

# Online Appendix for Market Power and Income Taxation

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## Appendix 1

*Proof of Proposition 1.* To complete the proof of Proposition 1, begin with the government's budget constraint (2), which is assumed to be satisfied in economy  $E$  with income tax schedule  $T$ . We can, as with individuals' budget constraints, make the substitution on the left side,  $p_i = c_i/(1 - \lambda)$ , which yields

$$(A1) \quad \sum_{i=1}^n \frac{c_i}{1 - \lambda} x_i^G = \int T(y(w))f(w)dw.$$

Next, we can rearrange terms in expression (5) for the corresponding income tax schedule to isolate  $T(y)$  and then integrate both sides over  $w$ , which yields

$$(A2) \quad \int T(y(w))f(w)dw = \int \frac{\hat{T}(y(w)) - \lambda y(w)}{1 - \lambda} f(w)dw + \Pi.$$

Using equation (A2) to substitute on the right side of equation (A1) and multiplying both sides by  $1 - \lambda$  gives us

$$(A3) \quad \sum_{i=1}^n c_i x_i^G = \int (\hat{T}(y(w)) - \lambda y(w)) f(w)dw + (1 - \lambda)\Pi.$$

Using the expression for total income earned in the economy,  $Y$ , we have

$$(A4) \quad \sum_{i=1}^n c_i x_i^G = \int \hat{T}(y(w))f(w)dw - \lambda Y + (1 - \lambda)\Pi.$$

The economy-wide resource constraint in  $E$  is

$$(A5) \quad Y = \sum_{i=1}^n c_i X_i.$$

Furthermore, another manipulation of the Lerner index shows that  $c_i = ((1 - \lambda)/\lambda)\mu_i$ . Substituting this in equation (A5) allows us to state

$$(A6) \quad Y = \sum_{i=1}^n \frac{1-\lambda}{\lambda} \mu_i X_i = \frac{1-\lambda}{\lambda} \sum_{i=1}^n \pi_i = \frac{1-\lambda}{\lambda} \Pi,$$

where the latter two equalities follow from the definitions of  $\pi_i$  and  $\Pi$ , respectively. Finally, using expression (A6) to substitute for  $Y$  in expression (A4) and simplifying, we obtain expression (7) in the text of the proof for Proposition 1 in the main text. ■

## Appendix 2

*Proof of Proposition 3.* The proof will proceed in two steps. In the first, the portion  $1 - \alpha$  of the markups that constitutes true profits will be eliminated, using a variation of the proof of Proposition 1, with the resulting intermediate economy, denoted  $\check{E}$ , being equivalent to economy  $E$ . Second, the portion  $\alpha$  of the markups that constitutes real resource costs will be eliminated, with the ultimately resulting economy  $\hat{E}$  being the one referred to in the proposition.

To begin, define economy  $\check{E}$  as identical to economy  $E$  except for the price-cost margins, which are now given by  $\check{\mu}_i = \alpha \mu_i$ ; moreover, in  $\check{E}$ ,  $\check{\alpha} = 1$ , which is to say that all of the remaining margins involve the return to real investments. We can further state that  $\check{p}_i = c_i + \check{\mu}_i$  and  $\check{\lambda} = \check{\mu}_i / \check{p}_i$ , for all  $i$ . It will also sometimes be useful to make reference to an expression for  $\check{\lambda}$  in terms of  $\lambda$  (which can be derived by manipulating the definitions of these Lerner indexes):<sup>1</sup>

$$(A7) \quad \check{\lambda} = \frac{\alpha \lambda}{1 - (1 - \alpha) \lambda}.$$

Turning to the budget constraint (1), taken to hold in economy  $E$ , we can multiply both sides by  $(1 - \lambda)/(1 - \check{\lambda})$ , making use of the definitions of the Lerner indexes and expression (A7), as appropriate, to yield the following analogue to expression (4):

$$(A8) \quad \sum_{i=1}^n \check{p}_i x_i = y - (1 - \alpha) \lambda y - (1 - (1 - \alpha) \lambda) (T(y) - \theta(y) \Pi).$$

Paralleling expression (5), we can define the corresponding income tax schedule for economy  $\check{E}$  as

$$(A9) \quad \check{T}(y) \equiv (1 - \alpha) \lambda y + (1 - (1 - \alpha) \lambda) (T(y) - \theta(y) \Pi).$$

