Event-Studies with a Continuous Treatment: Online Appendix

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A Proof of Theorem 1

First, notice that part (i) is a special case of part (ii) with $d_1 = d_L$ and $d_2 = d_H$. Thus, the proof of part (i) follows directly from part (ii).

For part (ii), it suffices to show that, for every $g \in \mathcal{G} \setminus \{\infty\}$, and every time period $t = 2, \dots, T$, $\theta^o_{d_1,d_2}(g,t) = E[ATT(g,t,D)\big|G = g, d_1 \leq D \leq d_2]$, where $0 < d_1 \leq d_2$. Towards that end, note that under Assumptions 1 to 4, we have that, for any $d \in [d_1, d_2]$, $g \in \mathcal{G} \setminus \{\infty\}$,

$$\begin{aligned} ATT(g,t,d) =& E[Y_t - Y_{g-1}\big|G = g, D = d] \\ & - E[Y_t - Y_{g-1}\big|U_{\max\{t,g-1\}} = 1, d_1 \leq D \leq d_2 \cup D = 0], \end{aligned} \tag{A.1}$$

which follows from Theorem 3.1 and Appendix SA of Callaway, Goodman-Bacon and Sant'Anna (2024), and the fact that all units with $U_{\max\{t,g-1\}} = 1$ are untreated by time t. From (A.1) and the law of iterated expectations, it follows that

$$\begin{aligned} E[ATT(g,t,D)\big|G = g, d_1 \leq D \leq d_2] =& E[Y_t - Y_{g-1}\big|G = g, d_1 \leq D \leq d_2] \\ & - E[Y_t - Y_{g-1}\big|U_{\max\{t,g-1\}} = 1, d_1 \leq D \leq d_2 \cup D = 0], \end{aligned} \tag{A.2}$$

and the right-hand side of (A.2) is the definition of $\theta^o_{d_1,d_2}(g,t)$. This establishes that $\theta^o_{d_1,d_2}(g,t) = E[ATT(g,t,D)\big|G = g, d_1 \leq D \leq d_2]$ and concludes the proof of part (ii) of Theorem 1. As mentioned above, part (i) of Theorem 1 follows by taking $d_1 = d_L$ and $d_2 = d_H$.

Next, we prove part (iii) that $ATT^{es}_{e_1,e_2}(d) = E[\widetilde{Y}^{e_1,e_2}(G)\big|G + e_2 \in [2,T], D = d]$. Towards that end, first notice that, for every $d \in \mathcal{D}_+$, every group $g \in \mathcal{G} \setminus \{\infty\}$ and time period $t \in [2,T]$ such that $t \leq g + e_2, g + e_2 \in [2,T]$, $e_2 \geq 0$, we have from Appendix SA of Callaway, Goodman-Bacon and Sant'Anna (2024) that under Assumptions 1 to 4,

$$ATT(g,t,d) = E[Y_t - Y_{g-1}\big|G = g, D = d] - E[Y_t - Y_{g-1}\big|U_{g+e_2} = 1].$$

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Thus, it follows that, for a given group g that satisfies the above restrictions, and for any $0 \leq e_1 \leq e_2$,

$$\frac{\sum_{e=e_1}^{e_2} ATT(g, g+e, d)}{e_2 - e_1 + 1} = \sum_{e=e_1}^{e_2} \frac{E[Y_{g+e} - Y_{g-1} \big| G = g, D = d] - E[Y_{g+e} - Y_{g-1} \big| U_{g+e_2} = 1]}{e_2 - e_1 + 1}. \quad \text{(A.3)}$$

Next, notice that from the linearity property of conditional expectations and from the fact that the data is balanced in event-time (so there are no compositional changes across event-times), we have that, for every g such that $g + e_2 \in [2, T]$, and every $d \in \mathcal{D}_+$,

$$\begin{aligned} E[\widetilde{Y}^{e_1,e_2}(G) \big| G = g, D = d] &= E\left[\frac{\sum_{e=e_1}^{e_2} (Y_{g+e} - Y_{g-1}) - E[Y_{g+e} - Y_{g-1} \big| U_{g+e_2} = 1]}{e_2 - e_1 + 1} \middle| G = g, D = d \right] \\ &= \sum_{e=e_1}^{e_2} \frac{E[Y_{g+e} - Y_{g-1} \big| G = g, D = d] - E[Y_{g+e} - Y_{g-1} \big| U_{g+e_2} = 1]}{e_2 - e_1 + 1} \\ &= \frac{\sum_{e=e_1}^{e_2} ATT(g, g+e, d)}{e_2 - e_1 + 1}, \end{aligned} \quad \text{(A.4)}$$

where the last equality follows from (A.3).

