Online Appendix: Does school desegregation promote diverse interactions? An equilibrium model of segregation within schools

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Computational Details

The linear specification allows for utility functions involving network-level controls, when estimation is performed using multiple networks. This can be achieved by a specification of the parameters such as

$$\theta_p = \theta_{p0} + \sum_{c=1}^{C} \theta_{pc} Z_c \tag{A1}$$

where Z_c is a network-level variable. The estimation methodology presented above can be applied to this specification without any change. However, estimation of a model with unobserved heterogeneity would require significant additional computational effort (see Appendix C in Mele (2017)).

I choose somewhat vague priors for the parameters to extract most of the information from the data. I assume independent normal priors

$$p(\theta) = \mathcal{N}(\mathbf{0}, 3\mathbf{I}_P), \tag{A2}$$

where P is the number of parameters.

The proposal distribution for the posterior simulation is

$$q_\theta(\cdot|\theta) = \mathcal{N}(\mathbf{0}, \delta\mathbf{\Sigma}), \tag{A3}$$

where δ is a scaling factor and $\mathbf{\Sigma}$ is a covariance matrix.

I use an adaptive procedure to determine a suitable $\mathbf{\Sigma}$. I start the iterations with $\mathbf{\Sigma} = \lambda\mathbf{I}_P$, where λ is a vector of standard deviations. I choose λ so that the sampler accepts at least 20%-25% of the proposed parameters, as is standard in the literature (Gelman et al., 2003; Robert and Casella, 2005). I run the chain and monitor convergence using standard methods. Once the chains have reached approximate convergence, I estimate the covariance matrix of the chains and use it as an approximate $\mathbf{\Sigma}$ for the next set of simulations. The scaling factor is $\delta = 2.38^2/P$ as suggested in Gelman, Roberts and Gilks (1996).

The network sampler uses a proposal $q_g(g|g')$, that selects a link to be updated at each period according to a discrete uniform distribution. The probability of network inversion is $p_{inv} = 0.01$.

All the posterior distributions shown in the following graphs are obtained with a simulation of 100000 Metropolis-Hastings updates of the parameters. These simulations start from values found after extensive experimentation with different starting values and burn-in periods, monitoring convergence using standard methods. For each parameter update, I simulate the network for 5000 iterations to collect a sample from the stationary distribution. For the estimation with 16 schools I ran 20000 parameter simulations and 10000 network simulations for each parameter.

A1. Parallel estimation with multiple networks

When data from multiple independent networks are available the estimation routines are easily adapted. Assume the researcher has data from C networks: let g_c and X_c denote the network matrix and the individual controls for network c, $c = 1, ..., C$. The aggregate data are denoted as $g = \{g_1, ..., g_c\}$ and $X = \{X_1, ..., X_c\}$.

Assuming each network is drawn from the stationary equilibrium of the model, each network has distribution

$$\pi\left(g_c, X_c, \theta\right) = \frac{\exp\left[Q\left(g_c, X_c, \theta\right)\right]}{\sum\limits_{\omega \in \mathcal{G}_c} \exp\left[Q\left(\omega_c, X_c, \theta\right)\right]} \tag{A4}$$

Since each network is independent, the likelihood of the data (g, X) can be written as

$$\begin{aligned}
\pi\left(g, X, \theta\right) &= \prod_{c=1}^{C} \pi\left(g_c, X_c, \theta\right) = \prod_{c=1}^{C} \left\{ \frac{\exp\left[Q\left(g_c, X_c, \theta\right)\right]}{c\left(\mathcal{G}_c, X_c, \theta\right)} \right\} \\
&= \frac{\exp\left[\sum_{c=1}^{C} Q\left(g_c, X_c, \theta\right)\right]}{\prod_{c=1}^{C} c\left(\mathcal{G}_c, X_c, \theta\right)} = \frac{\exp\left[\sum_{c=1}^{C} Q\left(g_c, X_c, \theta\right)\right]}{\mathcal{C}\left(\mathcal{G}, X, \theta\right)}
\end{aligned}$$

where $\mathcal{G} = \bigcup_{c=1}^{C} \mathcal{G}_c$ and $X = \{X_1, ..., X_C\}$. The likelihood for multiple independent networks is of the same form as the likelihood for one network observation. The structure of this likelihood makes parallelization extremely easy: each network can be simulated independently using the network simulation algorithm; at the end of the simulation we collect the last network and compute the potential; then we compute the sum of potentials and use it to compute the probability of update.