Therefore,

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<sup>1</sup> The interpretation of  $\lambda$  and  $\check{\lambda}$  as Lerner indexes when some of the former and all of the latter constitute the recovery of prior investments views the rents from prices above marginal cost as quasi-rents to that extent.

$$(A10) \quad \sum_{i=1}^n \check{p}_i x_i = y - \check{T}(y).$$

Note that, from the above definition of profits and the definition of economy  $\check{E}$ , it follows that  $\check{\Pi} = 0$  in  $\check{E}$ , so expression (A10) indicates that individuals' have the same budget sets and, as explained previously, will make the same choices and achieve the same utility.

Next, we need to show that the government's budget constraint holds. Here, we will multiply both sides of expression (2) by  $(1 - \lambda)/(1 - \check{\lambda})$  and make use of the Lerner index definitions and expression (A7) to yield

$$(A11) \quad \sum_{i=1}^n \check{p}_i x_i^G = (1 - (1 - \alpha)\lambda) \int T(y(w))f(w)dw.$$

We can use definition (A9) for  $\check{T}(y)$  to solve for  $T(y)$  and then integrate accordingly to obtain

$$(A12) \quad \sum_{i=1}^n \check{p}_i x_i^G = \int \check{T}(y(w))f(w)dw - (1 - \alpha)\lambda Y + (1 - (1 - \alpha)\lambda)\Pi.$$

Using expression (22) for  $Y$  (the resource constraint for economy  $E$  in this version of the model) and the pertinent definition of  $\Pi$ , and making appropriate substitutions using manipulations of the Lerner index definitions and expression (A7), it is possible to show that the last two terms are equal. Accordingly, we have budget balance in economy  $\check{E}$ :

$$(A13) \quad \sum_{i=1}^n \check{p}_i x_i^G = \int \check{T}(y(w))f(w)dw.$$

This completes the proof that economy  $E$  is equivalent to the otherwise identical economy  $\check{E}$ , except that  $\check{\mu}_i = \alpha\mu_i$  and  $\check{\alpha} = 1$ .

In step 2, we now show that this economy  $\check{E}$  is, in turn, equivalent to the economy  $\hat{E}$  described in the proposition. An individual's budget constraint in economy  $\check{E}$  is given above, in expression (A10). Using the fact (from the definition of the Lerner index) that  $\check{p}_i = c_i/(1 - \check{\lambda})$ , multiplying both sides by  $1 - \check{\lambda}$ , and recalling that  $y = wl$  yields:

$$(A14) \quad \sum_{i=1}^n c_i x_i = (1 - \check{\lambda})wl - (1 - \check{\lambda})\check{T}(wl).$$

Next, define  $\hat{w} \equiv (1 - \check{\lambda})w$ , so we can restate equation (A14) as

$$(A15) \quad \sum_{i=1}^n c_i x_i = \hat{w}l - (1 - \check{\lambda})\check{T}\left(\frac{\hat{w}l}{1 - \check{\lambda}}\right).$$

Now, starting with the income tax schedule  $\check{T}$  for economy  $\check{E}$ , we can define the corresponding income tax schedule  $\hat{T}$  for economy  $\hat{E}$  as

$$(A16) \quad \hat{T}(\hat{w}l) \equiv (1 - \check{\lambda})\check{T}\left(\frac{\hat{w}l}{1 - \check{\lambda}}\right).$$

Inserting definition (A16) into equation (A15) yields

$$(A17) \quad \sum_{i=1}^n c_i x_i = \hat{w}l - \hat{T}(\hat{w}l),$$

confirming that individuals' budget constraints continue to hold in economy  $\hat{E}$ . Specifically, individuals choosing any  $l$  can just afford the same consumption bundles, the  $x_i$ . A further implication, discussed in connection with Proposition 1, is that individuals will indeed make the same choices and thereby achieve the same utility.