From the definition of conditional expectations and its linearity property, it follows from (A.4) that

$$E[\widetilde{Y}^{e_1,e_2}(G) \big| G + e_2 \in [2, T], D = d] = \frac{\sum_{e=e_1}^{e_2} E\left[ATT(G, G+e, d) \middle| G + e_2 \in [2, T], D = d \right]}{e_2 - e_1 + 1},$$

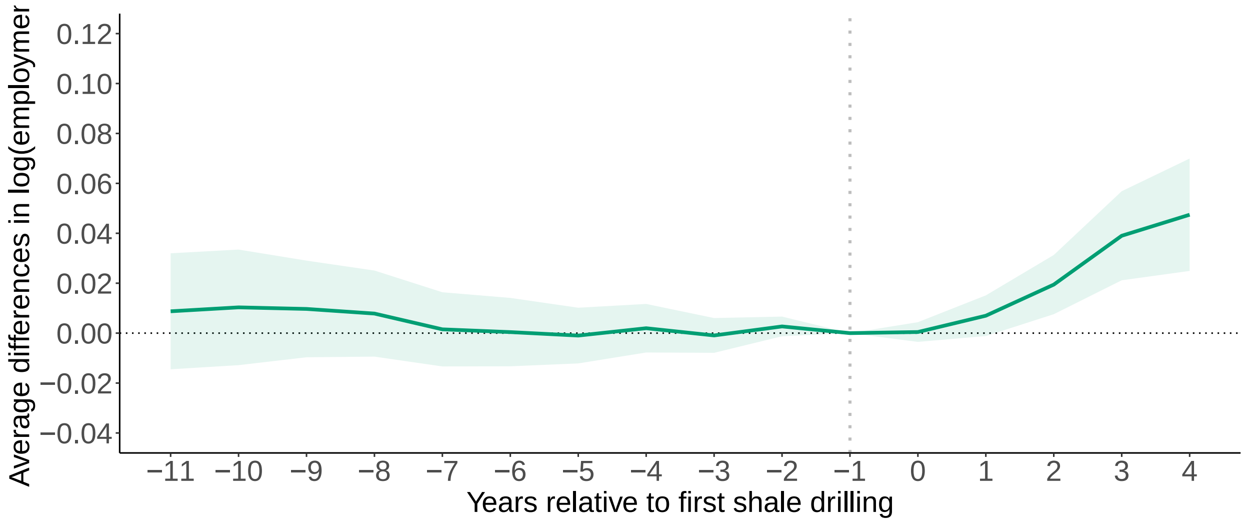
which is what we wanted to show. This concludes the proof of Theorem 1. □

B Additional plots for empirical application

We now complement the empirical analysis of our main text related to Bartik et al. (2019*a*). As discussed in Section IV, Bartik et al. (2019*a*) use a staggered and non-binary treatment variable to study the local economic effects of hydraulic fracking, and we slightly modify the DiD research design in their paper by exploiting variation in the timing of fracking activity across shale formations from 2001-2014 (G_i, hand-collected by the authors) and continuous variation in prospectivity score across counties (D_i, purchased from Rystad Energy); see Bartik et al. (2019*b*). We denote counties with zero prospectivity score as "never-treated" and set $G_i = \infty$ for them. We use the log of total county employment as the outcome of interest and use not-yet-treated units as the comparison group in all estimates below.

In the main text, we report in Figure 1 estimates of $ATT^{es}_{d_1,d_2}(e)$ using two sets of (d_1, d_2): the orange curve sets $d_1 = 0.20$ and $d_2 = 3.95$, where 0.20 and 3.95 are the minimum and the median fracking exposure among counties with positive exposure ("low dose"), whereas the blue curve sets d_1 slightly above 3.95 and $d_2 = 9.35$, where 9.35 is the maximum fracking exposure ("high dose"). In some applications, we expect researchers also to want to report an "overall" event-study aggregation, $ATT^{es}(e)$, as discussed in our main text. Figure B.1 presents estimates of such event-study coefficients using the event-study estimators proposed by Callaway and Sant'Anna (2021).