Therefore, the algorithm is modified as follows

ALGORITHM 1: ***(Parallel approximate exchange algorithm***

Fix the number of simulations R. Store each network data (g_c, X_c) in a different processor/core. At each iteration t, with current parameter $\theta_t = \theta$ and network data g

1) *Propose a new parameter θ' from a distribution $q_\theta(\cdot|\theta)$*

$$\theta' \sim q_\theta(\cdot|\theta) \tag{A5}$$

2) *For each processor c, start the network sampler at the observed network g_c, iterating for R steps using parameter θ' and collect the last simulated network g'_c*

$$g'_c \sim \mathcal{P}^{(R)}_{\theta'}(g'_c|g_c) \tag{A6}$$

3) *Update the parameter according to*

$$\theta_{t+1} = \begin{cases} \theta' & \textit{with prob. } \alpha_{pex}(\theta, \theta') \\ \theta & \textit{with prob. } 1 - \alpha_{pex}(\theta, \theta') \end{cases}$$

where

$$\alpha_{pex}(\theta, \theta') = \min \left\{ 1, \frac{\exp\left[\sum_{c=1}^{C} Q(g'_c, X_c, \theta)\right]}{\exp\left[\sum_{c=1}^{C} Q(g_c, X_c, \theta)\right]} \frac{p(\theta')}{p(\theta)} \frac{q_\theta(\theta|\theta')}{q_\theta(\theta'|\theta)} \frac{\exp\left[\sum_{c=1}^{C} Q(g_c, X_c, \theta')\right]}{\exp\left[\sum_{c=1}^{C} Q(g'_c, X_c, \theta')\right]} \right\} \tag{A7}$$

The speed of the algorithm depends on the largest network in the data. Since each parameter update requires the result of each processor simulation there is some idle time, since small networks are simulated much faster. However, one could easily modify the algorithm to have different number of network simulations for networks of different sizes, so for each c we would have a different R_c

Freeman Segregation Index

The Freeman segregation index measures the degree of segregation in a population with two groups (Freeman, 1972). Assume there are two groups, A and B. Let n_{AB} be the total number of links that individuals of group A form to individuals of group B. Let n_{BA}, n_{BB} and n_{AA} be analogously defined. The original index developed by Freeman (1972) is defined as

$$FSI = \frac{\mathbb{E}[n_{AB}] + \mathbb{E}[n_{BA}] - (n_{AB} + n_{BA})}{\mathbb{E}[n_{AB}] + \mathbb{E}[n_{BA}]} \tag{B1}$$

When the link formation does not depend on the identity of individuals, then the links should be randomly distributed with respect to identity. Therefore, the index measures the difference between the expected and actual number of links among individuals of different groups, as a fraction of the expected links. An index of 0 means that the actual network closely resembles one in which links are formed at random. Higher values indicate more segregation. In this paper segregation is measured using the index[1]

$$SEG = \max\{0, FSI\} \tag{B2}$$

The index varies between 0 and 1, where the maximum corresponds to a network in which there are no cross-group links.

