}_{w_b \times L_{\tilde{b}}}$$

If we index the larger or major sector (with white capital, white labor, and some Black labor) as M and the smaller or minor sector (with only Black capital and some Black labor) as m, we can think of w_M as the marginal product of labor in the larger sector and w_m as the marginal product in the minor sector.

Taking the first derivative and setting it to zero yields, after some manipulation, the still-clumsy equation

$$\frac{w_M}{w_m} = \frac{(L_b - (1-\alpha)L_{\tilde{b}})}{(L_b - L_{\tilde{b}})}$$

There does not appear to be a closed form solution for $L_{\tilde{b}}$ but it can be solved numerically for given parameters. So long as capital per person is substantially lower in the Black sector than the white, the left-hand expression declines in a nonlinear but steady way as $L_{\tilde{b}}$ increases from 0 to L_b. The right hand side expression increases steadily until $\frac{L_{\tilde{b}}}{L_b} \approx 1 - \alpha$ at which point it increases rapidly.

TABLE A1—THE PAYOFF MATRIX FOR WHITE CAPITAL AND WHITE LABOR

	White Labor		
White Capital	Integration	Segregation	Expropriation
Integration	r_{b+w}, w_{b+w}	$(r_{b+w} + r_w)/2 - \varepsilon$, $(w_{b+w} + w_w)/2 - \varepsilon$	$(r_{b+w} + r_{\tilde{b}+w})/2 - \varepsilon$, $(w_{b+w} + w_{\tilde{b}+w} + t)/2 - \varepsilon$
Segregation	$(r_{b+w} + r_w)/2 - \varepsilon$, $(w_{b+w} + w_w)/2 - \varepsilon$	r_w, w_w	$(r_w + r_{\tilde{b}+w})/2 - \varepsilon$, $(w_w + w_{\tilde{b}+w} + t)/2 - \varepsilon$
Expropriation	$(r_{b+w} + r_{\tilde{b}+w})/2 - \varepsilon$, $(w_{b+w} + w_{\tilde{b}+w} + t)/2 - \varepsilon$	$(r_w + r_{\tilde{b}+w})/2 - \varepsilon$, $(w_w + w_{\tilde{b}+w} + t_w)/2 - \varepsilon$	$r_{\tilde{b}+w}$, $w_{\tilde{b}+w} + t_w$

A5. *Expropriation as an Equilibrium*

Table A1 presents a simple payoff matrix for the strategic interaction of White capital and White labor. The assumptions behind it are (a) there is no Black political power so only White interests are relevant, (b) White capital competes with White capital in the market but cooperates politically, and similarly White labor cooperates politically, (c) in case White capital and labor disagree on which of the three approaches, there is a 50% chance of the coalition of either factor succeeding and (d) disagreement imposes a cost, ε. If ε is substantial, this becomes a coordination game, where (E, E) is a stable equilibrium.

A6. *Human Capital Extension*

INTEGRATED WORK, COMPLETELY SEGREGATED HUMAN CAPITAL DEVELOPMENT

In this scenario, the local economy has moved back to integration. All capital and labor are pooled in the Cobb-Douglas production function. However, there are two twists: first, human capital is now a third factor of production, starting with a very small share that increases over time. Second, human capital development is segregated and restricted, so only whites are able to acquire it.

The new production function, adapted from **?**, is

$$Y = AK^{\alpha}L^{(1-\alpha)\omega}H^{(1-\alpha)(1-\omega)}$$

As before, α is parameterized as 1. It seems likely that ω steadily increased over the 20th century [CITE]. A value of around 0.25 seems appropriate to model Southern regional economies circa 1950 (**?**, See for example the growth of white collar and professional work force in Table 6.2).

The literature on human capital and skills development is vast but for the purposes of this section I focus on a specific form of the model of human capital development. In particular, I assume there are not major differences in endowment nor in potential development. Major differences in human capital development are largely about differential access to schooling or training.

For an economy where human capital is largely about achieving a high school level education, and where only a few percent are professionals, this seems reasonable. If extremely high performance jobs such as surgery become central than clearly this would need to be adjusted and some definition of native talent might need to be included.

In the simplest version of the segregated human capital model, ALL human capital is acquired by whites, and normalized to equal to their labor (so $L_w = H$).

Payout to capital in this model looks roughly as it did in full integration - white and Black capital are pooled and each earn:

$$r = \frac{dY}{dK} = \alpha AK^{\alpha-1}L^{(1-\alpha)\omega}H^{(1-\alpha)(1-\omega)}$$

Although there is no longer a "white wage" and a "Black wage", Black workers are only paid the marginal product of their labor input, while white workers are paid that plus a return for their human capital.

$$w_b = (1-\alpha)(1-\omega)AK^{\alpha}L^{(1-\alpha)(1-\omega)-1}H^{(1-\alpha)\omega}$$

White workers receive this plus a payout for their human capital:

$$w_w = (1-\alpha)(1-\omega)AK^{\alpha}L^{(1-\alpha)(1-\omega)-1}H^{(1-\alpha)\omega} + (1-\alpha)\omega AK^{\alpha}L^{(1-\alpha)(1-\omega)}H^{(1-\alpha)\omega-1}$$

Integrated Work, Partial Segregation in Human Capital Development

In this situation work is integrated, and there is now partial access to human capital development for all individuals, but it is uneven.

Using the previous production function:

$$Y = AK^{\alpha}L^{(1-\alpha)\omega}H^{(1-\alpha)(1-\omega)}$$

While resources and opportunities for human capital development are now accessible to everybody, they are unevenly distributed, where Black workers can only access some fraction, $\xi > 1$, of the opportunities available to white workers.

There is still no "White wage" or "Black wage", and all workers are able to accumulate some human capital. However, while for white workers $H_w = L_w$, for Black workers $H_b = \xi L_b$.

The wages for the two groups thus varies:

$$w_b = (1-\alpha)(1-\omega)AK^{\alpha}L^{(1-\alpha)(1-\omega)-1}H^{(1-\alpha)\omega} + (1-\alpha)\omega\xi AK^{\alpha}L^{(1-\alpha)(1-\omega)}H^{(1-\alpha)\omega-1}$$

White workers receive this plus a payout for their human capital:

$$w_w = (1-\alpha)(1-\omega)AK^{\alpha}L^{(1-\alpha)(1-\omega)-1}H^{(1-\alpha)\omega} + (1-\alpha)\omega AK^{\alpha}L^{(1-\alpha)(1-\omega)}H^{(1-\alpha)\omega-1}$$

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