The government's budget constraint in economy  $\check{E}$  is given by expression (A13). Here too we can use the fact that  $\check{p}_i = c_i/(1 - \check{\lambda})$ , multiply both sides by  $1 - \check{\lambda}$ , and recall that  $y = wl$  to yield

$$(A18) \quad \sum_{i=1}^n c_i x_i^G = (1 - \check{\lambda}) \int \check{T}(wl(w))f(w)dw.$$

Now, define  $\hat{f}(\hat{w}) \equiv f(\hat{w}/(1 - \check{\lambda}))$ . That is, we take a grossed-up magnitude for the original ability distribution in order to determine the density for a particular ability level in the new distribution. Running in the opposite direction may be more intuitive: for any ability level in the original distribution for equivalent economies  $E$  and  $\check{E}$ , we consider a scaled down ability level (wage) in the distribution for economy  $\hat{E}$  (recalling that  $\hat{w} = (1 - \check{\lambda})w$ ), reflecting that a fraction of everything that labor produces is paying for the investment costs associated with the markups in  $\check{E}$  (or  $\alpha$  of the markup in  $E$ ) and thus is not available to pay the costs  $c_i$  associated with the  $x_i$ .

We can use this definition of  $\hat{f}(\hat{w})$ , the definition of  $\hat{w}$ , and expression (A16) to restate the integrand on the right side of equation (A18):

$$(A19) \quad \check{T}(wl(w))f(w) = \check{T}\left(\frac{\hat{w}l(\hat{w})}{1 - \check{\lambda}}\right)f\left(\frac{\hat{w}}{1 - \check{\lambda}}\right) = \frac{\hat{T}(\hat{w}l(\hat{w}))}{1 - \check{\lambda}}\hat{f}(\hat{w}),$$

where the first equality makes use of the fact that  $l(w) = l(\hat{w})$ , as discussed after expression (A17). Substituting into equation (A18), and returning to the definition of labor income  $y$ , gives us

$$(A20) \quad \sum_{i=1}^n c_i x_i^G = \int \hat{T}(y(\hat{w}))\hat{f}(\hat{w})d\hat{w}.$$

Therefore, the government's budget constraint holds in economy  $\hat{E}$ , which completes the proof of equivalence.

Finally, it is useful to restate the definition  $\hat{f}(\hat{w}) \equiv f(\hat{w}/(1 - \check{\lambda}))$  in the notation of the original economy  $E$ . Substituting from expression (A7) for  $\check{\lambda}$  yields

$$(A21) \quad \hat{f}(\hat{w}) = f\left(\frac{1 - (1 - \alpha)\lambda}{1 - \lambda}\hat{w}\right),$$

which is the same as expression (23) in the proposition's claim. ■

### Appendix 3

*Proof of Proposition 4.* The steps of the proof and pertinent equations are the same as in the proof of Proposition 2 until we reach expression (14) for  $d\Pi/d\gamma$ , reflecting that in this section's model profits are now given by expression (24), taking into account as well that, for this parameterized reform, the  $j^{\text{th}}$  element of that summation is now  $(1 - \rho(\gamma)\alpha_j)\gamma\mu_j X_j$ . The resulting analogue to expression (14), evaluated at  $\gamma = 1$ , is

$$(A22) \quad \frac{d\Pi}{d\gamma} = (1 - \alpha_j)\mu_j \frac{dX_j}{d\gamma} + \sum_{i \neq j} (1 - \alpha_i)\mu_i \frac{dX_i}{d\gamma} + \left(1 - \alpha_j - \frac{d\rho(\gamma)}{d\gamma}\alpha_j\right)\mu_j X_j.$$

Substituting this derivative into expression (13) for the effect of the reform on the government's budget surplus and cancelling terms yields

$$(A23) \quad \frac{d\sigma(\gamma)}{d\gamma} = (1 - \alpha_j)\mu_j \frac{dX_j}{d\gamma} + \sum_{i \neq j} (1 - \alpha_i)\mu_i \frac{dX_i}{d\gamma} - \alpha_j \left(1 + \frac{d\rho(\gamma)}{d\gamma}\right)\mu_j X_j.$$

Using the fact that  $\mu_i = \lambda_i p_i$ , we have

$$(A24) \quad \frac{d\sigma(\gamma)}{d\gamma} = (1 - \alpha_j)\lambda_j p_j \frac{dX_j}{d\gamma} + \sum_{i \neq j} (1 - \alpha_i)\lambda_i p_i \frac{dX_i}{d\gamma} - \alpha_j \left(1 + \frac{d\rho(\gamma)}{d\gamma}\right)\lambda_j p_j X_j.$$