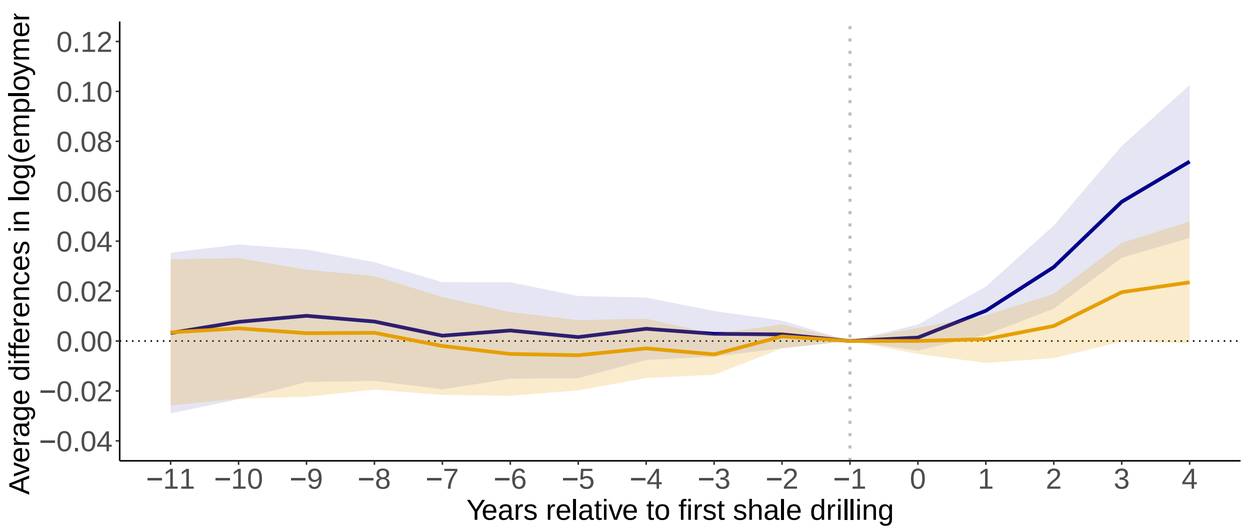
Figure B.1: Overall event-study estimates



Notes: Solid lines denotes estimates of $ATT^{es}(e)$ using Callaway and Sant'Anna (2021). Shaded areas are 95% pointwise confidence intervals.

As one should expect, the event-study estimates in Figure B.1 are an average of the "high dose" and "low dose" event-study estimates in Figure 1 from the main text (which we reproduce as Figure B.2 to facilitate comparisons). From Figure B.1, one can see that non-parallel pre-trends are not a major concern, and that longer-run effects are stronger than shorter-run ones.

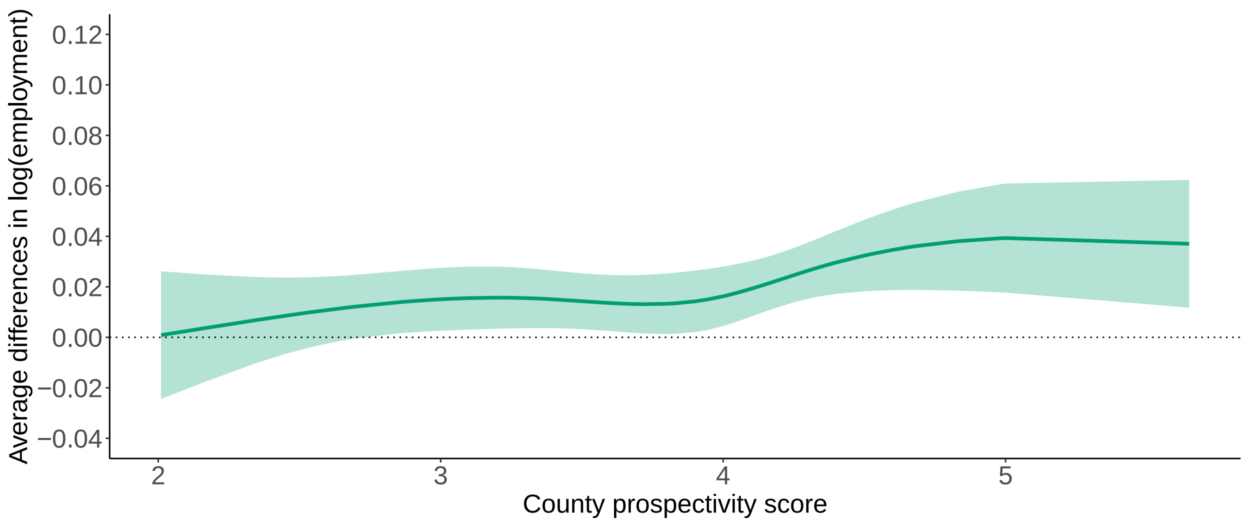
Figure B.2: Event Study estimates for high and low-dose groups



Notes: Solid lines denotes estimates of $ATT^{es}_{d_1,d_2}(e)$ using Callaway and Sant'Anna (2021). Shaded areas are 95% pointwise confidence intervals. The orange (blue) curve sets d_1 and d_2 to 0.2 and 3.95 (3.96 and 9.35).