To complete the derivation of the index, the expected number of cross-group links is computed as

$$\begin{aligned} \mathbb{E}[n_{AB}] &= \frac{(n_{AA}+n_{AB})(n_{AB}+n_{BB})}{n_{AA}+n_{AB}+n_{BA}+n_{BB}} \\ \mathbb{E}[n_{BA}] &= \frac{(n_{BA}+n_{BB})(n_{AA}+n_{BA})}{n_{AA}+n_{AB}+n_{BA}+n_{BB}} \end{aligned}$$

*

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Posterior estimates (complete tables)

[1] The index (B1) varies between -1 and 1. However, the interpretation of the index when it assumes negative values is not clear. Therefore Freeman (1972) suggests to use only when it is nonnegative, to measure the presence of segregation

Table D1—Model (2) in Table 3

	mean	median	std. dev.	5 pctile	95 pctile
A. Direct utility (u_{ij})					
CONSTANT	-5.5381	-5.5384	0.0261	-5.5805	-5.4942
SAME GENDER	0.1850	0.1850	0.0133	0.1631	0.2069
SAME GRADE	2.2384	2.2384	0.0030	2.2334	2.2434
WHITE-WHITE	0.5414	0.5413	0.0048	0.5335	0.5494
BLACK-BLACK	0.3660	0.3661	0.0173	0.3375	0.3943
HISP-HISP	1.6794	1.6794	0.0322	1.6267	1.7324
ATTRACTIVE i (Physical)	0.3068	0.3070	0.0270	0.2623	0.3509
ATTRACTIVE j (Physical)	0.2322	0.2322	0.0076	0.2198	0.2448
ATTRACTIVE i (Personality)	0.0063	0.0061	0.0128	-0.0145	0.0275
ATTRACTIVE j (Personality)	0.8678	0.8679	0.0173	0.8391	0.8959
Income i - Income j (logs)	0.1462	0.1461	0.0068	0.1351	0.1574
Income i + Income j(logs)	-0.0806	-0.0806	0.0049	-0.0885	-0.0725
FRACTION WHITES	-0.4814	-0.4814	0.0320	-0.5338	-0.4284
FRACTION BLACKS	3.0985	3.0984	0.0156	3.0730	3.1242
FRACTION HISP	2.4440	2.4439	0.0207	2.4100	2.4781
WHITE-WHITE * FRACTION WHITES	1.0094	1.0095	0.0267	0.9659	1.0532
BLACK-BLACK * FRACTION BLACKS	0.1478	0.1478	0.0095	0.1322	0.1633
HISP-HISP * FRACTION HISP	-1.4255	-1.4258	0.0309	-1.4758	-1.3744
SCHOOL 1	-2.1181	-2.1186	0.0616	-2.2192	-2.0164
SCHOOL 2	1.5065	1.5066	0.0105	1.4891	1.5236
SCHOOL 3	-0.0532	-0.0531	0.0299	-0.1022	-0.0040
SCHOOL 4	1.1044	1.1043	0.0154	1.0793	1.1297
SCHOOL 5	1.3247	1.3246	0.0091	1.3098	1.3397
SCHOOL 6	0.4666	0.4667	0.0224	0.4300	0.5037
SCHOOL 7	2.3742	2.3741	0.0261	2.3313	2.4171
SCHOOL 8	0.2750	0.2750	0.0382	0.2125	0.3374
SCHOOL 9	-1.3631	-1.3631	0.0281	-1.4090	-1.3171
SCHOOL 10	1.7814	1.7817	0.0339	1.7255	1.8369
SCHOOL 11	-1.4060	-1.4059	0.0094	-1.4216	-1.3905
SCHOOL 12	2.9026	2.9027	0.0241	2.8627	2.9422
SCHOOL 13	0.3076	0.3076	0.0420	0.2388	0.3770
B. Mutual utility (m_{ij})					
CONSTANT	1.1853	1.1852	0.0388	1.1218	1.2484
SAME GENDER	1.1652	1.1653	0.0121	1.1452	1.1849
SAME GRADE	-1.6882	-1.6883	0.0210	-1.7228	-1.6537
WHITE-WHITE	0.0073	0.0074	0.0132	-0.0147	0.0289
BLACK-BLACK	0.7468	0.7468	0.0318	0.6943	0.7992
HISP-HISP	0.7779	0.7778	0.0089	0.7635	0.7925
C. Indirect utility and Popularity (v_{ij})					
CONSTANT	-0.2891	-0.2890	0.0129	-0.3105	-0.2680
SAME GENDER	0.1721	0.1721	0.0053	0.1635	0.1808
SAME GRADE	-0.3145	-0.3145	0.0066	-0.3254	-0.3038
WHITE-WHITE	0.2239	0.2238	0.0085	0.2099	0.2379
BLACK-BLACK	-0.1364	-0.1364	0.0146	-0.1605	-0.1124
HISP-HISP	0.4328	0.4327	0.0105	0.4157	0.4501

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 100000 parameter simulations, and 5000 network simulations for each parameter proposal.