We can now restate expression (A24) as indicating the presence of a government budget surplus if and only if

$$(A25) \quad p_j \frac{dX_j}{d\gamma} (\lambda_j^\alpha - \bar{\lambda}_j^\alpha) > \alpha_j \left(1 + \frac{d\rho(\gamma)}{d\gamma}\right)\lambda_j p_j X_j,$$

which is the same as expression (26) in the proposition. As with Proposition 2, if this inequality holds, it is possible to further adjust the income tax schedule to rebate the budget surplus so as to generate a strict Pareto improvement. ■

## Appendix 4

*Proof of claim that a proportional reduction in all markups (when those markups are not proportional), combined with an offsetting adjustment to the income tax schedule, is Pareto improving.* The proof tracks closely much of the proof of Proposition 2, which characterizes when reducing a *single* markup is Pareto improving. Once again, we will consider a reform parameterized by  $\gamma$ : in economy  $E$ , set  $\gamma = 1$  and this time restate the price-cost margins on *all* of the goods  $i$  as  $\gamma\mu_i$ , so we now have  $p_i = c_i + \gamma\mu_i$ , for all  $i$ . The analogue to expression (9) for the adjustment to the income tax schedule is

$$(A26) \quad \frac{\partial T(y, \gamma)}{\partial \gamma} = - \sum_{i=1}^n \mu_i x_i(y) + \theta(y) \frac{d\Pi}{d\gamma}.$$

The subsequent demonstration in the proof of Proposition 2 that labor effort and overall utility will be unchanged is the same and hence is omitted here.

Proceeding to consider the impact of increasing  $\gamma$  on the government's budget, the expression for the surplus (or deficit) is the same as in expression (11), reproduced here:

$$(A27) \quad \sigma(\gamma) = \int T(y(w), \gamma) f(w) dw - \sum_{i=1}^n p_i(\gamma) x_i^G.$$

Making use of expression (A26) for  $\partial T(y, \gamma)/\partial \gamma$ , the effect of the reform on the budget surplus (which equals 0 when  $\gamma = 1$ ) is given by this analogue to expression (12):

$$(A28) \quad \frac{d\sigma(\gamma)}{d\gamma} = - \sum_{i=1}^n \left( \int \mu_i x_i(y(w)) f(w) dw \right) + \frac{d\Pi}{d\gamma} - \sum_{i=1}^n \mu_i x_i^G.$$

(As before, these derivatives and others are evaluated at  $\gamma = 1$ , with explicit notation to this effect omitted.) Combining the first and third terms on the right side and recalling the definition of  $X_i$ , we have

$$(A29) \quad \frac{d\sigma(\gamma)}{d\gamma} = \frac{d\Pi}{d\gamma} - \sum_{i=1}^n \mu_i X_i.$$

Differentiating the expression for total profits,  $\Pi = \sum_{i=1}^n \gamma \mu_i X_i$ , in this parameterized experiment, we have

$$(A30) \quad \frac{d\Pi}{d\gamma} = \sum_{i=1}^n \mu_i \left( \frac{dX_i}{d\gamma} + X_i \right).$$

Combining equations (A29) and (A30) yields:

$$(A31) \quad \frac{d\sigma(\gamma)}{d\gamma} = \sum_{i=1}^n \mu_i \frac{dX_i}{d\gamma},$$

which, note, is identical to expression (15) in the proof of Proposition 2, but it now carries a different interpretation. Specifically, because the parameterized experiment raises all the price-cost margins and not just one, the right side of expression (A31) represents the product of the rate of price increase on each good and the rate of change in aggregate consumption of that good. From the (Hicksian) compensated law of demand, the value of the right side is negative.<sup>2</sup> Intuitively, it measures the total increase in resource use on account of the now-more-distorted prices due to the nonproportional markups being higher. (If the markups were proportional, the price ratios would not change, so the value of the right side would be zero rather than negative.) Hence, a marginal proportional increase in all markups produces a budget deficit. Likewise, a proportional reduction generates a surplus that can be rebated so as to raise everyone's utility. ■

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<sup>2</sup> The compensated law of demand arises from applying the axiom of revealed preference twice, requiring that the pre-reform bundle not be affordable at post-reform prices and that the post-reform bundle not be affordable at pre-reform prices (noting that, because this is a compensated exercise, utility is the same before and after).