The Slanted-L Phillips Curve

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Online Appendix

Figure 1

Figure 1 shows scatter plots of the annual inflation rate and the labor-market tightness (v/u) for the United States and for the sample 2009 Q1–2023 Q2. Inflation rate (core) is at annual rates and computed using the quarterly CPI core. CPI core quarterly observations are the average of the relevant monthly observations. Data are from U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items Less Food and Energy in U.S. City [CPILFESL], retrieved from FRED, Federal Reserve Bank of St. Louis. The variable v/u is computed as the ratio between the job openings and the unemployment level. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations. Job openings are from U.S. Bureau of Labor Statistics, Job Openings: Total Nonfarm [JTSJOL], retrieved from FRED, Federal Reserve Bank of St. Louis. Unemployment level is from U.S. Bureau of Labor Statistics, Unemployment Level [UNEMPLOY], retrieved from FRED, Federal Reserve Bank of St. Louis. Each plot shows also the fit of the linear regression model with 95% confidence bounds conditional on $v/u < 1$ and $v/u \geq 1$.

Figure 2

Figure 2 presents scatter plots of unemployment rate and core inflation rate at quarterly frequency for the period 2009 Q1 – 2023 Q3, with core inflation represented at annual rates. The 'L' function is shown with the vertical line indicating the average of the unemployment rate computed on the observations between its minimum and (minimum + 0.2). The flat segment of the 'L' function corresponds to the fitted line derived from linear regression (OLS) between inflation and unemployment for each country. Observations used to draw the vertical line are excluded from this regression analysis. Unemployment Rate: Harmonized Unemployment, Monthly Rates, Total, All Persons, obtained for each country (Australia, Canada, Germany, France, United Kingdom, Italy, Japan, United States) from the Organization for Economic Co-operation and Development via FRED, Federal Reserve Bank of St. Louis. Inflation Rate: Annual percentage change in the Core CPI, corresponding to the series 'core CPI (starndardized) SADJ,' retrieved for each country from Thomson Reuters Datastream, respectively given by AUCCOR..E, CNCCOR..E, BDCCOR..E, UKCCOR..E, FRCCOR..E, ITCCOR..E, JPCCOR..E, USCCOR..E.

Figure 3

Figure 3 presents scatter plots of adjusted unemployment rate and core inflation rate at quarterly frequency for the period 2009 Q1 – 2023 Q3, with core inflation represented at annual rates. For each country, the adjusted unemployment rate is derived from the time-series data on the unemployment rate of each country by subtracting the unemployment rate at maximum employment, u_f, derived in Figure 2 to draw the vertical line and then adding the average unemployment rate (at maximum employment) across all countries. The 'L' function is illustrated, featuring a vertical line representing the average of the (adjusted) unemployment rate computed on the observations between its minimum and (minimum + 0.2). The flat segment of the 'L' model corresponds to the fitted line obtained through linear regression (OLS) between inflation and (adjusted) unemployment for all countries. Observations used to draw the vertical line are excluded from this regression analysis. The hyperbolic function corresponds to the non-linear least-squares estimates of the model

$$\pi_{i,t} = a + b\left(\frac{1}{u_{i,t}^{dev}}\right)^{c}$$

with the following estimated coefficients $a = 1.3909$, $b = 1.3531e+09$ and $c = 13.3963$.

Derivation of equation (5).

Consider labor demand

$$L_t^d = \left(\frac{1}{\alpha A_t}\frac{W_t}{P_t}\right)^{\frac{1}{\alpha-1}}$$

and take the log, to obtain:

$$l_t^d = \frac{1}{\alpha-1}(w_t - \ln\alpha - a_t), \tag{A1}$$

in which $l_t^d = \ln L_t^d$, $w_t = \ln W_t/P_t$ and $a_t = \ln A_t$. Using equation (3) when the wage norm is binding, the (log) real wage is given by

$$w_t = \lambda w_{t-1} + \gamma\lambda\pi_t^e - \lambda\pi_t + (1-\lambda)w_t^f,$$

in which $\pi_t^e = \ln\Pi_t^e$, $\pi_t = \ln\Pi_t$ and

$$w_t^f = \ln\alpha + (\alpha-1)\bar{l}, \tag{A2}$$

for $\bar{l} = \ln\bar{L}$. Consider the assumption

$$w_{t-1} = \ln\phi + \ln\alpha + (\alpha-1)\bar{l},$$

it follows that

$$w_t = \lambda\ln\phi + \gamma\lambda\pi_t^e - \lambda\pi_t + w_t^f$$

Therefore we can write (A1) as

$$l_t^d = \frac{1}{\alpha-1}(\lambda\ln\phi + \gamma\lambda\pi_t^e - \lambda\pi_t + w_t^f - \ln\alpha - a_t),$$

from which, using (A2), it follows

$$l_t^d - \bar{l} = \frac{1}{\alpha-1}(\lambda\ln\phi + \gamma\lambda\pi_t^e - \lambda\pi_t - a_t).$$

The above equation can be also rewritten as

$$\pi_t = \frac{(1-\alpha)}{\lambda}(l_t^d - \bar{l}) + \ln\phi + \gamma\pi_t^e - \frac{1}{\lambda}a_t.$$

Note that

$$u_t = 1 - \frac{L_t^d}{\bar{L}}$$

and that for small u, we can use the approximation $u_t \approx -\ln L_t^d/\bar{L}$. We can then write

$$\pi_t = -\kappa u_t + \gamma\pi_t^e + \upsilon_t,$$

which is equation (5), having defined

$$\kappa \equiv \frac{(1-\alpha)}{\lambda},$$
$$\upsilon_t \equiv \ln\phi - \frac{1}{\lambda}a_t.$$