Table D2—Model (1) in Table 3

	mean	median	std. dev.	5 pctile	95 pctile
CONSTANT	-6.9201	-6.9196	0.0393	-6.9854	-6.8562
SAME GENDER	-0.4545	-0.4546	0.0204	-0.4884	-0.4213
SAME GRADE	2.3124	2.3124	0.0030	2.3075	2.3174
WHITE-WHITE	0.3504	0.3505	0.0061	0.3402	0.3603
BLACK-BLACK	0.1443	0.1444	0.0183	0.1135	0.1742
HISP-HISP	1.8597	1.8598	0.0297	1.8109	1.9085
ATTRACTIVE i (Physical)	0.2757	0.2760	0.0267	0.2314	0.3191
ATTRACTIVE j (Physical)	-0.0410	-0.0411	0.0104	-0.0583	-0.0241
ATTRACTIVE i (Personality)	-0.4402	-0.4399	0.0152	-0.4657	-0.4158
ATTRACTIVE j (Personality)	1.0672	1.0672	0.0179	1.0378	1.0966
Income i - Income j (logs)	0.1793	0.1792	0.0071	0.1676	0.1911
Income i + Income j (logs)	-0.0882	-0.0882	0.0050	-0.0963	-0.0800
FRACTION WHITES	0.9070	0.9067	0.0465	0.8312	0.9843
FRACTION BLACKS	3.2238	3.2237	0.0153	3.1989	3.2491
FRACTION HISP	2.5240	2.5237	0.0211	2.4900	2.5593
WHITE-WHITE * FRACTION WHITES	1.3962	1.3959	0.0271	1.3526	1.4409
BLACK-BLACK * FRACTION BLACKS	0.4664	0.4665	0.0127	0.4457	0.4875
HISP-HISP * FRACTION HISP	-1.5643	-1.5643	0.0305	-1.6135	-1.5135
SCHOOL 1	-3.4873	-3.4871	0.0739	-3.6103	-3.3653
SCHOOL 2	1.8278	1.8278	0.0115	1.8089	1.8469
SCHOOL 3	-0.5626	-0.5623	0.0317	-0.6159	-0.5110
SCHOOL 4	0.4159	0.4159	0.0219	0.3796	0.4516
SCHOOL 5	1.4366	1.4365	0.0082	1.4232	1.4503
SCHOOL 6	1.3884	1.3882	0.0311	1.3376	1.4399
SCHOOL 7	2.8597	2.8592	0.0283	2.8139	2.9071
SCHOOL 8	1.2675	1.2672	0.0446	1.1948	1.3421
SCHOOL 9	-1.9436	-1.9431	0.0306	-1.9940	-1.8944
SCHOOL 10	1.7678	1.7679	0.0352	1.7091	1.8254
SCHOOL 11	-0.9222	-0.9224	0.0144	-0.9455	-0.8981
SCHOOL 12	3.5492	3.5491	0.0297	3.5004	3.5986
SCHOOL 13	-0.3995	-0.3996	0.0445	-0.4720	-0.3257

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 100000 parameter simulations, and 5000 network simulations for each parameter proposal.