We next move to estimates of time-averaged dose-response curves, $ATT^{es}_{e_1,e_2}(d)$. Figure 2 in the main text displays results for time-averaged dose-response curves, $ATT^{es}_{e_1,e_2}(d)$ using Callaway, Goodman-Bacon and Sant'Anna (2024)'s estimators with cubic splines: the orange curve sets e_1 and e_2 to 0 and 2 ("short-run"), and the blue curve uses 3 and 4 ("long-run"). We reproduce Figure 2 as Figure B.4 below to facilitate comparisons. Similar to the above, we expect that some researchers may be interested in reporting an "overall" dose-response curve, e.g., by setting $e_1 = 0$ and $e_2 = 4$. We report estimates of this in Figure B.3.

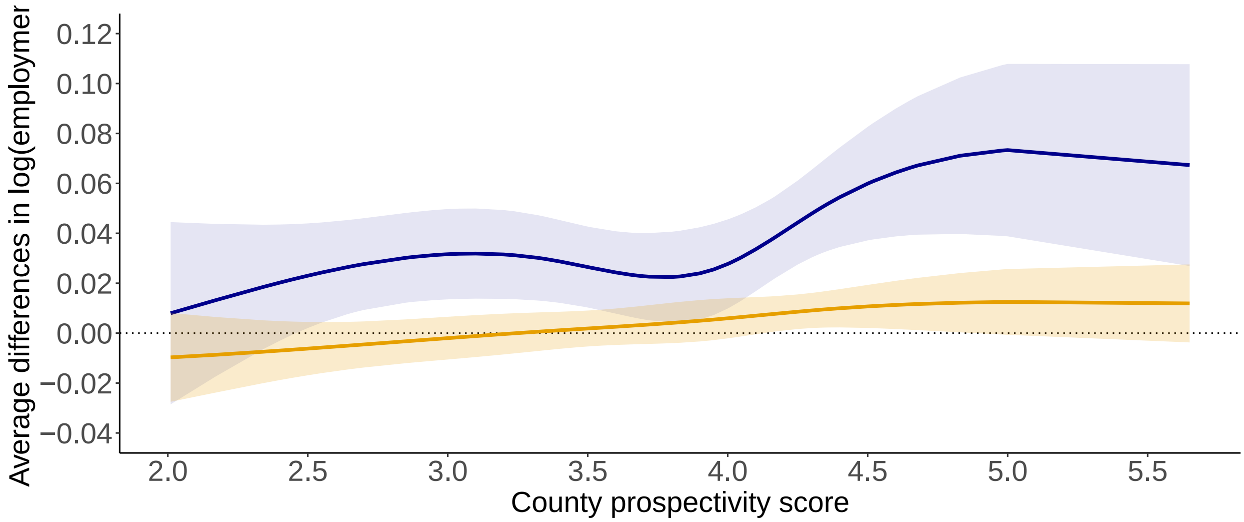
Figure B.3: Time-averaged estimated dose-response curves



Notes: Solid lines denotes estimates of $ATT^{es}_{e_1,e_2}(d)$ using Callaway, Goodman-Bacon and Sant'Anna (2024), with $e_1 = 0$ and $e_2 = 4$. Shaded areas are 95% pointwise confidence intervals.

Figure B.3 echoes the conclusions from the "long-term" dose-response results in Figure B.4 that counties with higher prospectivity scores have larger employment effects from fracking. Figure B.3 also highlights that average employment effects in the first 4 years after fracking are similarly large for all counties with scores above about 2.5; it is not just the most fracking-amenable counties that drive its labor market effects.

Figure B.4: Estimated dose-response curves for short and long-run effects



Notes: Solid lines denotes estimates of $ATT^{es}_{e_1,e_2}(d)$ using Callaway, Goodman-Bacon and Sant'Anna (2024). Shaded areas are 95% pointwise confidence intervals. The orange (blue) curve sets e_1 and e_2 to 0 and 2 (3 and 4).

References

Bartik, Alexander W., Janet Currie, Michael Greenstone, and Christopher R. Knittel. 2019*a*. "The Local Economic and Welfare Consequences of Hydraulic Fracturing." *American Economic Journal: Applied Economics*, 11(4): 105–55.

Bartik, Alexander W., Janet Currie, Michael Greenstone, and Christopher R. Knittel. 2019*b*. "Replication package for: The Local Economic and Welfare Consequences of Hydraulic Fracturing." American Economic Association. `https://www.aeaweb.org/journals/dataset?id=10.1257/app.20170487`.

Callaway, Brantly, and Pedro H. C. Sant'Anna. 2021. "Difference-in-differences with multiple time periods." *Journal of Econometrics*, 225(2): 200–230.

Callaway, Brantly, Andrew Goodman-Bacon, and Pedro H. C. Sant'Anna. 2024. "Difference-in-Differences with a Continuous Treatment." *arXiv:2107.02637*.