Table D3—Model (4) in Table 3

	mean	median	std. dev.	5 pctile	95 pctile
A. Direct utility (u_{ij})					
CONSTANT	-5.9132	-5.9131	0.0146	-5.9373	-5.8894
MALE	0.0463	0.0464	0.0119	0.0264	0.0657
WHITE	0.0044	0.0045	0.0163	-0.0224	0.0310
BLACK	1.1310	1.1311	0.0063	1.1208	1.1414
HISP	2.2806	2.2804	0.0303	2.2308	2.3306
INCOME	-1.6492	-1.6490	0.0381	-1.7122	-1.5869
SAME GENDER	0.4851	0.4851	0.0155	0.4597	0.5107
SAME GRADE	2.0113	2.0113	0.0264	1.9674	2.0545
WHITE-WHITE	0.5720	0.5719	0.0129	0.5510	0.5933
BLACK-BLACK	1.1445	1.1442	0.0147	1.1208	1.1691
HISP-HISP	-0.2269	-0.2270	0.0172	-0.2553	-0.1986
BEAUTY i	-2.2413	-2.2411	0.0382	-2.3044	-2.1790
BEAUTY j	1.5861	1.5857	0.0207	1.5525	1.6204
PERSONALITY i	-0.1570	-0.1570	0.0100	-0.1736	-0.1404
PERSONALITY j	-0.7390	-0.7388	0.0185	-0.7698	-0.7088
Income i - Income j	0.9012	0.9010	0.0208	0.8672	0.9356
Income i + Income j	0.9244	0.9242	0.0220	0.8885	0.9607
FRACTION WHITES	-1.4420	-1.4420	0.0091	-1.4569	-1.4269
FRACTION BLACKS	1.8309	1.8309	0.0119	1.8114	1.8504
FRACTION HISP	0.7798	0.7798	0.0106	0.7624	0.7970
WHITE-WHITE * FRACTION WHITE	2.7840	2.7831	0.0504	2.7028	2.8685
BLACK-BLACK * FRACTION BLACKS	0.4028	0.4028	0.0063	0.3923	0.4130
HISP-HISP * FRACTION HISP	-1.3630	-1.3629	0.0075	-1.3754	-1.3508
SCHOOL 1	-0.0766	-0.0766	0.0279	-0.1220	-0.0307
SCHOOL 2	1.3889	1.3890	0.0273	1.3436	1.4337
SCHOOL 3	1.8308	1.8306	0.0185	1.8005	1.8610
SCHOOL 4	1.4277	1.4276	0.0173	1.3996	1.4565
SCHOOL 5	1.9201	1.9201	0.0145	1.8961	1.9440
SCHOOL 6	-0.7518	-0.7519	0.0197	-0.7841	-0.7191
SCHOOL 7	0.0355	0.0355	0.0135	0.0129	0.0576
SCHOOL 8	-0.5121	-0.5122	0.0228	-0.5494	-0.4746
SCHOOL 9	-2.6615	-2.6613	0.0559	-2.7538	-2.5701
SCHOOL 10	1.1371	1.1374	0.0345	1.0796	1.1937
SCHOOL 11	-0.8724	-0.8725	0.0274	-0.9173	-0.8271
SCHOOL 12	1.6418	1.6419	0.0207	1.6078	1.6758
SCHOOL 13	1.3257	1.3248	0.0526	1.2412	1.4140
B. Mutual utility (m_{ij})					
CONSTANT	6.1668	6.1659	0.0408	6.1010	6.2346
SAME GENDER	1.0716	1.0716	0.0153	1.0462	1.0967
SAME GRADE	-3.0514	-3.0510	0.0220	-3.0882	-3.0160
WHITE-WHITE	-0.6017	-0.6016	0.0186	-0.6322	-0.5711
BLACK-BLACK	1.1177	1.1175	0.0261	1.0750	1.1613
HISP-HISP	-1.4659	-1.4655	0.0229	-1.5033	-1.4287
C. Indirect utility and Popularity (v_{ij})					
CONSTANT	-0.4705	-0.4705	0.0071	-0.4823	-0.4587
SAME GENDER	-0.4074	-0.4072	0.0069	-0.4188	-0.3962
SAME GRADE	0.1136	0.1136	0.0095	0.0981	0.1293
WHITE-WHITE	0.1856	0.1857	0.0090	0.1708	0.2004
BLACK-BLACK	0.1372	0.1371	0.0081	0.1239	0.1507
HISP-HISP	-0.5067	-0.5066	0.0111	-0.5249	-0.4886

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 100000 parameter simulations, and 5000 network simulations for each parameter proposal.

Table D4—Model (3) in Table 3

	mean	median	std. dev.	5 pctile	95 pctile
CONSTANT	-6.6500	-6.6499	0.0189	-6.6812	-6.6191
MALE	-0.1517	-0.1516	0.0105	-0.1690	-0.1349
WHITE	-0.1710	-0.1709	0.0139	-0.1938	-0.1484
BLACK	1.0451	1.0451	0.0048	1.0371	1.0530
HISP	2.0990	2.0991	0.0235	2.0604	2.1375
INCOME	-2.0543	-2.0542	0.0319	-2.1075	-2.0025
SAME GENDER	0.2067	0.2068	0.0131	0.1848	0.2281
SAME GRADE	2.3817	2.3814	0.0211	2.3469	2.4166
WHITE-WHITE	1.0138	1.0136	0.0133	0.9921	1.0358
BLACK-BLACK	1.6491	1.6489	0.0159	1.6233	1.6754
HISP-HISP	0.3186	0.3184	0.0166	0.2914	0.3463
ATTRACTIVE i (Physical)	-2.3568	-2.3567	0.0296	-2.4057	-2.3084
ATTRACTIVE j (Physical)	2.5166	2.5163	0.0255	2.4750	2.5590
ATTRACTIVE i (Personality)	-0.4964	-0.4964	0.0087	-0.5108	-0.4821
ATTRACTIVE j (Personality)	-1.0932	-1.0930	0.0165	-1.1205	-1.0664
Income i - Income j (logs)	0.8883	0.8883	0.0141	0.8654	0.9116
Income i + Income j (logs)	1.0947	1.0947	0.0177	1.0660	1.1242
FRACTION WHITES	-1.7088	-1.7087	0.0074	-1.7210	-1.6966
FRACTION BLACKS	1.3416	1.3419	0.0128	1.3205	1.3625
FRACTION HISP	0.8397	0.8397	0.0084	0.8260	0.8535
WHITE-WHITE * FRACTION WHITES	4.3915	4.3908	0.0526	4.3059	4.4785
BLACK-BLACK * FRACTION BLACKS	0.2528	0.2529	0.0061	0.2428	0.2627
HISP-HISP * FRACTION HISP	-1.6908	-1.6907	0.0088	-1.7053	-1.6766
SCHOOL 1	-0.2439	-0.2439	0.0222	-0.2807	-0.2075
SCHOOL 2	1.7809	1.7807	0.0217	1.7450	1.8169
SCHOOL 3	1.7858	1.7858	0.0146	1.7616	1.8097
SCHOOL 4	1.9064	1.9061	0.0176	1.8780	1.9355
SCHOOL 5	2.2429	2.2428	0.0127	2.2221	2.2642
SCHOOL 6	-1.4227	-1.4226	0.0179	-1.4523	-1.3933
SCHOOL 7	-0.2224	-0.2223	0.0124	-0.2428	-0.2024
SCHOOL 8	0.2460	0.2457	0.0232	0.2084	0.2842
SCHOOL 9	-2.7969	-2.7967	0.0425	-2.8673	-2.7275
SCHOOL 10	0.8911	0.8914	0.0262	0.8480	0.9341
SCHOOL 11	-1.0609	-1.0608	0.0206	-1.0949	-1.0270
SCHOOL 12	0.9857	0.9859	0.0206	0.9516	1.0191
SCHOOL 13	2.9091	2.9085	0.0523	2.8237	2.9954

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 100000 parameter simulations, and 5000 network simulations for each parameter proposal.

Table D5—Model (6) in Table 3

	mean	median	std. dev.	5 pctile	95 pctile
A. Direct utility (u_{ij})					
CONSTANT	-5.8070	-5.8070	0.0068	-5.8185	-5.7958
MALE	0.2350	0.2348	0.0081	0.2212	0.2483
WHITE	0.3023	0.3023	0.0120	0.2826	0.3219
BLACK	1.1801	1.1801	0.0028	1.1755	1.1847
HISP	2.0295	2.0297	0.0197	1.9967	2.0616
INCOME	-1.4645	-1.4645	0.0259	-1.5072	-1.4216
SAME GENDER	0.7644	0.7644	0.0093	0.7491	0.7797
SAME GRADE	2.1800	2.1798	0.0162	2.1533	2.2067
WHITE-WHITE	0.2739	0.2739	0.0063	0.2637	0.2839
BLACK-BLACK	0.9405	0.9405	0.0098	0.9246	0.9570
HISP-HISP	-0.1394	-0.1395	0.0098	-0.1554	-0.1226
BEAUTY i	-1.9430	-1.9432	0.0258	-1.9855	-1.9001
BEAUTY j	1.2609	1.2609	0.0121	1.2412	1.2812
PERSONALITY i	-0.1631	-0.1633	0.0057	-0.1725	-0.1533
PERSONALITY j	-0.3939	-0.3939	0.0135	-0.4164	-0.3716
Income i - Income j	0.7403	0.7401	0.0134	0.7178	0.7621
Income i + Income j	0.6892	0.6894	0.0149	0.6643	0.7136
FRACTION WHITES	-1.6126	-1.6124	0.0060	-1.6228	-1.6029
FRACTION BLACKS	1.9618	1.9618	0.0063	1.9514	1.9722
FRACTION HISP	0.7731	0.7731	0.0066	0.7623	0.7839
WHITE-WHITE * FRACTION WHITE	2.3272	2.3271	0.0340	2.2719	2.3837
BLACK-BLACK * FRACTION BLACKS	0.2516	0.2515	0.0066	0.2410	0.2624
HISP-HISP * FRACTION HISP	-1.1400	-1.1399	0.0059	-1.1494	-1.1302
SCHOOL 1	-0.1335	-0.1336	0.0163	-0.1603	-0.1067
SCHOOL 2	1.4996	1.4994	0.0169	1.4722	1.5276
SCHOOL 3	1.8785	1.8785	0.0102	1.8620	1.8954
SCHOOL 4	1.3724	1.3724	0.0106	1.3553	1.3898
SCHOOL 5	1.6828	1.6827	0.0088	1.6686	1.6973
SCHOOL 6	-1.0683	-1.0679	0.0128	-1.0902	-1.0481
SCHOOL 7	-0.9817	-0.9815	0.0280	-1.0285	-0.9346
SCHOOL 8	-0.5932	-0.5929	0.0203	-0.6274	-0.5602
SCHOOL 9	0.2444	0.2442	0.0109	0.2267	0.2624
SCHOOL 10	-1.1949	-1.1948	0.0168	-1.2230	-1.1679
SCHOOL 11	-2.3824	-2.3821	0.0379	-2.4446	-2.3196
SCHOOL 12	1.2316	1.2318	0.0248	1.1911	1.2720
SCHOOL 13	-1.4722	-1.4719	0.0203	-1.5061	-1.4389
SCHOOL 14	1.8479	1.8480	0.0084	1.8339	1.8617
SCHOOL 15	0.5666	0.5663	0.0301	0.5176	0.6166
B. Mutual utility (m_{ij})					
CONSTANTm	5.3139	5.3137	0.0257	5.2721	5.3572
SAME GENDERm	1.1539	1.1536	0.0088	1.1397	1.1688
SAME GRADEm	-3.0575	-3.0575	0.0158	-3.0831	-3.0317
WHITE-WHITEm	-0.4960	-0.4959	0.0120	-0.5162	-0.4766
BLACK-BLACKm	0.7067	0.7068	0.0178	0.6771	0.7362
HISP-HISPm	-1.4639	-1.4639	0.0120	-1.4839	-1.4442
C. Indirect utility and Popularity (v_{ij})					
CONSTANTv	-0.4308	-0.4309	0.0048	-0.4386	-0.4230
SAME GENDERv	-0.3987	-0.3987	0.0045	-0.4061	-0.3914
SAME GRADEv	0.3266	0.3266	0.0072	0.3148	0.3384
WHITE-WHITEv	0.2978	0.2978	0.0042	0.2909	0.3047
BLACK-BLACKv	0.1202	0.1203	0.0088	0.1057	0.1343
HISP-HISPv	-0.2859	-0.2860	0.0059	-0.2958	-0.2759

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 20000 parameter simulations, and 10000 network simulations for each parameter proposal.

Table D6—Model (5) in Table 3

	mean	median	std. dev.	5 pctile	95 pctile
A. Direct utility (u_{ij})					
CONSTANT	-7.2182	-7.2151	0.0329	-7.2761	-7.1685
MALE	-0.2718	-0.2717	0.0301	-0.3208	-0.2232
WHITE	0.0440	0.0445	0.0455	-0.0286	0.1136
BLACK	0.7074	0.7049	0.0138	0.6880	0.7323
HISP	1.4590	1.4588	0.0213	1.4250	1.4946
INCOME	-1.8738	-1.8740	0.0279	-1.9215	-1.8279
SAME GENDER	0.3154	0.3153	0.0156	0.2898	0.3406
SAME GRADE	2.5185	2.5173	0.0297	2.4713	2.5689
WHITE-WHITE	0.9959	0.9832	0.0534	0.9271	1.0975
BLACK-BLACK	1.5347	1.5251	0.0437	1.4755	1.6159
HISP-HISP	0.7130	0.7030	0.0530	0.6427	0.8099
BEAUTY i	-1.9291	-1.9295	0.0266	-1.9732	-1.8841
BEAUTY j	2.7615	2.7616	0.0242	2.7218	2.8005
PERSONALITY i	-0.8646	-0.8571	0.0401	-0.9359	-0.8087
PERSONALITY j	-0.6361	-0.6332	0.0238	-0.6817	-0.6017
Income i - Income j	0.9938	0.9943	0.0141	0.9695	1.0169
Income i + Income j	0.8977	0.8979	0.0164	0.8704	0.9243
FRACTION WHITES	-1.5748	-1.5614	0.0661	-1.6958	-1.4910
FRACTION BLACKS	0.7645	0.7742	0.0534	0.6684	0.8375
FRACTION HISP	1.0078	1.0023	0.0319	0.9660	1.0638
WHITE-WHITE * FRACTION WHITE	4.7269	4.7281	0.0509	4.6417	4.8081
BLACK-BLACK * FRACTION BLACKS	0.1172	0.1171	0.0125	0.0974	0.1382
HISP-HISP * FRACTION HISP	-1.3872	-1.3915	0.0297	-1.4288	-1.3364
SCHOOL 1	-0.4403	-0.4408	0.0232	-0.4783	-0.4007
SCHOOL 2	2.4641	2.4648	0.0204	2.4303	2.4969
SCHOOL 3	1.3139	1.3041	0.0418	1.2578	1.3919
SCHOOL 4	2.4282	2.4233	0.0356	2.3778	2.4915
SCHOOL 5	2.8177	2.8181	0.0191	2.7867	2.8487
SCHOOL 6	-1.7375	-1.7362	0.0249	-1.7802	-1.6982
SCHOOL 7	-0.7972	-0.7982	0.0387	-0.8597	-0.7306
SCHOOL 8	-1.6076	-1.6071	0.0584	-1.7031	-1.5125
SCHOOL 9	0.4031	0.4053	0.0193	0.3675	0.4317
SCHOOL 10	-0.9558	-0.9558	0.0405	-1.0210	-0.8891
SCHOOL 11	-2.5207	-2.5209	0.0388	-2.5865	-2.4563
SCHOOL 12	0.7806	0.7750	0.0429	0.7171	0.8594
SCHOOL 13	-1.4684	-1.4657	0.0440	-1.5433	-1.3997
SCHOOL 14	0.0815	0.0818	0.0257	0.0388	0.1237
SCHOOL 15	3.6894	3.6896	0.0506	3.6055	3.7699

Estimated posterior distribution for the full structural model. The estimates are obtained with a sample of 20000 parameter simulations, and 10000 network simulations for each parameter proposal.